

Problem 4.3

0.0/8.0 points (ungraded)

Consider the function $f(z)$, obtained by analytical continuation of arithmetical value of $\sqrt{1+z^2}$ from the real line $z > 0$ into the regions D_i , see three options corresponding to $i = 1, 2, 3$ on the Fig. 1.

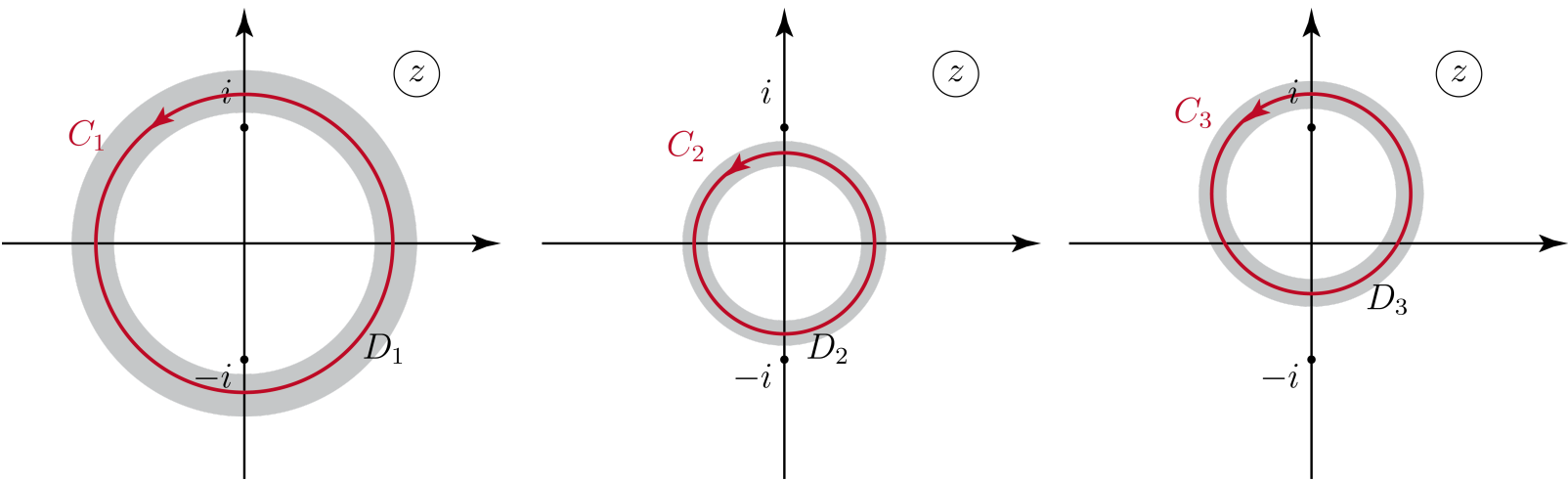


Fig. 1

(i) For $i = 1$ and $i = 2$ check that the function $f(z)$ is single-valued and compute the integrals $I_i = \int_{C_i} f(z) dz$. For $i = 3$, check that the analytical continuation into D_3 delivers a multi-valued function and the cuts are needed to render it single-valued. Use i for complex unity, $\text{sqrt}(\#)$ for $\sqrt{\#}$, $e^{(\#)}$ for the exponential and pi for π .

$I_1 =$

$I_2 =$

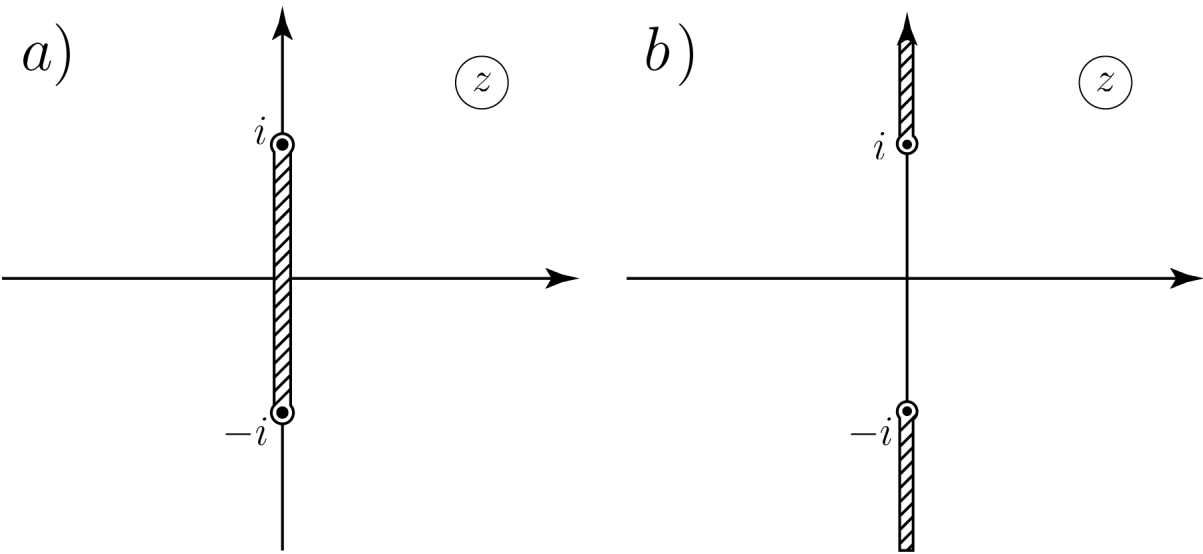


Fig. 2

(ii) Which choice of the cuts of those shown on the Fig. 2 leads to $f(-1) = -\sqrt{2}$?

☐ Cuts on the Fig. 2a

☐ Cuts on the Fig. 2b