MEASURING THE EFFECT OF INVENTORY CENTRALIZATION/ DECENTRALIZATION ON AGGREGATE SAFETY STOCK: THE "SQUARE ROOT LAW" REVISITED

by

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HEADNOTE

The "Square Root Law" has been used to approximate the changes in aggregate safety stock resulting from changes in the number of stocking locations used in the distribution of a product. The "Square Root Law" is a special case of a more comprehensive model which is developed in this article. The relationship between aggregate safety stock and the number of stocking locations used in the distribution of a product is a function of the relative sizes of the standard deviations of demand (Magnitude) and the correlation coefficient of sales between stocking locations. Two rules are derived to be easily applied by managers when deciding whether to add or delete stocking locations to the distribution of a product.

INTRODUCTION

The Square Root Law (SRL)[1] approximates the impact on a product's aggregate safety stock caused by a change in the number of inventory stocking locations used in the distribution for that product. The SRL is based on a number of restrictive and unrealistic assumptions, i.e., the demands at each stocking location are uncorrelated and the variability of demand is the same at all locations. This article develops a general model which eliminates the need for these assumptions. The SRL, then, becomes a special case of the general model.

In general, the greater the number of stocking locations, the greater the safety stock required to maintain a given customer service level, as measured by inventory availability. Conversely, aggregate safety stock is reduced whenever inventories are centralized into fewer stocking locations. The specific questions addressed in this article are: By how much does inventory increase or decrease as stocking locations are added or deleted, and what variables determine this quantity? The variables required to determine this amount are the same as those assumed away in the SRL. For ease of explication, the case for centralization rather than decentralization of stocking locations is discussed.

The percent reduction in aggregate safety stock made possible by centralization of inventories is defined here as the Portfolio Effect (PE). It is a function of two variables: sales correlation (Pearson product-moment correlation coefficient) of past sales between two stocking locations, as suggested by Eppen,[2] and Magnitude (M), which is the quotient of the standard deviation of sales of two stocking locations. The underlying principle explaining the Portfolio Effect is based on risk-pooling which has been applied in the commonality literature.[3,4,5] The issue of commonality deals with the effect of the degree of component part standardization on aggregate inventory. Unlike the commonality literature, however, the problem considered here concerns the centralization of inventory. Eppen[6] showed that the relationship between inventory centralization and aggregate safety stock is a function of the correlation of demands between stocking locations. This article adds the effect of Magnitude to this functional relationship and broadens the range of analysis by including the case of negative sales correlations. This model provides a more precise estimation of aggregate safety stock resulting from a change in the number of stocking locations than was previously possible by using either the SRL or Eppen's formulation.

The model is based on a few reasonable assumptions: (1) Inventory transfers between stocking locations at the same level are not common practice. (2) Inventory centralization is not affected by inbound supply uncertainty; i.e., the standard deviation of lead time is assumed to be zero. (3) Customer service level, as measured by inventory availability, is constant regardless of the number of stocking locations. (4) Demand at each stocking location is normally distributed.

This article develops two basic decision rules which can be easily applied by managers when making decisions regarding centralization versus decentralization of inventories. First, the potential of aggregate safety stock reduction resulting from inventory centralization is greater when sales correlation between stocking locations is small to negative and Magnitude (M) is small. Second, the incremental safety stock required to decentralize inventories is smaller as sales correlation approaches one.

In polar extremes, inventory centralization may eliminate safety stock altogether whenever Magnitude (M) is equal to one and the sales correlation is perfectly negative. Inversely, no additional safety stock is required to add a stocking location if the sales correlation between two stocking locations is perfectly positive.

The Portfolio Effect is defined in the first section. The second section explores the impact of the sales correlation and Magnitude on the Portfolio Effect. The third section demonstrates the impact of sales correlation and magnitude on aggregate safety stock. The fourth section generalizes the case of multiple locations. In the fifth section, the special case of the Square Root Law is delimited. An illustrative example of the Portfolio Effect is presented in the final section.

THE PORTFOLIO EFFECT

The portfolio effect (PE) measures the percent reduction in aggregate safety stock made possible by the consolidation of inventories from multiple locations into one location. It is defined in Equation (1).

$$PE = 1 - \frac{SS_a}{\sum_{i=1}^{n} SS_i}, \text{ for } 0 \leq PE \leq 1 \qquad (1)$$

where:

SS_a = aggregate safety stock for a given product if inventory is consolidated.

SS_i = safety stock for a given product at location i.

The bounds of the PE are 0 and 1. The closer the PE is to the upper bound, the greater the percent reduction in aggregate safety stock if inventories are consolidated. When PE = 0, there is no reduction in aggregate safety stock as a result of consolidation. A portfolio effect of 0.42, for instance, means that aggregate safety stock is reduced by 42 percent if inventories are consolidated into one location.

With two locations:

$$PE = 1 - \frac{SS_a}{SS_1 + SS_2}, \text{ for } 0 \leq PE \leq 1 \tag{2}$$

where:

SS_1 = safety stock at location 1.

SS_2 = safety stock at location 2.

The centralized aggregate safety stock in units is provided by manipulating Equation (2):

$$SS_a = (SS_1 + SS_2)\ (1 - PE)$$

MAGNITUDE, SALES CORRELATION AND THE PORTFOLIO EFFECT

It is necessary to review the definition of safety stock in order to express the PE in terms of Magnitude and sales correlation.

Safety stock is defined as:[7]

$$SS_i = k\ \sigma_i \tag{3}$$

where:

k = a managerially determined factor that indicates the number of standard deviations of demand to be kept as safety stock.

σ_i = standard deviation of demand at location i.

If k is equal in all stocking locations (Assumption (3)):

$$PE = 1 - \frac{\sigma_a}{\sigma_1 + \sigma_2} \tag{4}$$

where:

σ_a = standard deviation of aggregate demand if inventories are consolidated.

σ_1 = standard deviation of demand at location 1.

σ_2 = standard deviation of demand at location 2.

The equation needed to compute σ_a for n locations is well known.[8]

$$\sigma_a = \sqrt{\sum_{i=1}^{n} \sigma_i^2 + 2 \sum_{i<j} \sum \sigma_i \sigma_j \rho_{ij}} \tag{5}$$

where:

ρ_{ij} = correlation coefficient of sales between two locations.

For two locations, then:

$$\sigma_a = \sqrt{\sigma_1^2 + \sigma_2^2 + 2 \sigma_1 \sigma_2 \rho_{12}} \tag{6}$$

Substituting equation (6) for σ_a in Equation (4).

$$PE = 1 - \frac{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho_{12}}}{\sigma_1 + \sigma_2} \tag{7}$$

Magnitude is defined as:

$$M = \frac{\sigma_1}{\sigma_2}, \text{ for } \sigma_1 \geq \sigma_2 \text{ and } \sigma_2 \neq 0 \tag{8}$$

Substituting equation (8) for σ_1 in Equation 7.

$$PE = 1 - \frac{\sqrt{M^2\sigma_2^2 + \sigma_2^2 + 2M\sigma_2\sigma_2\rho_{12}}}{M\sigma_2 + \sigma_2}$$

Thus,

$$PE = 1 - \frac{\sqrt{M^2 + 1 + 2M\rho_{12}}}{M + 1} \tag{9}$$

By examining equation (4), one might suspect that the absolute value of the standard deviation of demand affects the PE. Equation (9), however, shows that it is the <u>relative</u> values of the standard deviations between locations which is important. Table 1 demonstrates this relationship for different sales correlations and Magnitudes of 1,2,4, and 8.

TABLE 1

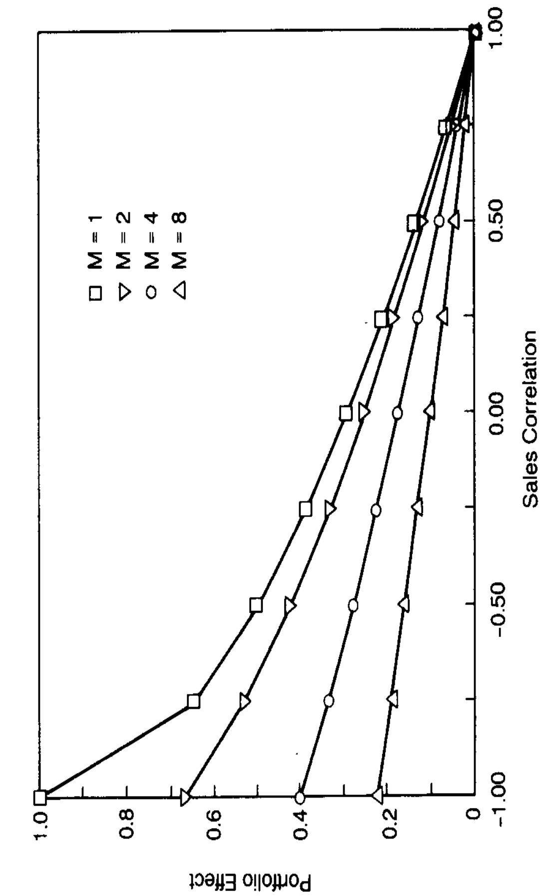
IMPACT OF SALES CORRELATION AND MAGNITUDE ON THE PORTFOLIO EFFECT

SALES CORR.	PORTFOLIO EFFECT M=1	M=2	M=4	M=8
-1.00	1.00	0.67	0.40	0.22
-.75	0.65	0.53	0.34	0.19
-.50	0.50	0.42	0.28	0.16
-.25	0.39	0.33	0.23	0.13
.00	0.29	0.25	0.18	0.10
.25	0.21	0.18	0.13	0.08
.50	0.13	0.12	0.08	0.05
.75	0.06	0.06	0.04	0.03
1.00	0.00	0.00	0.00	0.00

Figure 1 illustrates the relationship graphically. More specifically, it presents the percent reduction in aggregate safety stock made possible by consolidating inventories from two stocking locations given different sales correlations and Magnitudes. Note that Eppen[9] considered the correlation of sales between stocking locations, but not the Magnitude. The higher the correlation of sales between stocking locations, the smaller the PE--the smaller the percent reduction of aggregate safety stock when inventories are consolidated. Magnitude moderates the effect of sales correlation on the PE. Specifically, the impact of Magnitude on PE becomes less significant when sales correlations are positive and approaching one. However, as sales correlations go negative and approach minus one, the effect of Magnitude on PE becomes quite large. In fact, when sales correlation is -1, and Magnitude 1, PE is also 1 and aggregate safety stock is eliminated altogether.

The rationale for this elimination of safety stocks is provided by compensating sales between two stocking locations. When sales from one stocking location go up, sales for the other stocking location go down by the same amount. Aggregate sales are hence constant, the aggregate standard deviation of demand is zero, and aggregate safety stock is also zero.

FIGURE 1
IMPACT OF SALES CORRELATION AND MAGNITUDE
ON THE PORTFOLIO EFFECT



MULTIPLE LOCATIONS

The basic relationships derived in the previous section can be generalized for multiple locations. Equation (5) is used to compute the aggregate safety stock if inventories in all locations are centralized in one location.

$$\sigma_a = \sqrt{\sum_{i=1}^{n} \sigma_i^2 + 2 \sum_{i<j} \sum \sigma_i \sigma_j \rho_{ij}} \tag{5}$$

Assume three locations: 1, 2 and 3. The aggregate stock resulting from the centralization of these inventories is:

$$\sigma_a = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2\rho_{12} + 2\sigma_1\sigma_3\rho_{13} + 2\sigma_2\sigma_3\rho_{23}}$$

And the portfolio effect is:

$$PE = 1 - \frac{\sigma_a}{\sigma_1 + \sigma_2 + \sigma_3}$$

While the basic relationships developed for two locations are valid for multiple locations, the number of combination possibilities to consolidate inventories given multiple locations expands exponentially. Although there are only two possible combinations given two locations (centralize or leave decentralized); with three locations the number of combinations increases to five (centralize all locations, leave all three decentralized, or centralize any two of three locations). With four locations the number increases to twelve, and with five to twenty-seven!

To delimit the problem arising from the number of centralization possibilities existent with multiple locations, a PE matrix for four locations is indicated and illustrated in a later section. Similar in form to a common correlation matrix, the PE matrix contains the Portfolio Effects for every possible pair of locations. Inventory decentralization opportunities exist in locations with a low PE, while centralization opportunities exist in locations with a high PE.

THE "SQUARE ROOT LAW": A SPECIAL CASE OF THE PORTFOLIO EFFECT

The effect of sales correlation and Magnitude on the Portfolio Effect has been defined. This section provides a proof that the Square Root Law (SRL) is a restrictive special case of the Portfolio Effect. The SRL is an approximation of how much safety stock varies when inventory is centralized or decentralized.[10] Specifically, the SRL is defined as:

$$\sigma_a = \sigma_i \sqrt{n}$$

To facilitate this proof, however, note that the Portfolio Effect is the percent reduction in safety stock when inventories are centralized; whereas, the SRL defines the same reduction in units. To begin, Equation (5) provides the aggregate safety stock when the inventories for n locations are centralized.

$$\sigma_a = \sqrt{\sum_{i=1}^{n} \sigma_i^2 + 2 \sum_{i<j} \sum \sigma_i \sigma_j \rho_{ij}} \tag{5}$$

Maister, in defining the SRL made two restricting assumptions. First, "demands at each location are uncorrelated,"[11] i.e., $\rho_{ij} = 0$. Therefore,

$$\sigma_a = \sqrt{\sum_{i=1}^{n} \sigma_i^2}$$

Second, "the variability of demand is the same at all locations."[12] Therefore,

$$\sigma_a = \sqrt{n\,\sigma_i^2}$$

Clearly,

$\sigma_a = \sigma_i \sqrt{n}$; the same as Maister's SRL formulation.

The SRL, being an approximation and a special case of the Portfolio Effect, may lead to significant errors when estimating the aggregate safety stock. Table 1 shows the savings which result from centralizing inventories from two locations, given different Magnitudes and sales correlations. With

Maister's assumption that the variability of demands are equal at all locations, Magnitude will always be 1. In Table 1, with Magnitude = 1, and sales correlation = 0, the savings in aggregate safety stock is 29 percent. The range of savings in Table 1, however, is from 0 to 100 percent!

AN ILLUSTRATIVE EXAMPLE

Data from four stores of a department store chain were used to illustrate the Portfolio Effect. Unit sales per store were obtained for a typical product--men's white Jockey underwear, size 36. The data represent sales from 1985 to 1987. An analysis of 1987 data only produced similar results. The four stores are within overnight driving distance from a central distribution center. Tables 2 to 4 present sales correlations, Magnitudes, and the PEs for these four stores, respectively.

TABLE 2

SALES CORRELATIONS

	STORE			
	1	2	3	4
1	----			
2	-.1250	----		
3	-.3117	.4451	----	
4	-.1024	.7587	.5626	----

The sales correlation between stores 1 and 2, for instance, is computed as the Pearson product-moment correlation coefficient of past sales for the two stores.

The Magnitude between stores 1 and 2, for instance, is obtained by the ratio of the standard deviation of demand for each store, with the largest standard deviation as the numerator.

Finally, the PE is obtained by applying data from Tables 2 and 3 to Equation 9. For instance, the PE for stores 1 and 2 is:

TABLE 3

MAGNITUDES

	STORE 1	2	3	4
1	----			
2	2.32	----		
3	1.27	1.84	----	
4	1.34	1.74	1.06	----

TABLE 4

PE MATRIX

	STORE 1	2	3	4
1	----			
2	.274	----		
3	.406	.136	----	
4	.321	.058	.116	----

$$PE = 1 - \frac{\sqrt{M^2 + 1 + 2M\rho_{12}}}{M + 1} \tag{9}$$

with $\rho_{12} = -.125$ (from Table 2) and $M_{12} = 2.32$ (from Table 3). Thus,

$$PE = 1 - \frac{\sqrt{(2.32)^2 + 1 + 2(2.32)(-.125)}}{2.32 + 1} = .274$$

Table 4 demonstrates that centralizing inventories from stores 1 and 3, for instance, will reduce aggregate safety stock by 40.6%. On the other hand, aggregate safety stock reduction from centralization of inventories from stores 2 and 4 is only 5.8%. There are, thus, opportunities to centralize "backroom" inventories at stores with a higher portfolio effect. Decentralization of inventories between stores with a low portfolio effect is achieved without much additional safety stock.

SUMMARY

The Portfolio Effect is the percent reduction in aggregate safety stock made possible by centralizing inventories. It is independent of specific values of the standard deviation of demand in locations being considered for centralization or decentralization of inventories. It is, instead, a function of the sales correlation and the Magnitude.

The Square Root Law approximates the effect of inventory centralization or decentralization on aggregate safety stock. It has been demonstrated that the Square Root Law is a special case of the Portfolio Effect when Magnitude is equal to 1 and sales correlation is equal to 0.

The Portfolio Effect is maximized whenever the sales correlation between two inventory locations is highly negative and the Magnitude small. The opposite is also true. The PE is minimized whenever sales correlation between two stocking locations is highly positive, regardless of the Magnitude. This makes it possible for a firm to improve customer service by adding stocking locations without adding a significant amount of aggregate safety stock. The precise relationship is provided in Equation (9).

The PE matrix contains the Portfolio Effects for every possible pair of locations. It is useful to identify safety stock reduction opportunities in an environment of multiple locations.

ENDNOTES

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6. Same Reference as Endnote 2.

7. Starr, M.K. and D.W. Miller, Inventory Control: Theory and Practice, Prentice-Hall Inc., Englewood Cliffs, N.J., 1962, pp. 112-131.

8. Mood, A.M., F.A. Graybill and D.C. Boes, Introduction to the Theory of Statistics, McGraw-Hill Book Co., New York, 1974, pp. 178-179.

9. Same Reference as Endnote 2.

10. Maister, same Reference as Endnote 1.

11. Maister, same Reference as Endnote 1, p. 129.

12. Maister, same Reference as Endnote 1, p. 130.

ACKNOWLEDGEMENTS

Note: This research was partially supported by a grant from the University of Miami (Florida) School of Business.

The authors gratefully acknowledge David J. Closs of Michigan State University, John T. Mentzer (the Systems Section Editor), and an anonymous reviewer for helpful comments on drafts of this article. We also thank Howard Kreitzman, Vice President and General Merchandise Manager of Burdines Department Stores, for help in securing data used in the illustrative example.

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