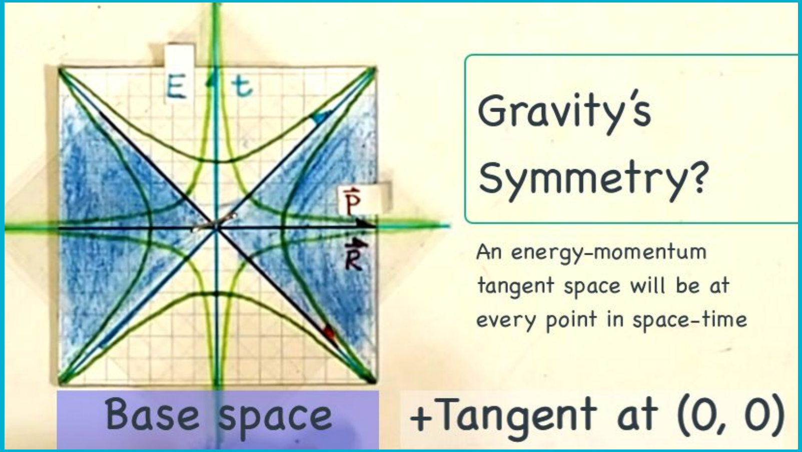
Special Relativity in Space-Time and Gravity in its Tangent Space Energy-Momentum



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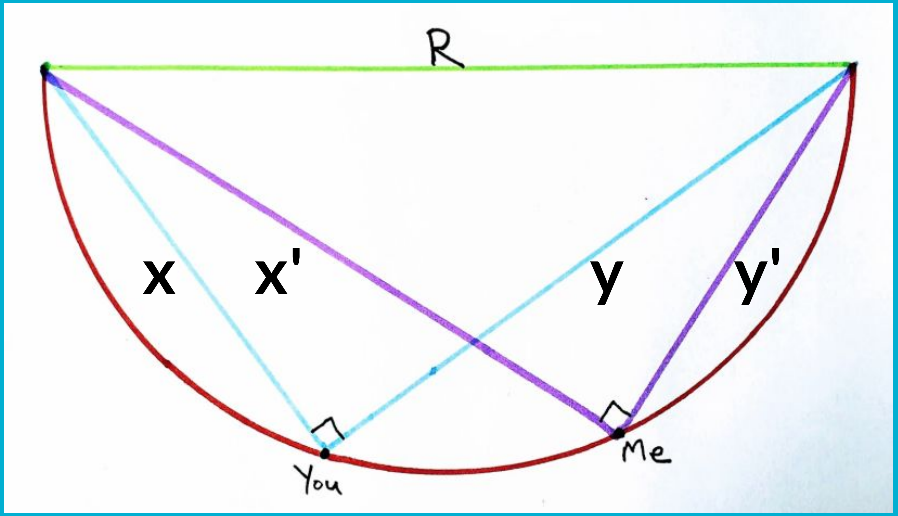
A novel proposal for gravity must explicitly show its symmetry, that it effects light, and passes tests.

The 4 fundamental forces: EM, the weak force, the strong force and gravity have
4 gauge symmetries: U(1), SU(2), SU(3), and Diff(M)

Gravity alters the path of light which is mildly remarkable given it has no rest mass. Requiring an effect on light eliminates a large block of alternative gravity proposals.

There are weak field and strong field experimental tests that can only be passed if the form of the proposal looks like a metric theory.

The invariant of ruler relativity (rotations, no boosts) can be understood by everyone

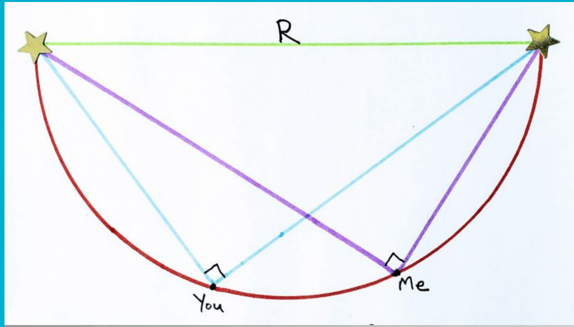


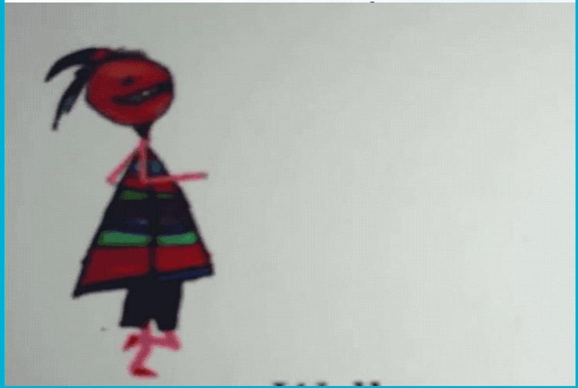
$$x \neq x'$$

$$y \neq y'$$

$$x^2 + y^2 = x'^2 + y'^2 = R^2$$

The invariant of special relativity is ruler relativity plus a teeny tiny fix need for a moving observer.





Moving Girl

$$R \neq R'$$
$$t \neq t'$$
$$t^2 - R^2 = t'^2 - R'^2 = \tau^2$$

The phase space of space-time can be transformed into an energy-momentum tangent space.

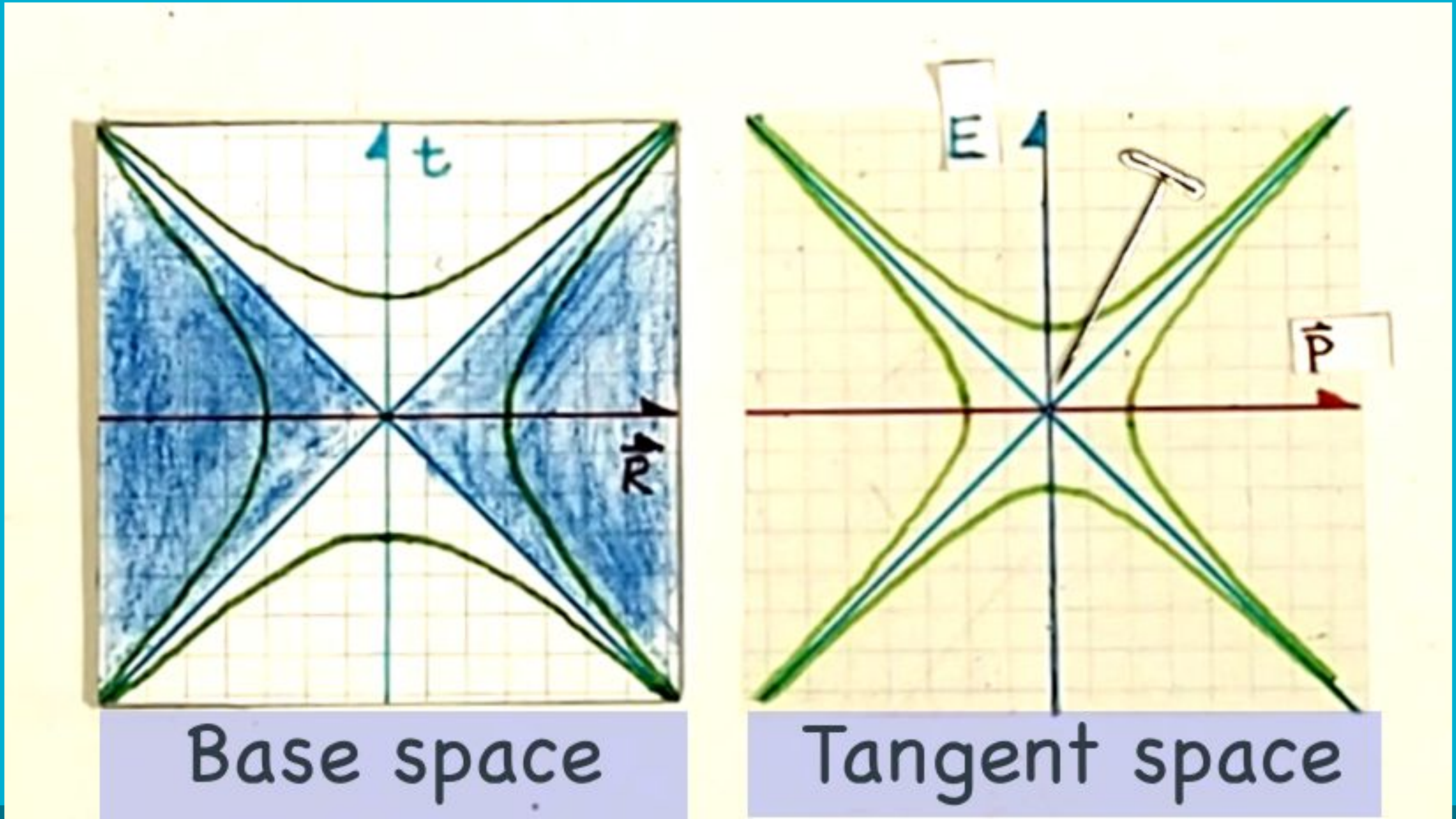
$$\left(d\tau^2 = dt^2 - d\vec{R}^2 \right) * \frac{m^2}{d\tau^2} \leftarrow \text{Test mass}$$

$$m^2 = \left(m\frac{dt}{\partial\tau} \right)^2 - \left(m\frac{d\vec{R}}{\partial\tau} \right)^2 = E^2 - P^2$$

Work with energy **E** and momentum **P** instead of phase terms **dt** and **dR**.

This transformation requires the equivalence principle be true, a good thing.

Visualize the base space of space-time and one of its tangent spaces, energy-momentum.



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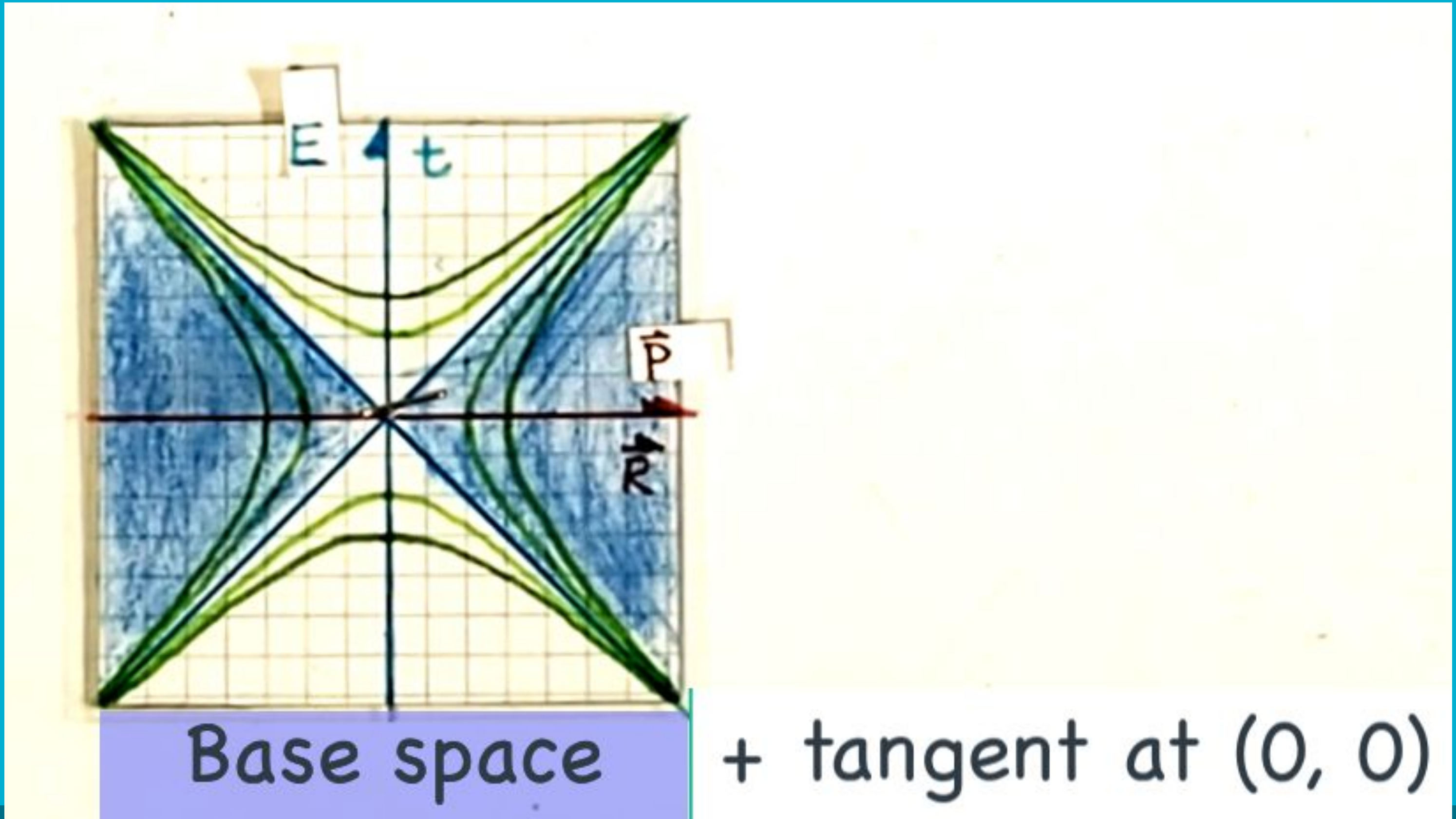

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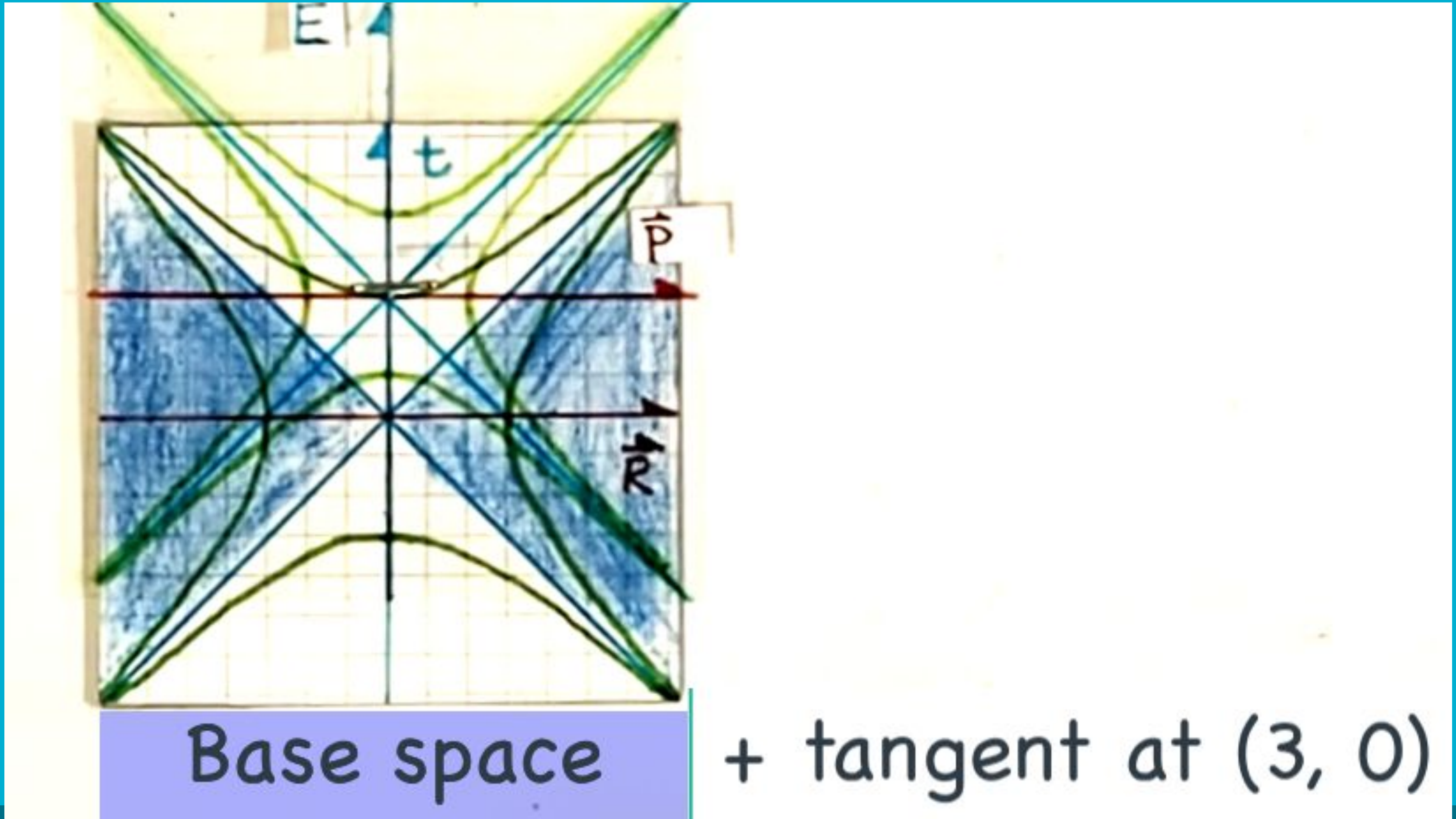

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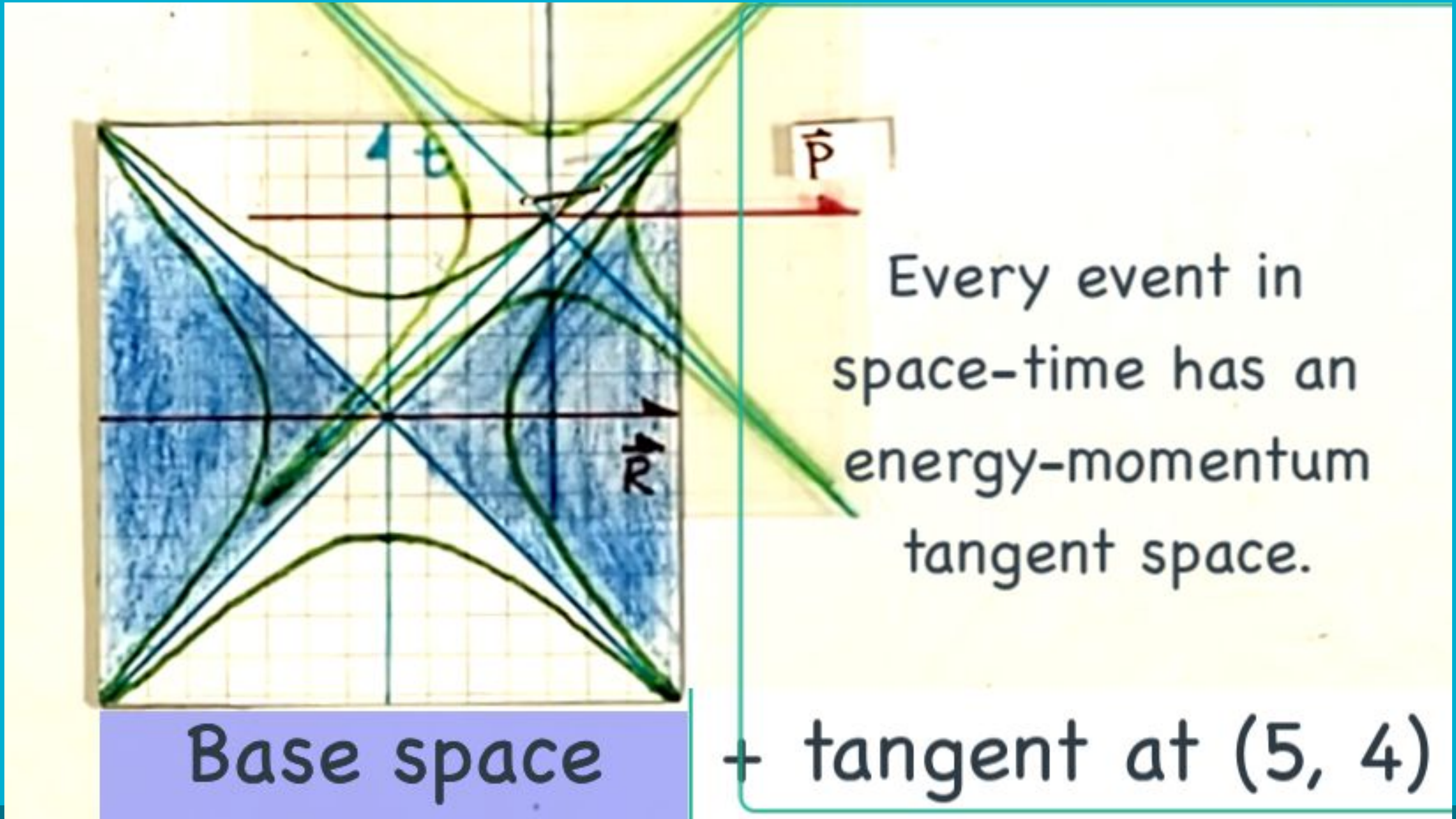


Special relativity can handle accelerations.

Special relativity accelerations **do not** bend light.

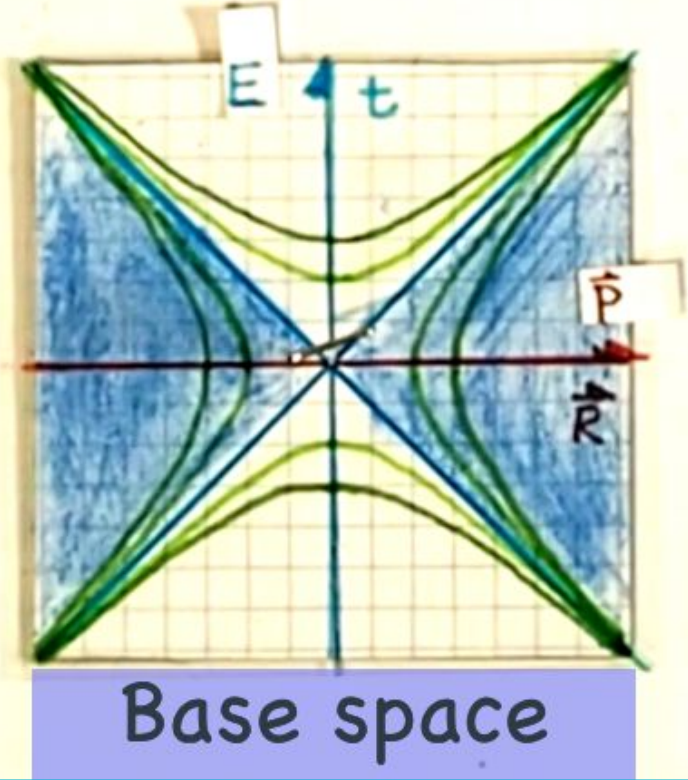
The doppler shift for energy will be the same as the doppler shift for momentum.

The ratio stays constant, so the speed of light remains the same.

This is special relativity after all!

Look for a **new** symmetry for energy-momentum, one that requires **no energy.**

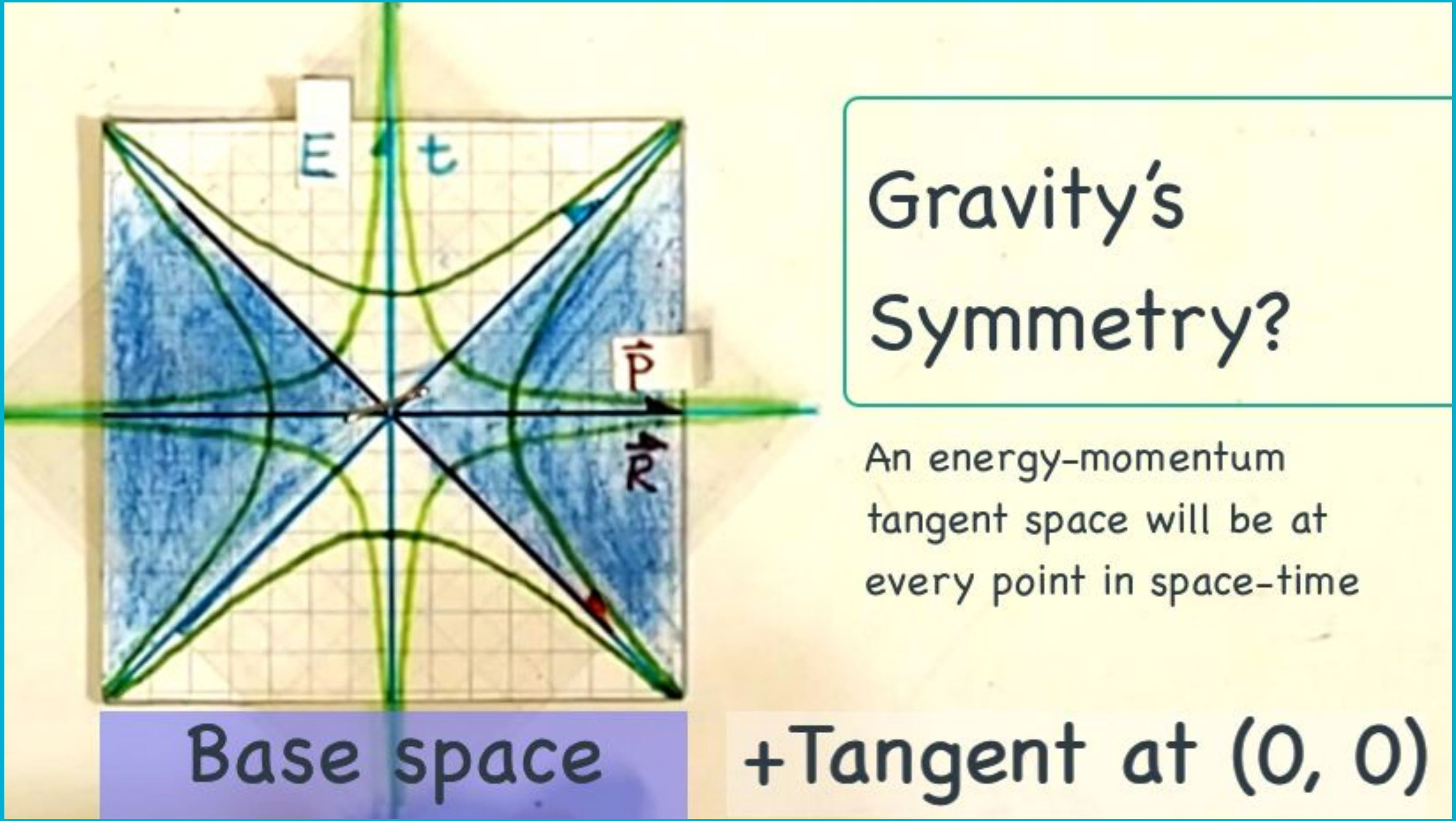
Rotate the hyperbolas by 45 degrees.



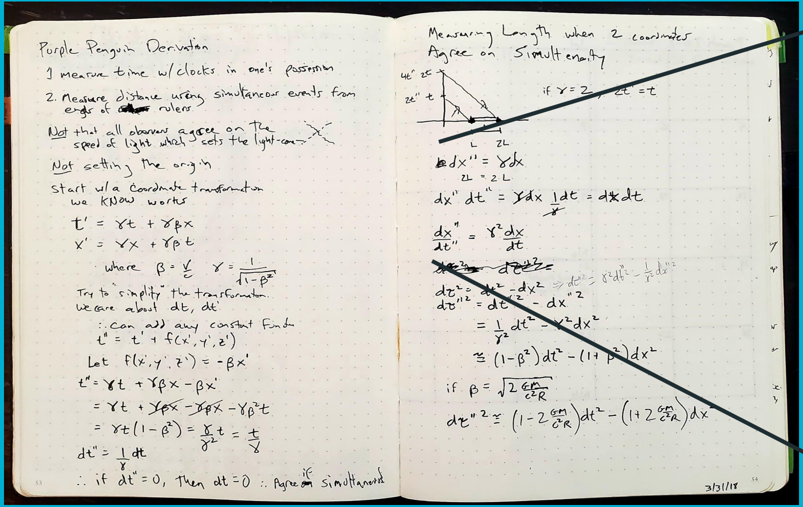
Find a zero-energy, zero-momentum symmetry in the energy-momentum tangent space.

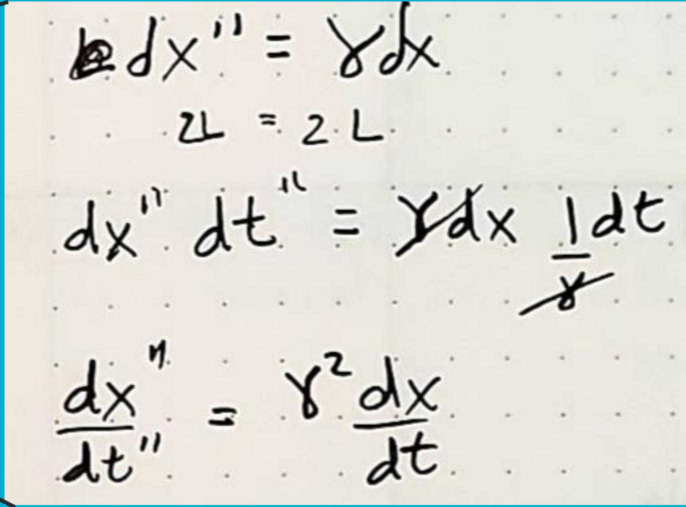
+Tangent at (0, 0)

Rotate the hyperbolas by 45 degrees.



Will the speed of light measurement **as seen by** another observer = c? NO, a good thing.

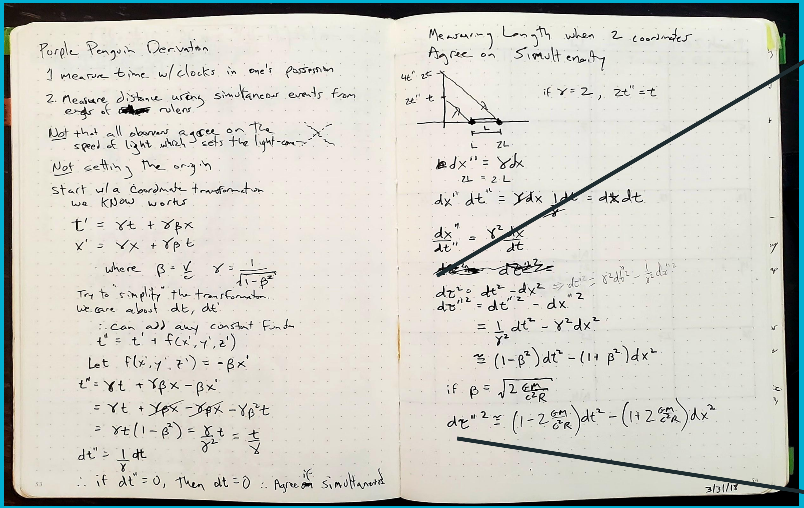


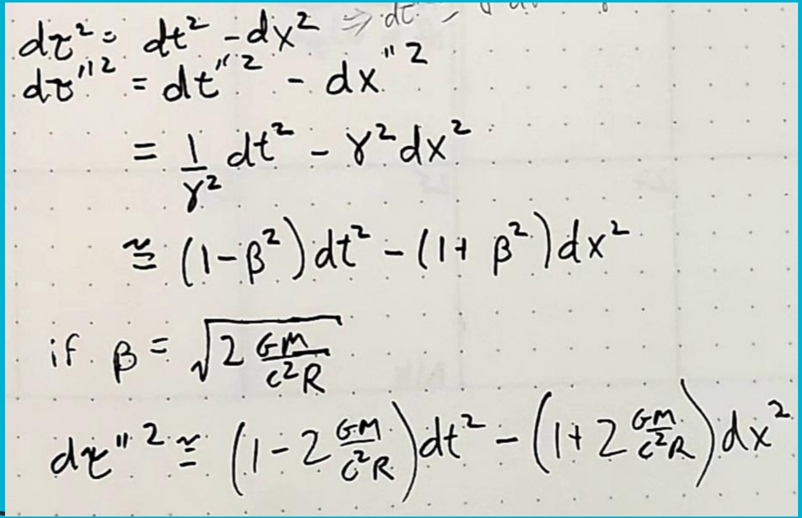


What is constant: energy times momentum.

What varies: the other observers measurement of the speed of light.

Use the Newtonian escape velocity to calculate how the interval squared changes for this transformation.





$$
\begin{aligned}
ds^2 &= -\left[1 - 2\frac{M_\odot}{r} + 2\left(\frac{M_\odot}{r}\right)^2\right] dt^2 + \left[1 + 2\frac{M_\odot}{r}\right]\left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \\
&= -\left[1 - 2\frac{M_\odot}{r} + 2\left(\frac{M_\odot}{r}\right)^2\right] dt^2 + \left[1 + 2\frac{M_\odot}{r}\right]\left[dx^2 + dy^2 + dz^2\right].
\end{aligned}
\qquad (40.1)
$$

(2) in isotropic coordinates

WHERE ARE THE LAGRANGIAN AND FIELD EQUATIONS???

Einstein has 10 non-linear, second order differential field equations that in special cases can be solved. To eliminate the constants of integration, one uses Newton's law of gravity as a boundary condition.

For this symmetry, Newton's escape velocity is used as a boundary condition.

I do think there may be a way to express the escape velocity field using a Lagrangian formalism, but I do not have it at this time.

Fini