McGill University

Numerical Analysis

MATH 578

Homework 4 and 5

Name: Doug Shi-Dong

Student ID: 260466662

November 17, 2015

Question 10

The trajectories are plotted in Figures 1, 3. Notice that the Forward-Euler (FE) is not stable in both cases. The amplitude of the trajectories increase over time for FE. On the other hand, Backward-Euler (BE) seems to have a damping mechanism, decreasing the amplitude of the trajectories over time. Crank-Nicolson is neither unstable nor dissipative. A comparison of the amplitude is seen in Figure 2 and 4.

Solving the second ODE with FE and BE exhibits an interesting behavior. The FE method has a greater frequency than the exact solution, whereas the BE method has a smaller frequency. Therefore, both methods cause time dilation of the solution. Crank-Nicolson is once again the same as the MATLAB computed solution.

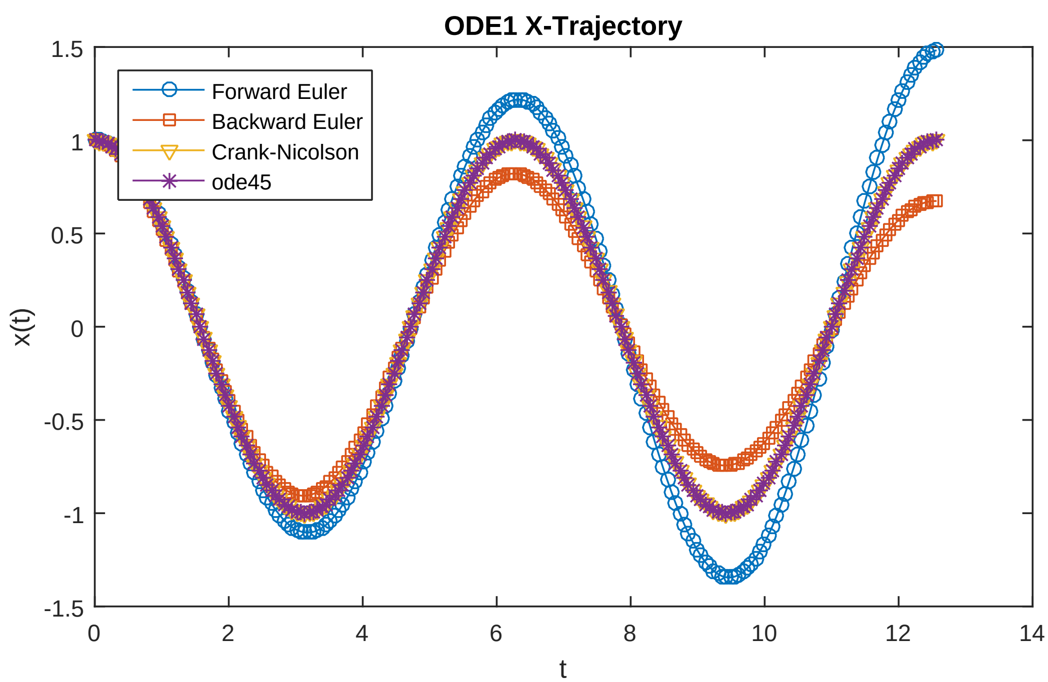

Figure 1: X Trajectory of ODE 1

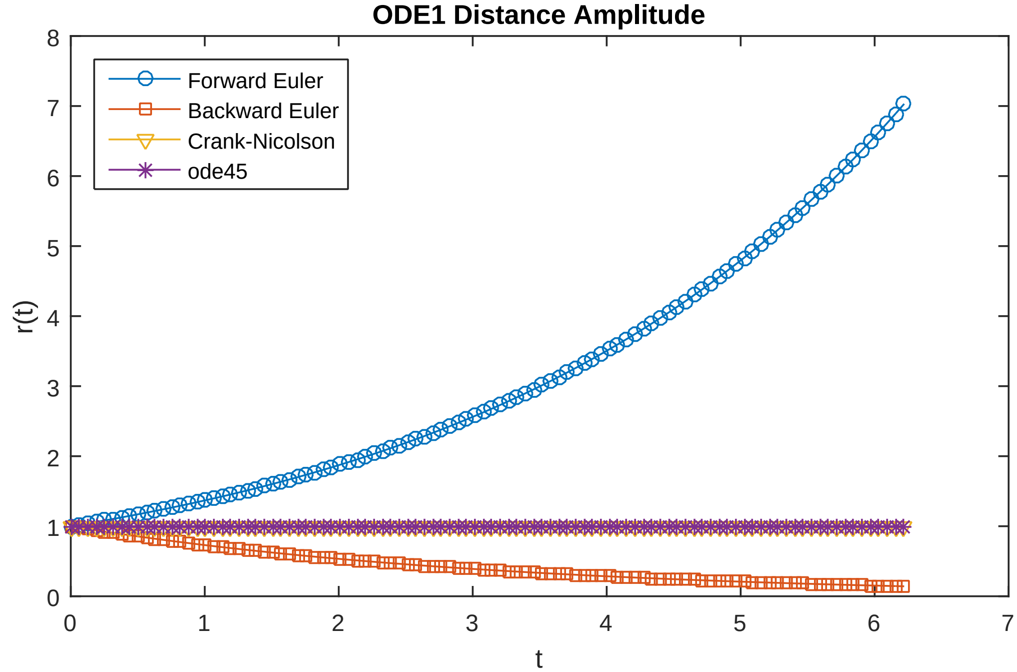

Figure 2: Trajectory Amplitude of ODE 1

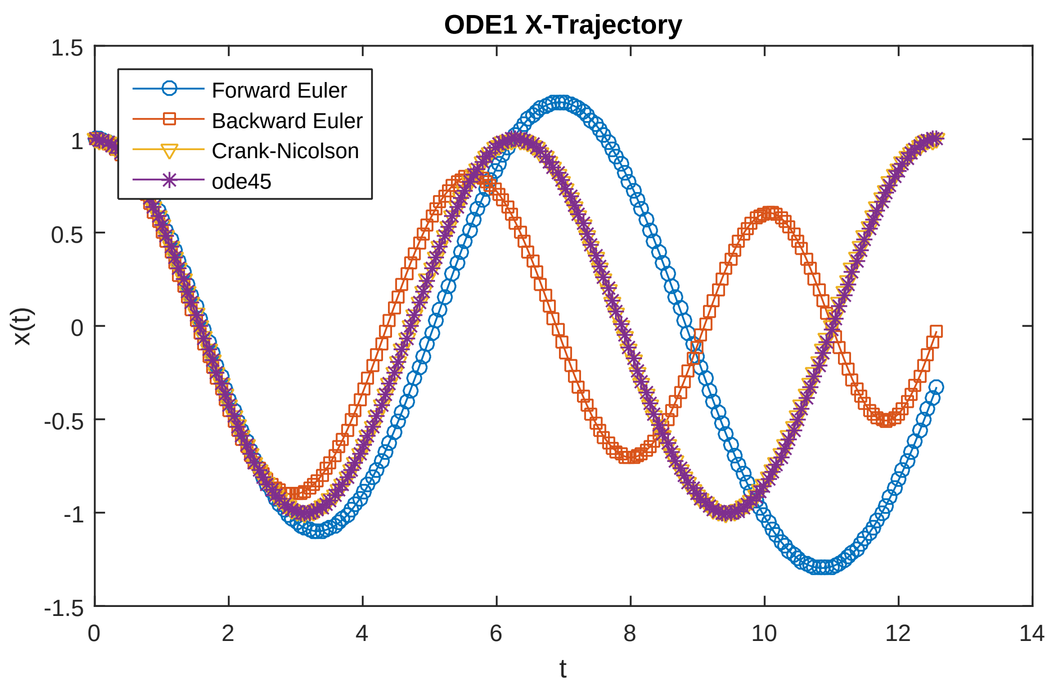

Figure 3: X Trajectory of ODE 2

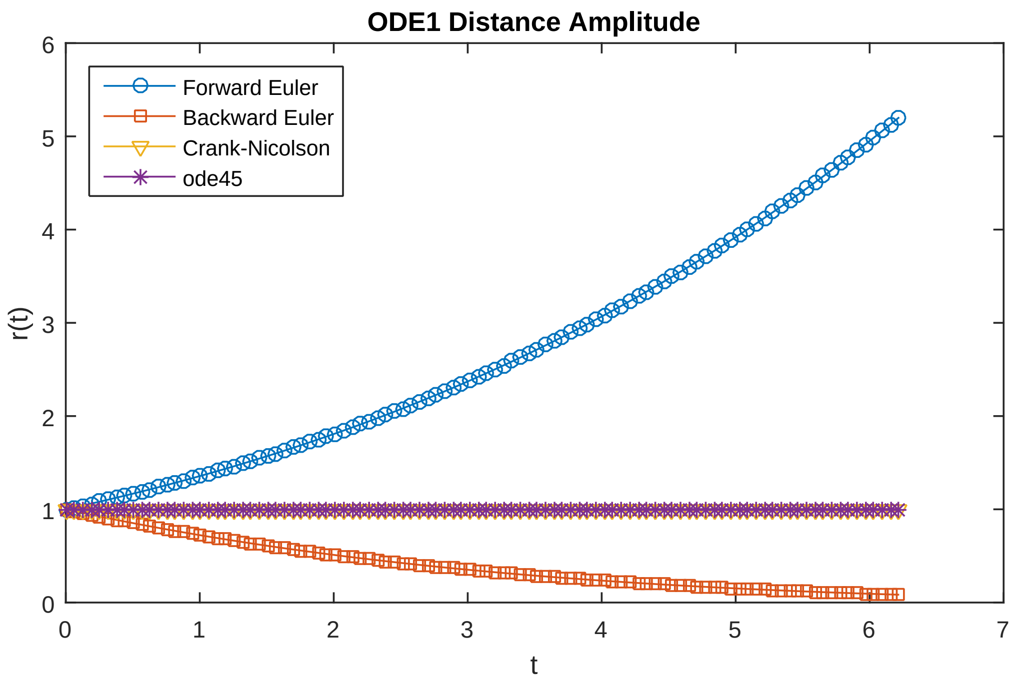

Figure 4: Trajectory Amplitude of ODE 2

Question 11

The polynomial is evaluated using CVX for $p = [1, 1.5, 2, 4, 8, inf]$. The maximum error between the exact polynomial and the evaluated polynomial is then recorded and plotted in Figure 5. As expected, using the infinity-norm reduces the maximum error.

Question 12

The results from CVX confirm the ones found by hand. CVX found $x = 1.6236e-12$ and $x = 1.3659e-09$ for the infinity-norm and two-norm respectively. The plot output is seen in Figure 6.

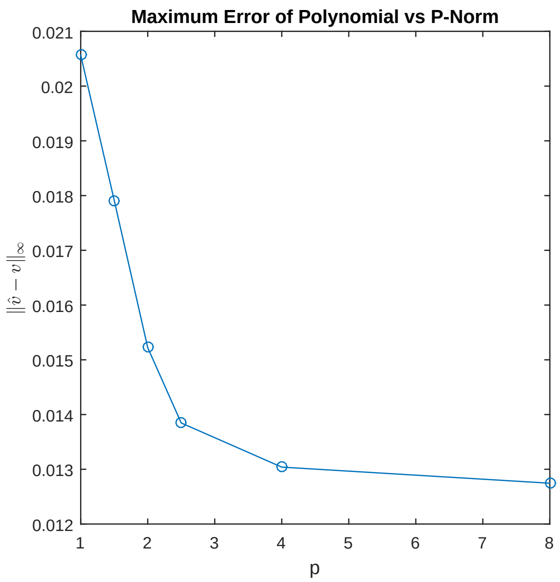

Figure 5: Maximum Error vs P-Norm

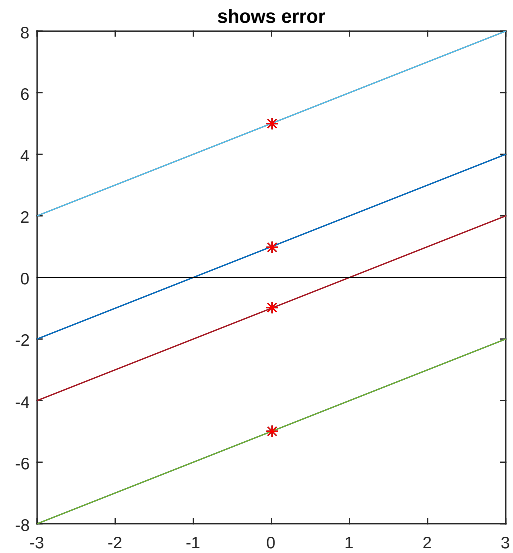

Figure 6: CVX Line Optimization

Question 13

The obvious way to formulate the problem is shown by Equation 1.

$$\underset{x}{\text{minimize}} \quad \left\| \begin{pmatrix} \|x - x_1\|_2 \\ \|x - x_2\|_2 \\ \dots \\ \|x - x_N\|_2 \end{pmatrix} \right\|_p \tag{1}$$

However, CVX requires the problem to be formulated in terms of inequalities in order to perform linear programming optimization. Equation 1 can be transformed into a linear system of inequalities as shown in Equation 2 where the RHS y are added to the design variables.

A few examples have been plotted in Figure 7.

$$\begin{aligned} \underset{x,y}{\text{minimize}} \quad & \|y\|_p \\ \text{subject to} \quad & \begin{pmatrix} \|x - x_1\|_2 \leq y_1 \\ \|x - x_2\|_2 \leq y_2 \\ \dots \\ \|x - x_N\|_2 \leq y_N \end{pmatrix} \end{aligned} \tag{2}$$

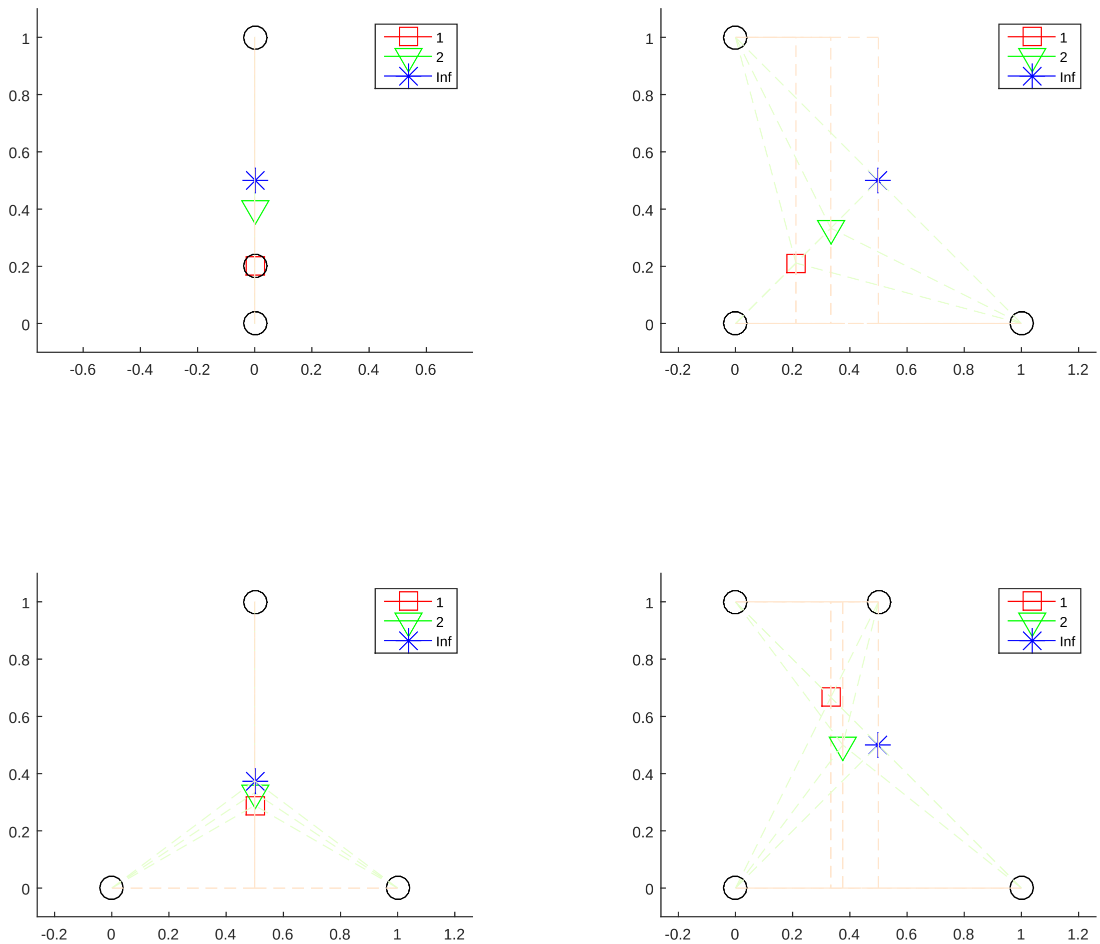

Figure 7: Facility Location Results from CVX

Question 14

Given two sets of point $U = \{u_1, \cdots, u_m\}$ and $V = \{v_1, \cdots, v_n\}$, we want a linear function $p(x) = c^T x - b$ which discriminates data. Once again, the problem is easily formulated into Equation 3.

$$\begin{aligned} c^T u_i - b > 0 \quad i = 1, m \\ c^T v_i - b < 0 \quad i = 1, n \end{aligned} \tag{3}$$

Since Equation 3 is a set of strict equalities and we require a set of inequalities, it is reformulated into Equation 4. The new formulation maximizes the gap between the linear function and the set of points.

$$\begin{aligned} \underset{c,b,\delta}{\text{maximize}} \quad & \delta \\ \text{subject to} \quad & c^T u_i - b \geq \delta \qquad i = 1, m \\ & c^T v_i - b \leq -\delta \qquad i = 1, n \end{aligned} \tag{4}$$

The resulting line is plotted in Figure 8 with the following parameters:

$$c^T = [148.3533, -157.4198] \quad b = -22.2961 \quad \delta = 1$$

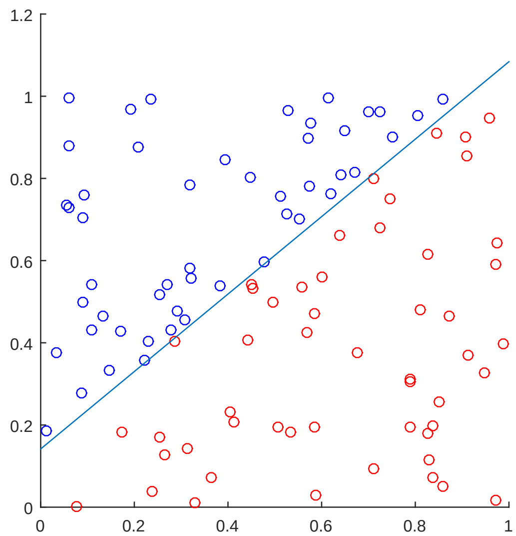

Figure 8: CVX Linear Discrimination

Codes

All codes are available on my GitHub:

`https://github.com/dougshidong/math578/tree/master/a4a5`