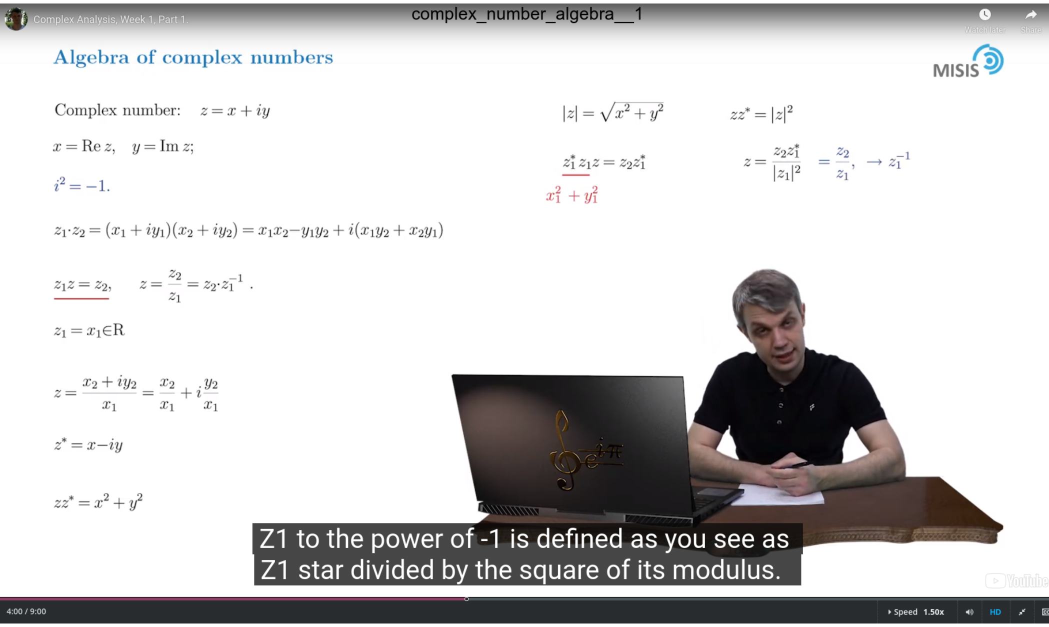
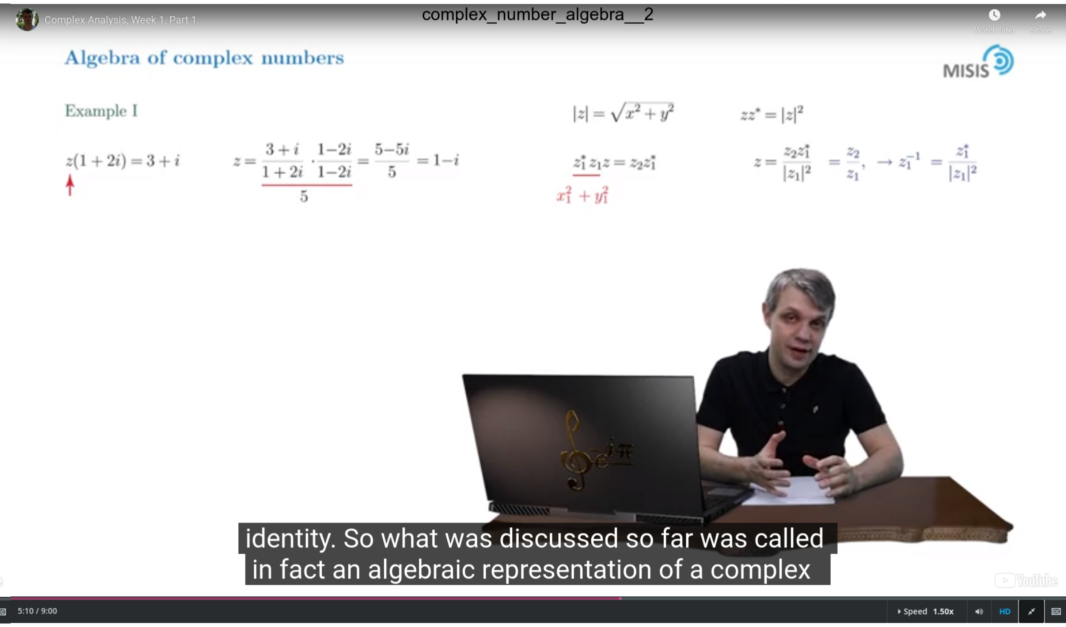
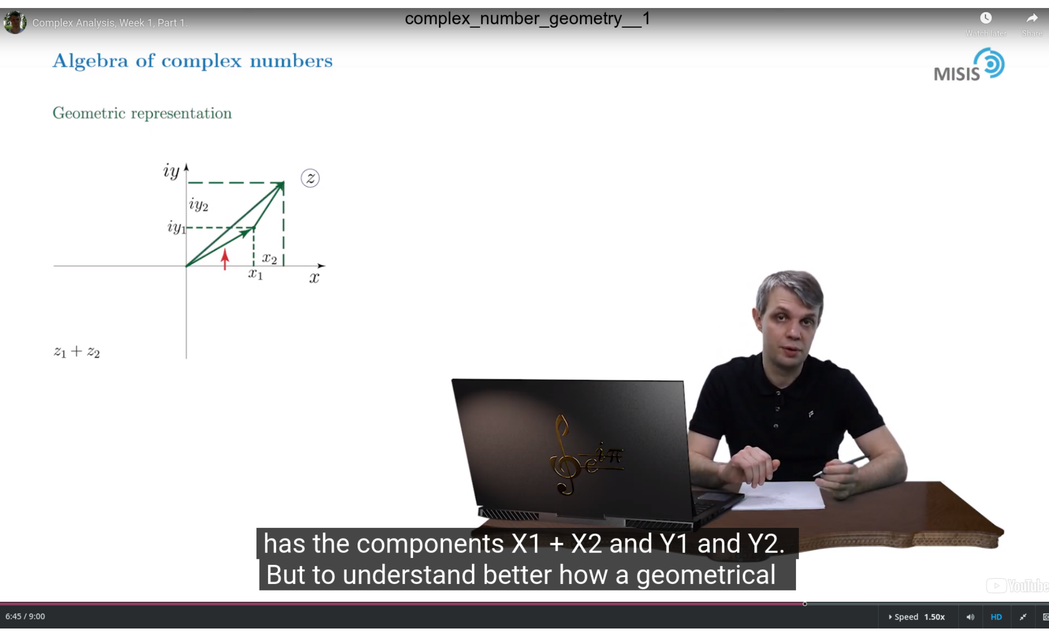
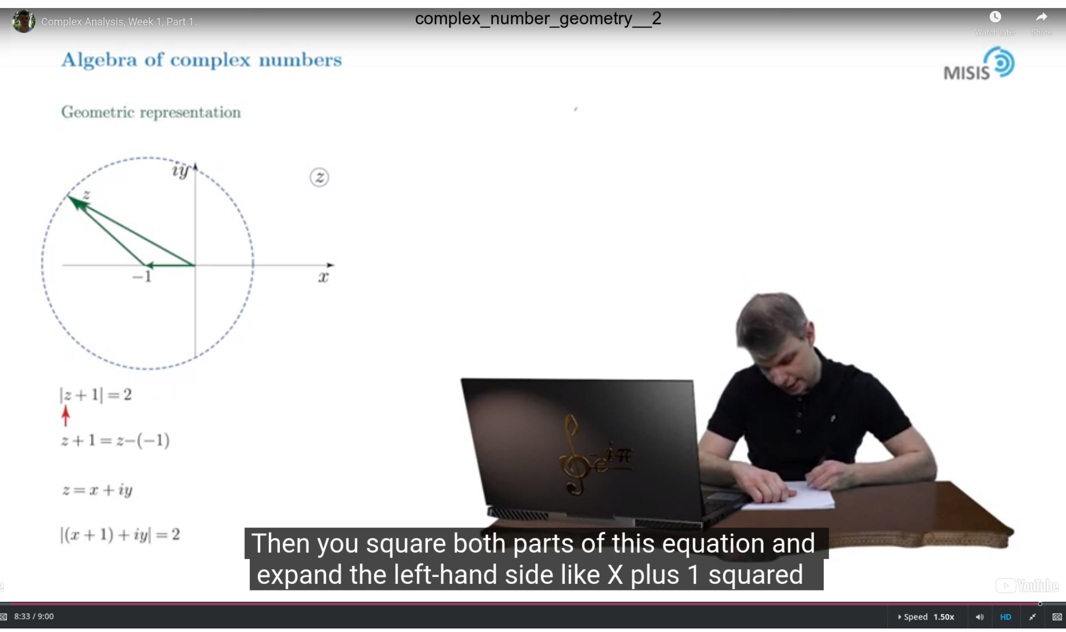
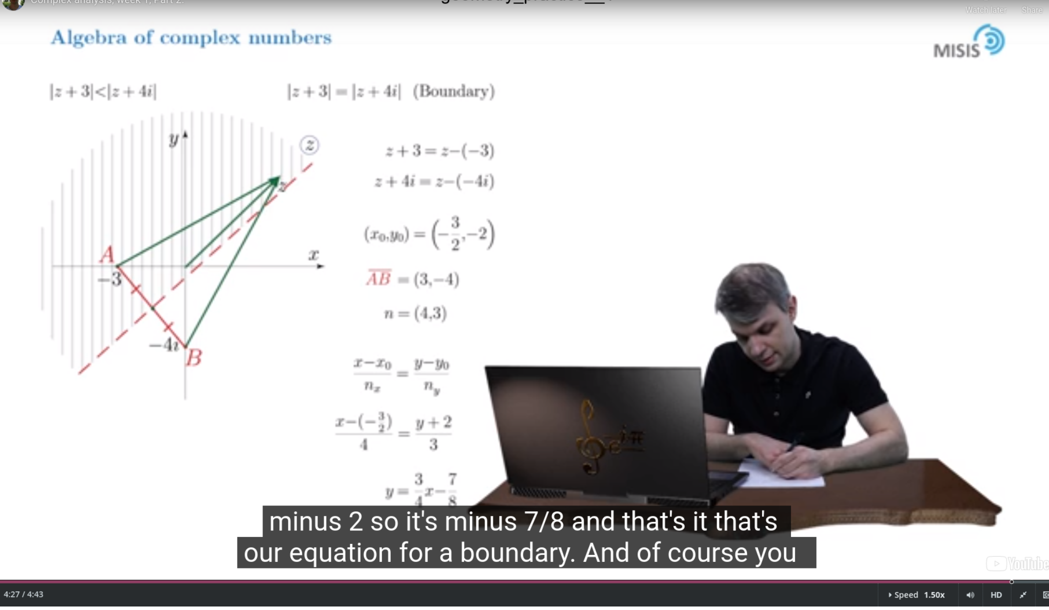

complex_number_algebra__1
Complex Analysis, Week 1, Part 1
Algebra of complex numbers
MISIS
Complex number: $z = x + iy$
$x = \mathrm{Re}\, z, \quad y = \mathrm{Im}\, z;$
$i^2 = -1.$
$z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1x_2 - y_1y_2 + i(x_1y_2 + x_2y_1)$
$z_1 z = z_2, \quad z = \frac{z_2}{z_1} = z_2 \cdot z_1^{-1}.$
$z_1 = x_1 \in \mathrm{R}$
$z = \frac{x_2 + iy_2}{x_1} = \frac{x_2}{x_1} + i\frac{y_2}{x_1}$
$z^* = x - iy$
$zz^* = x^2 + y^2$
$|z| = \sqrt{x^2 + y^2}$
$zz^* = |z|^2$
$z_1^* z_1 z = z_2 z_1^*$
$x_1^2 + y_1^2$
$z = \frac{z_2 z_1^*}{|z_1|^2} = \frac{z_2}{z_1}, \rightarrow z_1^{-1}$
Z1 to the power of -1 is defined as you see as
Z1 star divided by the square of its modulus.
4:00 / 9:00
Speed 1.50x
HD


complex_number_algebra__2
Complex Analysis, Week 1, Part 1
Algebra of complex numbers
MISIS
Example I
$z(1 + 2i) = 3 + i$
$z = \frac{3+i}{1+2i} \cdot \frac{1-2i}{1-2i} = \frac{5-5i}{5} = 1 - i$
5
$|z| = \sqrt{x^2 + y^2}$
$zz^* = |z|^2$
$z_1^* z_1 z = z_2 z_1^*$
$x_1^2 + y_1^2$
$z = \frac{z_2 z_1^*}{|z_1|^2} = \frac{z_2}{z_1}, \rightarrow z_1^{-1} = \frac{z_1^*}{|z_1|^2}$
identity. So what was discussed so far was called
in fact an algebraic representation of a complex
5:10 / 9:00
Speed 1.50x
HD

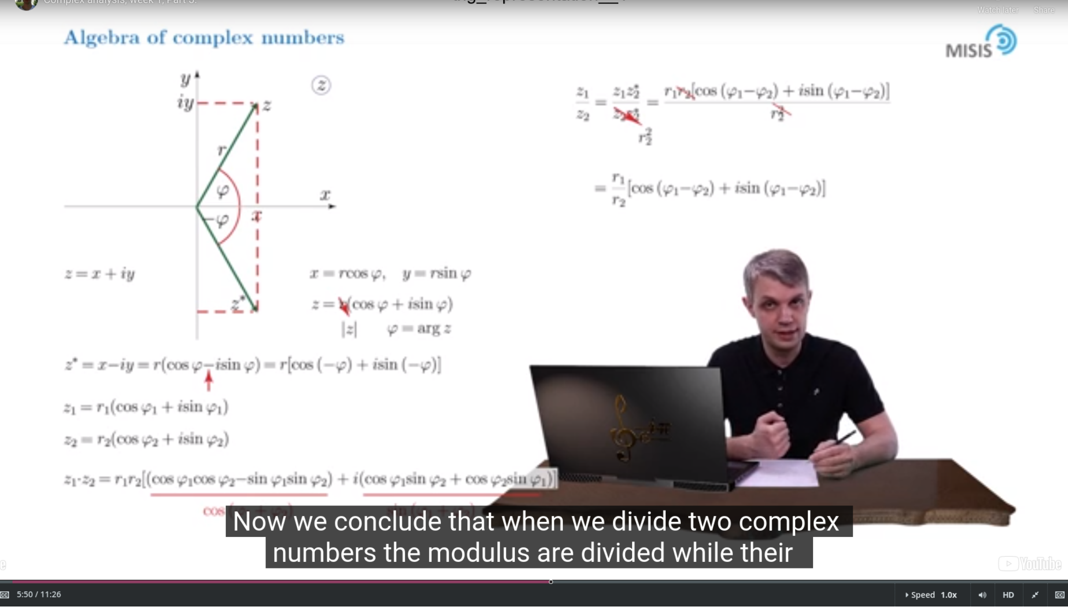

complex_number_geometry__1
Complex Analysis, Week 1, Part 1
Algebra of complex numbers
MISIS
Geometric representation
iy
iy_2
iy_1
x_2
x_1
x
z
$z_1 + z_2$
has the components X1 + X2 and Y1 and Y2.
But to understand better how a geometrical
6:45 / 9:00
Speed 1.50x
HD


complex_number_geometry__2
Complex Analysis, Week 1, Part 1
Algebra of complex numbers
MISIS
Geometric representation
iy
z
-1
x
$|z + 1| = 2$
$z + 1 = z - (-1)$
$z = x + iy$
$|(x + 1) + iy| = 2$
Then you square both parts of this equation and
expand the left-hand side like X plus 1 squared
8:33 / 9:00
Speed 1.50x
HD

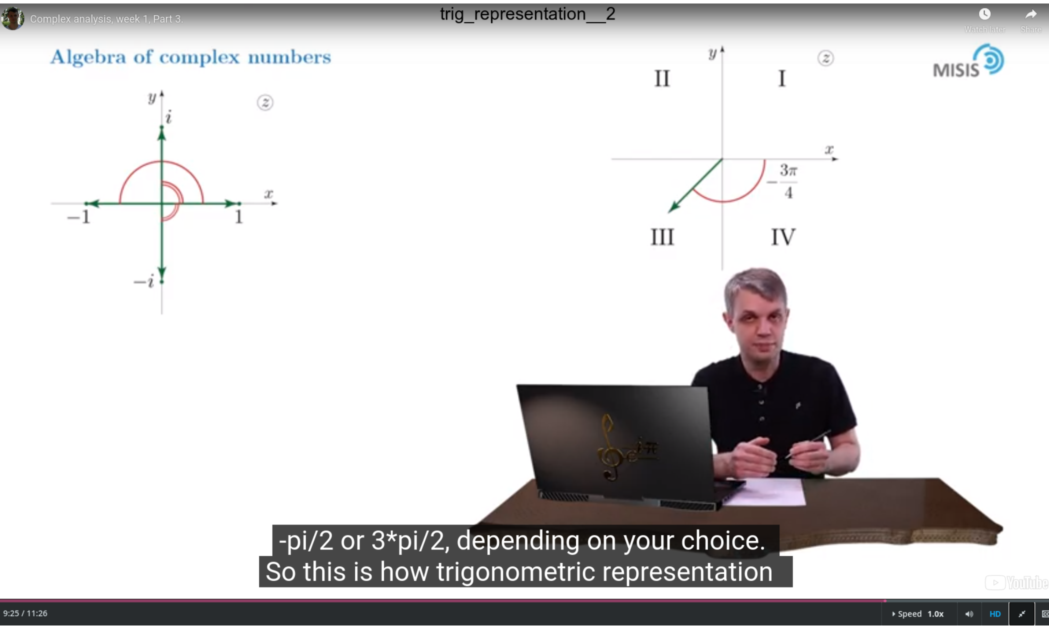
geometry_practice__1



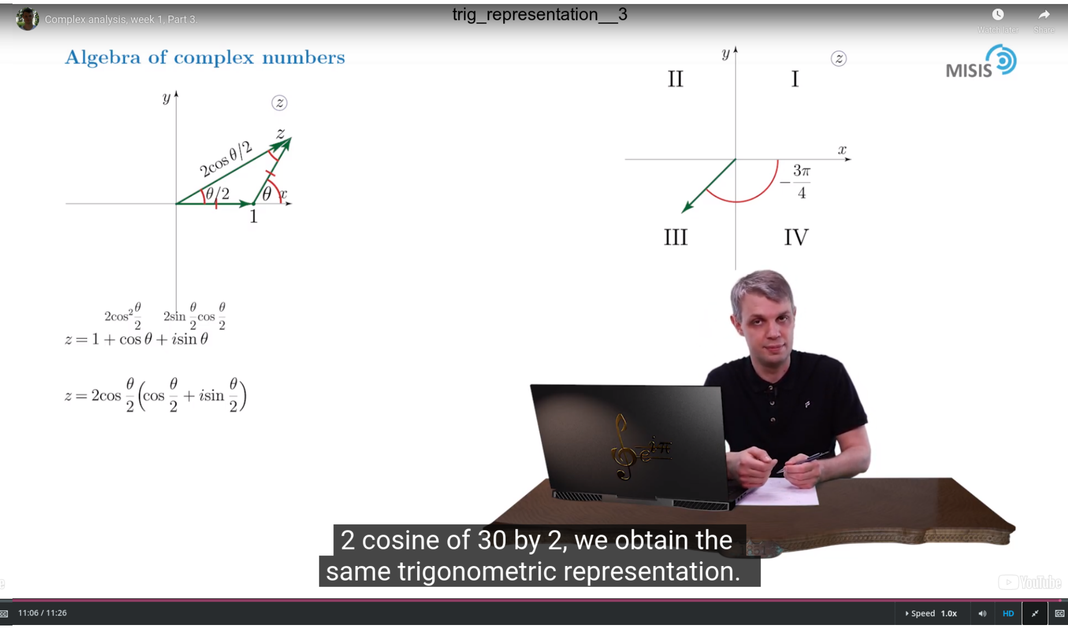
trig_representation__1

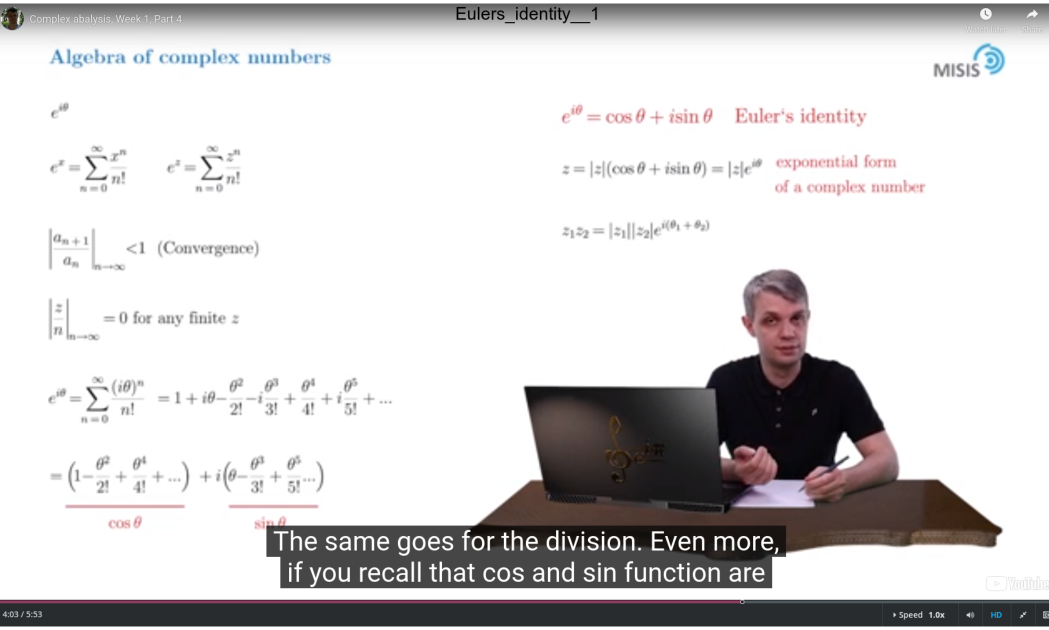


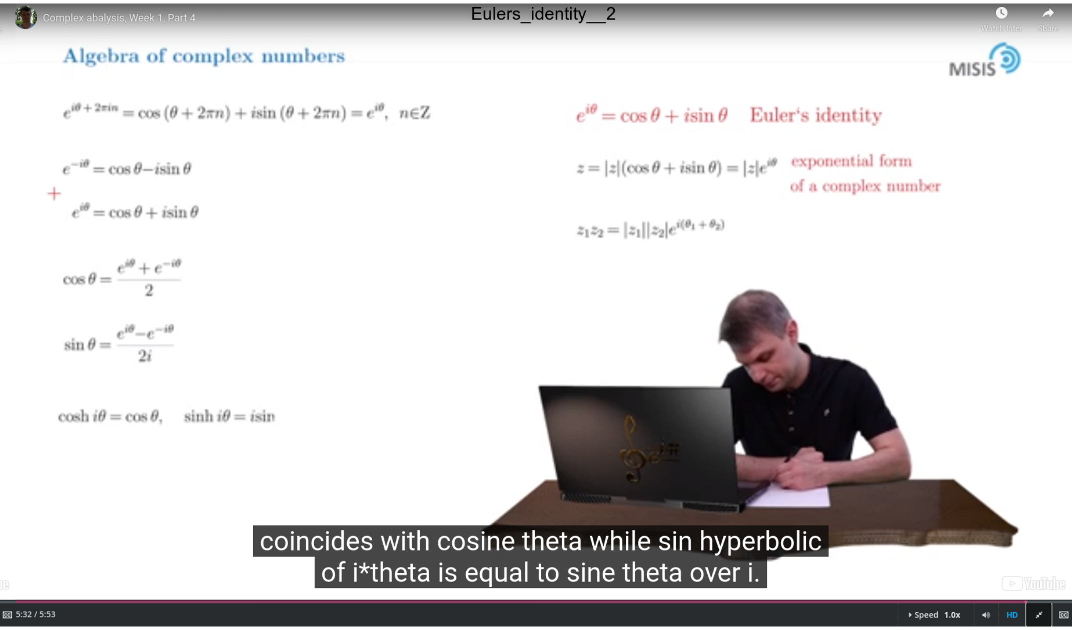
trig_representation__2

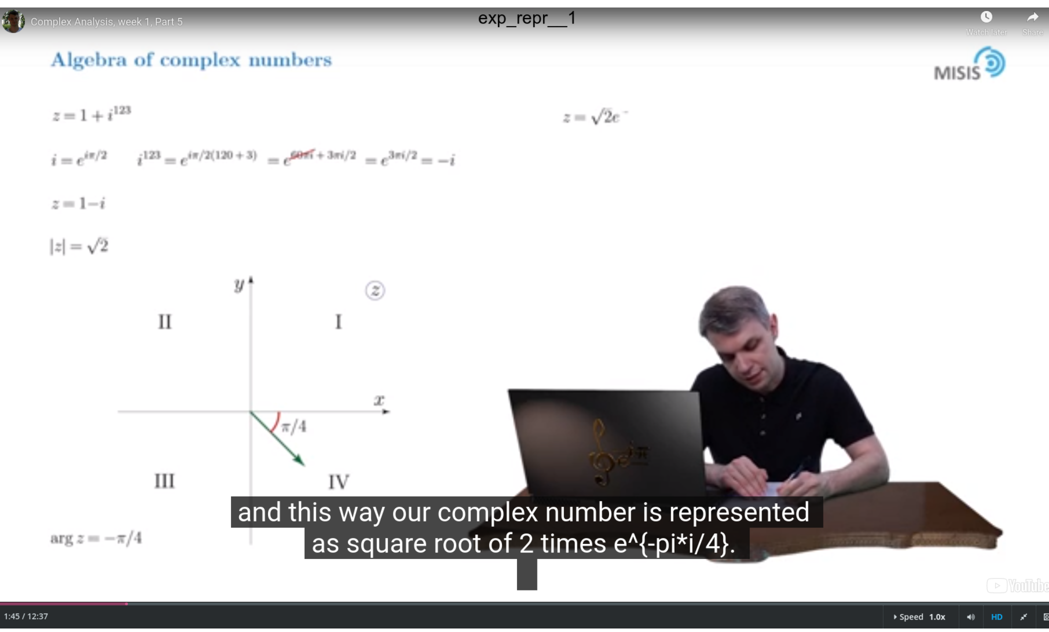


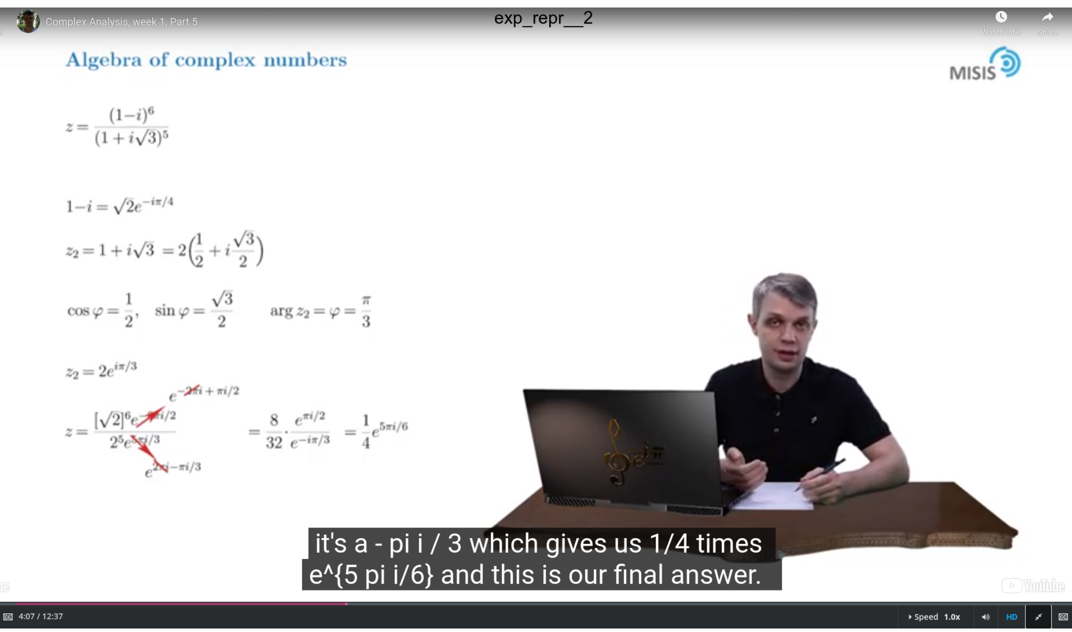
trig_representation__3



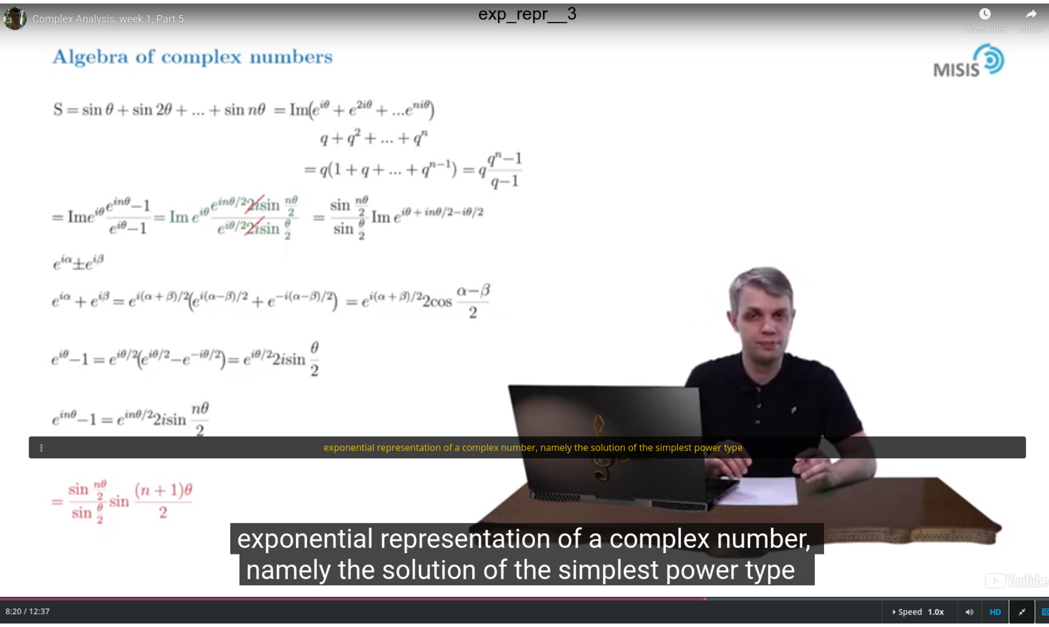

Complex abalysis. Week 1. Part 4
Eulers_identity__1
Algebra of complex numbers
MISIS
$e^{i\theta}$
$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$ $e^z=\sum_{n=0}^{\infty}\frac{z^n}{n!}$
$\left|\frac{a_{n+1}}{a_n}\right|_{n\to\infty}<1$ (Convergence)
$\left|\frac{z}{n}\right|_{n\to\infty}=0$ for any finite z
$e^{i\theta}=\sum_{n=0}^{\infty}\frac{(i\theta)^n}{n!}=1+i\theta-\frac{\theta^2}{2!}-i\frac{\theta^3}{3!}+\frac{\theta^4}{4!}+i\frac{\theta^5}{5!}+\dots$
$=\underbrace{\left(1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}+\dots\right)}_{\cos\theta}+i\underbrace{\left(\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}\dots\right)}_{\sin\theta}$
$e^{i\theta}=\cos\theta+i\sin\theta$ Euler's identity
$z=|z|(\cos\theta+i\sin\theta)=|z|e^{i\theta}$ exponential form of a complex number
$z_1z_2=|z_1||z_2|e^{i(\theta_1+\theta_2)}$
The same goes for the division. Even more,
if you recall that cos and sin function are
4:03 / 5:53
Speed 1.0x
HD


Complex abalysis. Week 1. Part 4
Eulers_identity__2
Algebra of complex numbers
MISIS
$e^{i\theta+2\pi in}=\cos(\theta+2\pi n)+i\sin(\theta+2\pi n)=e^{i\theta},\quad n\in Z$
$e^{-i\theta}=\cos\theta-i\sin\theta$
$+$
$e^{i\theta}=\cos\theta+i\sin\theta$
$\cos\theta=\frac{e^{i\theta}+e^{-i\theta}}{2}$
$\sin\theta=\frac{e^{i\theta}-e^{-i\theta}}{2i}$
$\cosh i\theta=\cos\theta,\quad \sinh i\theta=i\sin$
$e^{i\theta}=\cos\theta+i\sin\theta$ Euler's identity
$z=|z|(\cos\theta+i\sin\theta)=|z|e^{i\theta}$ exponential form of a complex number
$z_1z_2=|z_1||z_2|e^{i(\theta_1+\theta_2)}$
coincides with cosine theta while sin hyperbolic
of i*theta is equal to sine theta over i.
5:32 / 5:53
Speed 1.0x
HD

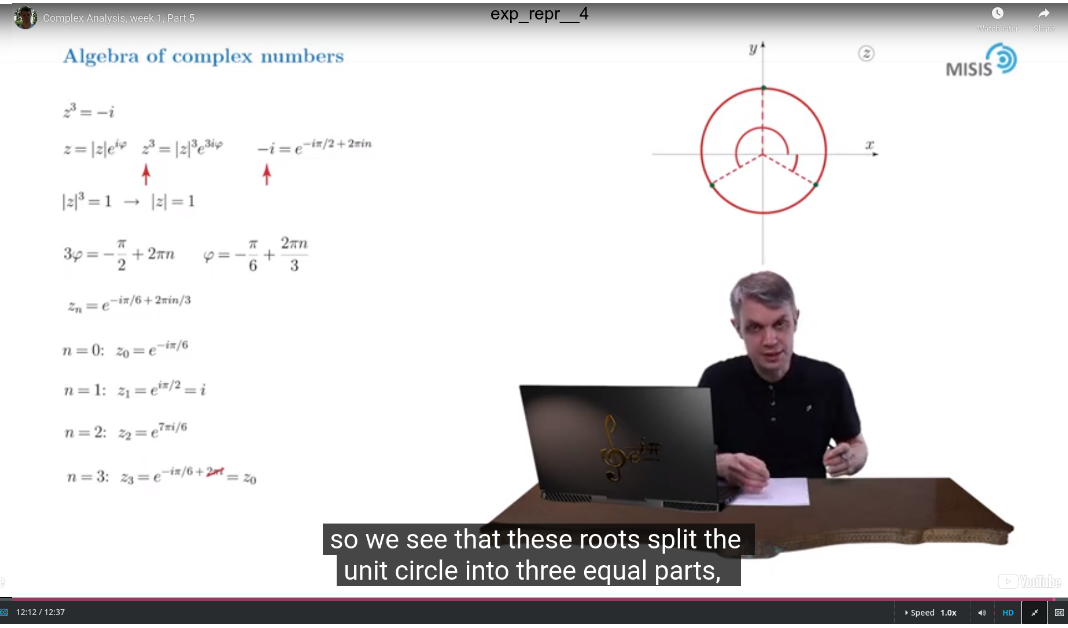

Complex Analysis. week 1. Part 5
exp_repr__1
Algebra of complex numbers
MISIS
$z=1+i^{123}$
$i=e^{i\pi/2}$ $i^{123}=e^{i\pi/2(120+3)}=e^{\cancel{60\pi i}+3\pi i/2}=e^{3\pi i/2}=-i$
$z=1-i$
$|z|=\sqrt{2}$
$z=\sqrt{2}e^{-}$
y
x
z
I
II
III
IV
$\pi/4$
$\arg z=-\pi/4$
and this way our complex number is represented
as square root of 2 times e^{-pi*i/4}.
1:45 / 12:37
Speed 1.0x
HD


Complex Analysis. week 1. Part 5
exp_repr__2
Algebra of complex numbers
MISIS
$z=\frac{(1-i)^6}{(1+i\sqrt{3})^5}$
$1-i=\sqrt{2}e^{-i\pi/4}$
$z_2=1+i\sqrt{3}=2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$
$\cos\varphi=\frac{1}{2},\quad \sin\varphi=\frac{\sqrt{3}}{2}\qquad \arg z_2=\varphi=\frac{\pi}{3}$
$z_2=2e^{i\pi/3}$
$z=\frac{[\sqrt{2}]^6 e^{\cancel{-6\pi i/4}}}{2^5 e^{\cancel{5\pi i/3}}}=\frac{8}{32}\cdot\frac{e^{\pi i/2}}{e^{-i\pi/3}}=\frac{1}{4}e^{5\pi i/6}$
$e^{-2\pi i+\pi i/2}$
$e^{2\pi i-\pi i/3}$
it's a - pi i / 3 which gives us 1/4 times
e^{5 pi i/6} and this is our final answer.
4:07 / 12:37
Speed 1.0x
HD

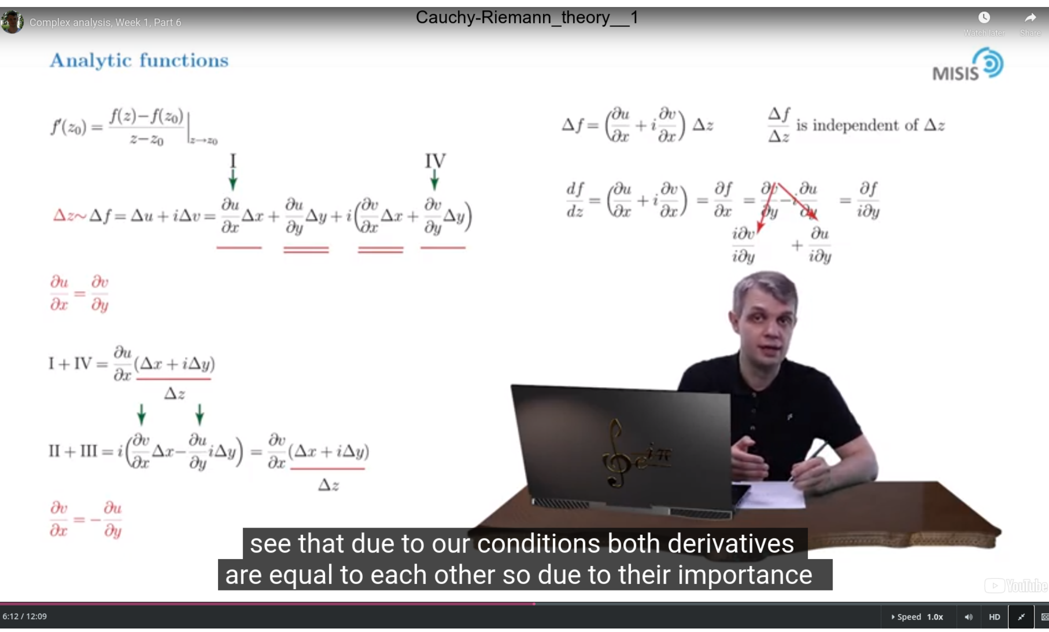
exp_repr__3



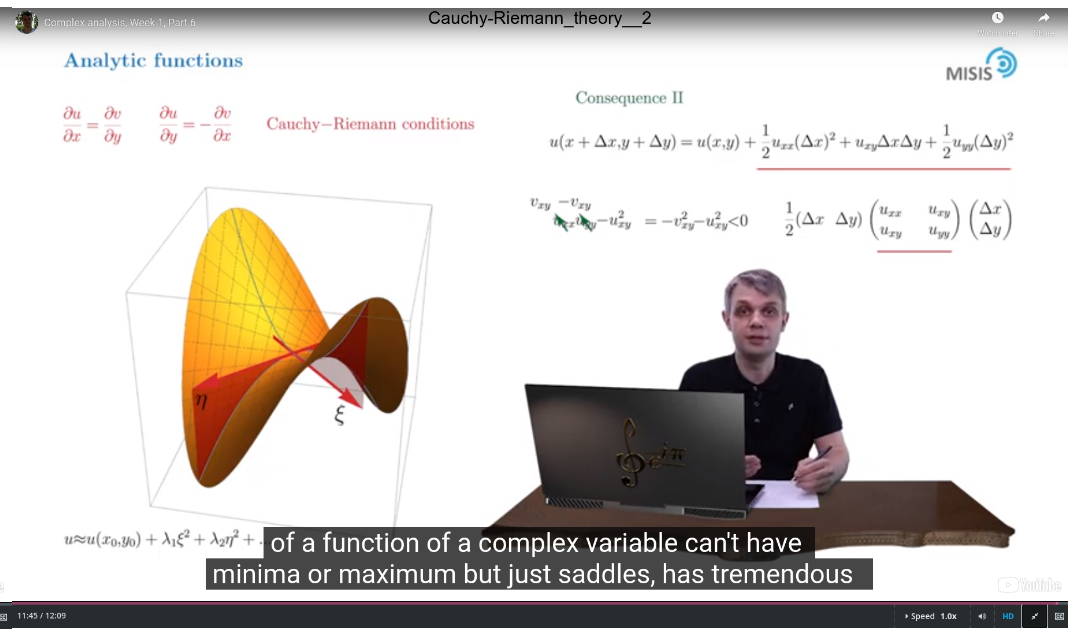
exp_repr__4

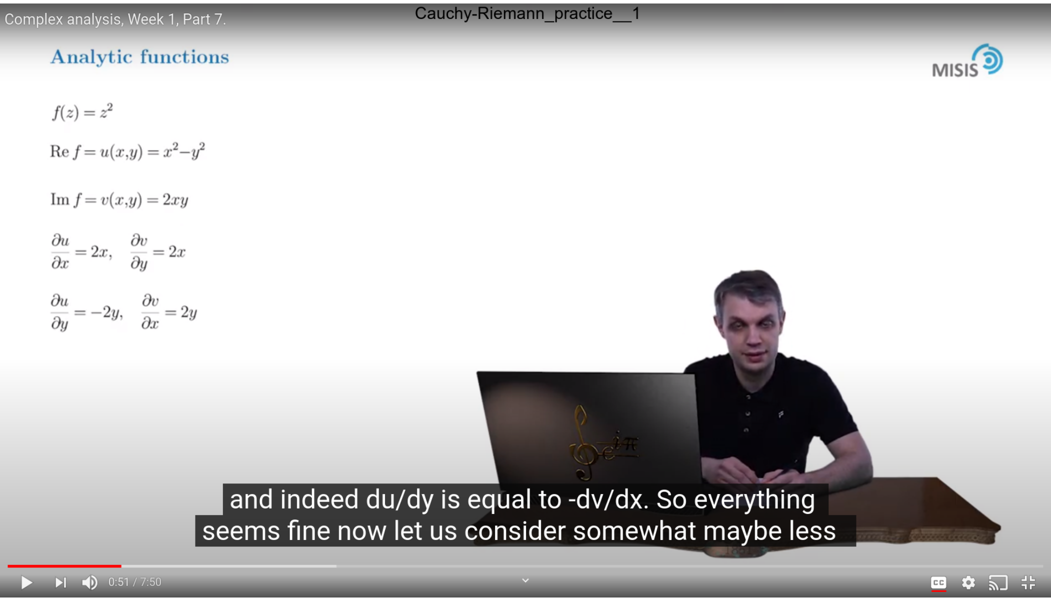


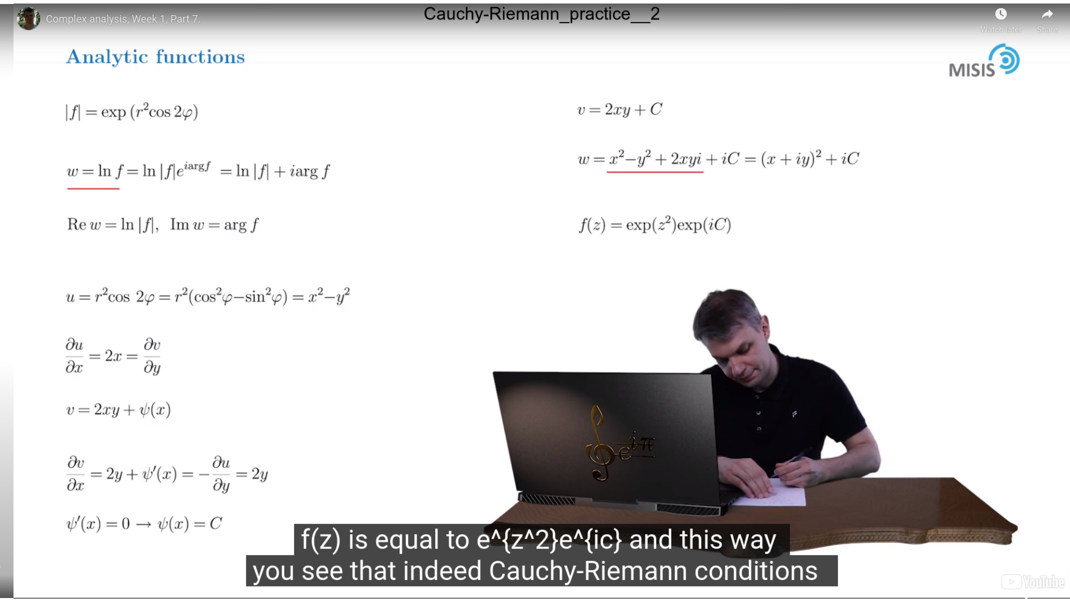
Cauchy-Riemann_theory__1

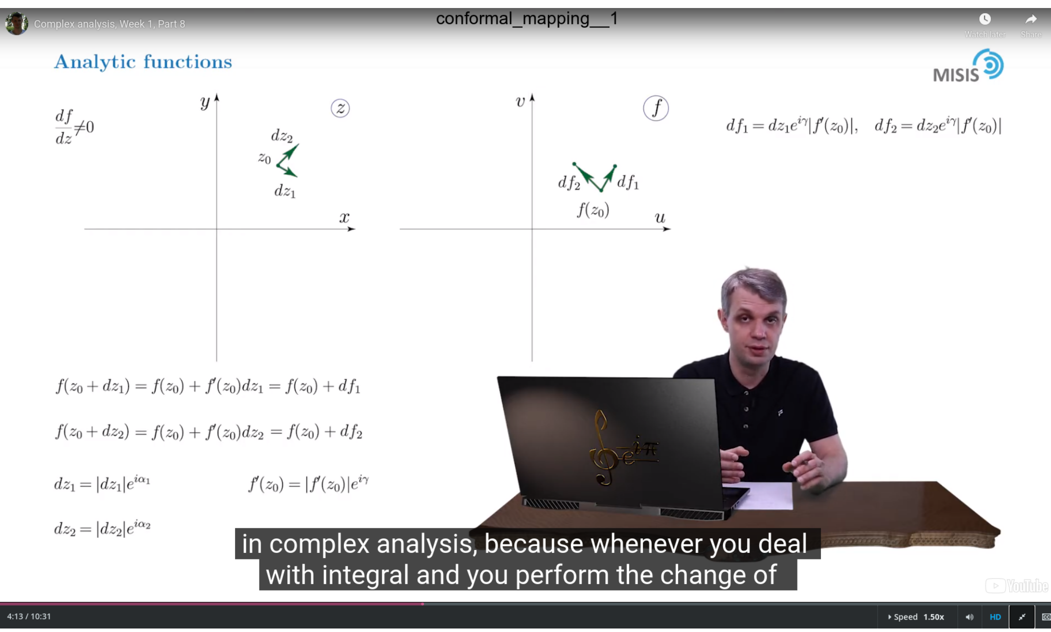


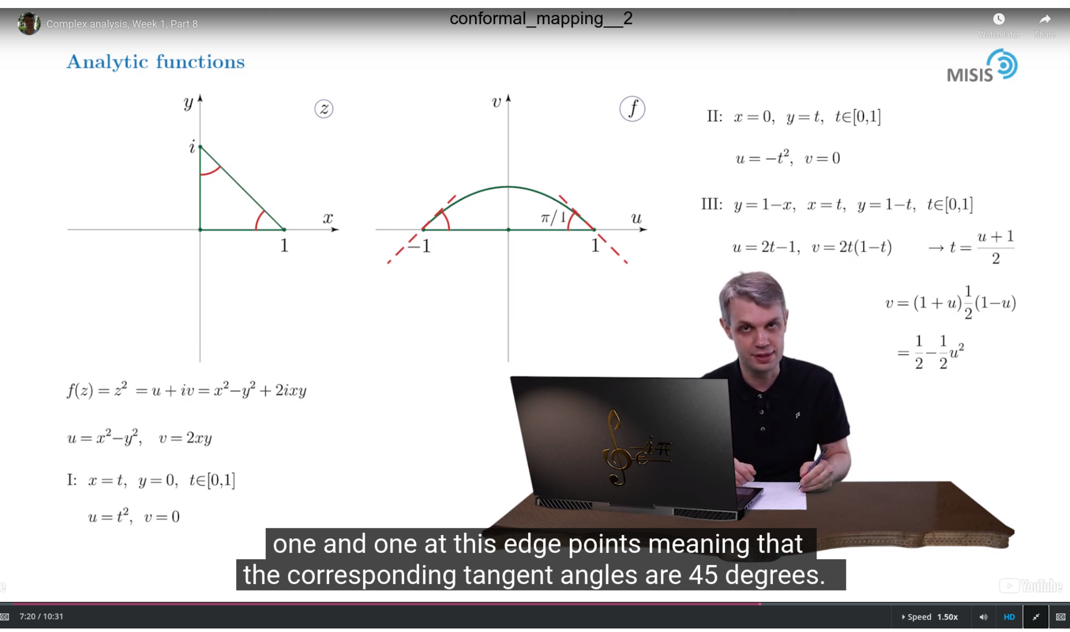
Cauchy-Riemann_theory__2



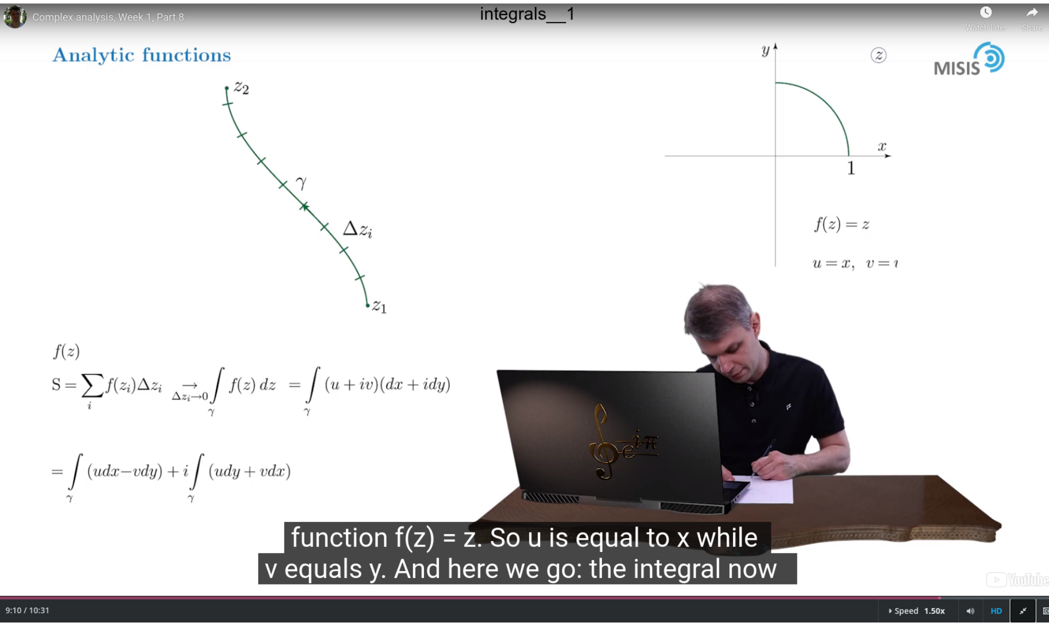

Complex analysis, Week 1, Part 7.
Cauchy-Riemann_practice__1
Analytic functions
MISIS
$f(z)=z^2$
$\mathrm{Re}\, f=u(x,y)=x^2-y^2$
$\mathrm{Im}\, f=v(x,y)=2xy$
$\frac{\partial u}{\partial x}=2x, \quad \frac{\partial v}{\partial y}=2x$
$\frac{\partial u}{\partial y}=-2y, \quad \frac{\partial v}{\partial x}=2y$
and indeed du/dy is equal to -dv/dx. So everything
seems fine now let us consider somewhat maybe less
0:51 / 7:50


Complex analysis, Week 1, Part 7
Cauchy-Riemann_practice__2
Analytic functions
MISIS
$|f|=\exp(r^2\cos 2\varphi)$
$w=\ln f=\ln|f|e^{i\arg f}=\ln|f|+i\arg f$
$\mathrm{Re}\, w=\ln|f|, \quad \mathrm{Im}\, w=\arg f$
$u=r^2\cos 2\varphi=r^2(\cos^2\varphi-\sin^2\varphi)=x^2-y^2$
$\frac{\partial u}{\partial x}=2x=\frac{\partial v}{\partial y}$
$v=2xy+\psi(x)$
$\frac{\partial v}{\partial x}=2y+\psi'(x)=-\frac{\partial u}{\partial y}=2y$
$\psi'(x)=0\rightarrow\psi(x)=C$
$v=2xy+C$
$w=x^2-y^2+2xyi+iC=(x+iy)^2+iC$
$f(z)=\exp(z^2)\exp(iC)$
f(z) is equal to e^{z^2}e^{ic} and this way
you see that indeed Cauchy-Riemann conditions


Complex analysis, Week 1, Part 8
conformal_mapping__1
Analytic functions
MISIS
$\frac{df}{dz}\neq 0$
$df_1=dz_1e^{i\gamma}|f'(z_0)|, \quad df_2=dz_2e^{i\gamma}|f'(z_0)|$
$f(z_0+dz_1)=f(z_0)+f'(z_0)dz_1=f(z_0)+df_1$
$f(z_0+dz_2)=f(z_0)+f'(z_0)dz_2=f(z_0)+df_2$
$dz_1=|dz_1|e^{i\alpha_1}$
$f'(z_0)=|f'(z_0)|e^{i\gamma}$
$dz_2=|dz_2|e^{i\alpha_2}$
in complex analysis, because whenever you deal
with integral and you perform the change of
4:13 / 10:31


Complex analysis, Week 1, Part 8
conformal_mapping__2
Analytic functions
MISIS
II: $x=0, \quad y=t, \quad t\in[0,1]$
$u=-t^2, \quad v=0$
III: $y=1-x, \quad x=t, \quad y=1-t, \quad t\in[0,1]$
$u=2t-1, \quad v=2t(1-t) \quad \rightarrow t=\frac{u+1}{2}$
$v=(1+u)\frac{1}{2}(1-u)$
$=\frac{1}{2}-\frac{1}{2}u^2$
$f(z)=z^2=u+iv=x^2-y^2+2ixy$
$u=x^2-y^2, \quad v=2xy$
I: $x=t, \quad y=0, \quad t\in[0,1]$
$u=t^2, \quad v=0$
one and one at this edge points meaning that
the corresponding tangent angles are 45 degrees.
7:20 / 10:31

integrals__1



integrals__2