Why quaternions

Why quaternions and not ...

Tor Sjöstrand - Feb 26
Doug Sweetser Sorry for not replying in a while. I have been absorbed by space-time algebra, which, believe it or not, has convinced me that complex quaternions are not that stupid after all. The imaginary unit is a consequence of constructing bivectors out of orthonormal unit vectors. Such bivectors v, which are unique to different plane orientations, satisfy the equation v^2=-1, like the imaginary unit. Why don't you like complex quaternions?

Hi Tor. I created my reply 4 months later, so I am slower. I was working on explaining my physics insights to a general audience, people in my Unitarian Universalist Church. That detour was most excellent because it force me to put into simple terms ideas that must be simple because I was describing SSIN, which is an acronym for the Simplest Shit In Nature.

I would never say that complex quaternions were a stupid approach to take to modern physics. There is a collection of technical books and papers one could start with and work out from there. Understanding physics is like appreciating sculpture. There is a value at looking at things from different perspectives.

Given that statement, shouldn't I take my own advice and work with complex quaternions? One belief I have is that someday in the future, there will be but one way to do all of physics. I say this because Nature appears to have one set of rules that we do no understand in full today. Each researcher needs to make their own strategic bet about which approach they think will be best in the long run. You are betting some amount of effort on complex quaternion. Most stick to tensors without questioning.

I can give specific reasons why I have chosen not to work with complex quaternions. Let's start with a physics principle and see if it has consequences to the math tools one choses to use. I have a deep respect for Prof. Leondard Susskind, a world class theorist who is trying to reach out to sincere folks like me that are not world class theorists themselves. In the book, "The theoretical minumum, what you need to know to start doing physics" by Susskind and George Hrabovsky, the first lecture is on the nature of classical physics. The first idea he talks about deals with reversibility. Let me read from the good book, page 9:
If every state has a single unique arrow leading into it, and a single arrow leading out of it, then it is a legal deterministic reversible law. Here is a slogan: there must be one arrow to tell you where you're going and one to tell you where you have come from. The rule that dynamical laws must be deterministic and reversible is so central to classical physics that we sometimes forget to mention is

when teaching the subject. In fact, it doesn't even have a name. We could call it the first law, but unfortunately there are already two first laws - Newton's and the first law of thermodynamics. There is even a zeroth law of thermodynamics. So we have to go back to a minus-first law to gain priority for what is undoubtedly the must fundamental of all physical laws - the conservation of information. The conservation of information is simply that every state has one arrow in and one arrow out. It ensures that you never lose track of where you started. END QUOTE.

The two most basic tools in the math drawer are addition and multiplication. Begin with the addition operator. Try 2 + 4 = 6. Draw the graph. The starting node is 2. The ending node is 6. The edge must be labeled with a 4. How does one go backwards? The starting node is 6, the ending node is 2. If we add 4 to 6 we get 10. The edge must be labeled differently: it needs to be the additive inverse of 4, minus 4.

Repeat the process for multiplication. To go from 2 to 6, the edge gets labeled with a 3. To go from 6 to 2, the multiplicative inverse of 3 is needed, 1/3.

This example used real numbers. Real numbers have been there from day 1 of the birth of modern physics.

The mathematicians had a rather nasty battle over complex numbers which have the property of a division algebra, meaning a multiplicative inverse always exists. While Newton was aware of complex numbers, these numbers only took the center stage with the development of quatum mechanics.

The next step of the progression of division algebras are quaternions. These days, even though I own quaternions.com, I prefer to relabel them as space-time numbers. My training in Python says names are of topmost importance. A quaternion is 4 Roman soldiers and only suggests something about numerology with a focus on 4. This may create more problems than it solves. Yes, quaternions can be represented using 4 numbers or symbols, but it is essential to always remember it is one number that has to travel together with its structure. Space-time numbers claim by their name alone to be the numbers to use in Minkowski space-time. I always know physically what a quaternion in space-time is: an event in space-time. Mathematicians know what complex numbers are and how to use them in esquisit detail. Mathematical physicist should demand to know what they mean physically because that is the physics part of the Universe. I doubt many physicists would admit that if they don't know what a complex number is physically, then as a consequence physical understanding is elusive.

A 4-vector $p^\mu = (1, 2, 3, 4)$ can be added or subtracted to another, say $q^\nu = (4, 3, 2, 1)$.

The exact same statemens holds for space-time numbers but without any Greek leters.

A 4-vector can be multiplicated by a scalar, say a 5. Hold on a second, what is a "scalar"? Mathematicians know, but what does a physicist think it is physically? You cannot pick a number out of nothing. There are not 5's floating out in the vaccuum of space to use as you wish. With space-time numbers, we have a precise idea that can be understood physically. A scalar is a space-time number that is only about time, either more or less of it. The space terms are all zero, or literally here, where the observer is in space. So:

$$5 * p = (1, 2, 3, 4)(5, 0, 0, 0) = (5, 10, 15, 20)$$

What multiplying the scalar 5 space-time number by P says is the resulting event is 5 times longer and 5 times more distant in every non-zero direction than the event P. This result with space-time numbers is indistinguishable from the 4-vector p^{μ} multiplied by the scalar 5. I can multiply any and all events in space-time together so long as one of them has the space values equal to three zeroes. Yuk.

Now there is a problem with the Mathematicians claim that one can multiply by a scalar. Not mathematically, they get to do what they define as legal. In a different reference frame, the values in $(5, 0, 0, 0)$ can and will change. That's relativitic physics. The zeros will become non-zero depending on the motion of the observer. That is not a problem for space-time numbers where all values can be non-zero and multiplication just works. What mathematicians are requiring by saying a 4-vector p^{μ} can be multiplied by a scalar is that one has to work in one reference frame where the spatial terms are equal to zero. I doubt physicists would like that.

Why are mathematicians satisfied with their definition of 4-vectors? Because it is easy to extend to 5 dimension, and 10, dimensions, and 11 or 24 dimensions. They love to play in higher dimensional space. For space-time problmes, I don't care about 5, 10, 11, or 24 dimensions. The Universe I can collect data in has 3 spatial dimensions and one time dimension.

Could I make a video just like this one that puts down space-time numbers? When I started back in 1997, in the first year or two, it was clear I could do Newtonian physics because it was easy enough to generate Newton's second law and also manage the rotational forces. The first test - and it was a test - was to see if I could solve every problem in a class on special relativity using real-valued space-time numbers. The test was passed. I then had to figure out the Maxwell equations. That took years, literally years, due to my isolation from the professional approach. By the mid 2000s I understood how to apply the Euler-Lagrange equations and bingo, that task was cleared. The literature by the way says one needs complex quaternions. That does work, as does real-values quaternions.

The big problem got me was gravity. For more than 10 years, I had an idea that did not use quaternions, but its hyper-complex variant. The rules for multiplication create all the same terms but there are no minus signs anywhere. Again, due to my isolation, it took a decade before someone pointed out my Lagrange density would be altered by a change in angle, so the proposal did not conserve angular momentum, and was thus wrong. I made sure to not delete my failures from the public record - hypercomplex numbers sounds neat. Instead, I put a big fat label: RESCINDED in front of it.

For some 3 years, I would have said that space-time numbers have no way to explain gravity. I tried a bunch of crap and could accept it was crap. In the spring of 2015, I saw a symmetry in a space-time diagram I had never heard discussed before. It is the Lorentz hyperbolas rotated by 45 degrees. There is no doubt on a math level that symmetry is a thing. I think if a decent physicist thought about that for a while, they would say this is a relativity symmetry, so it is 2 people looking at the same thing, so they will agree on somethings and disagree on others. We know with special relativity they agree on the interval and they agree on the speed of light. For this symmetry they do not agree on the interval and they do not agree to the same value of the speed of light. That is what goes on for gravity: intervals change and the path of light is bent.

I cannot make the video arguing against space-time numbers. If you would like to do so, feel free.