Problem 1.1

0.0/3.0 points (ungraded)

Provide a geometric desctiption of the described sets in the complex plane and derive it geometrically and algebraically.

1) Show that this inequality $2 \leq |z-i| \leq 4$ describes an annulus.
Find its center (a complex number)

Find its area (use pi for π)

2) Show that this equality $|z-4i|+|z+4i|=10$ describes an ellipse.
Find its center (a complex number)

Find its larger semiaxis

3) Show that this equality $\mathrm{Im}\,\frac{1}{z}=1$ describes a circle.
Find its center (a complex number)

Find its radius

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Problem 1.2

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Problem 1.2

0.0/2.0 points (ungraded)

Let ε be arbitrary n-th rooth of unity (distinct from 1). Prove the following equality.

$$1 + 2\varepsilon + 3\varepsilon^2 + \ldots + n\varepsilon^{n-1} = \frac{n}{\varepsilon - 1}$$

In order to prove it, compute this sum in the closed form. To this end, notice that the summed series can be obtained by differentiation of a more usual geometric series. What is the result?

$$1 + 2\varepsilon + 3\varepsilon^2 + \ldots + n\varepsilon^{n-1} = \frac{(n(\boxed{}) - 1)\varepsilon^{\boxed{}} -}{(\boxed{})^{\boxed{}}}$$

	$\varepsilon - 1$	$\varepsilon + 1$	n	2	1		

Notice that from this result the statment of the problem follows immediately.

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Problem 1.3

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Problem 1.3

0.0/4.0 points (ungraded)

(i) Determine the image of a line $\operatorname{Im} z = 1$ under the map $z \to w(z) = z^3 + 3z - i$.

This image can be characterized by the following function:

$$\boxed{} = \boxed{} + 3|\boxed{}|^{\boxed{}}$$

1	2	-1	-2	$\operatorname{Im} \omega$	$\operatorname{Re} \omega$	

(ii) Determine the image of a circle $|z - i| = 1$ under the map $z \to w(z) = \frac{1}{z-2i}$.

Show that this image is a straight line on the complex plane. Derive the equation describing this straight line.

$$\operatorname{Im} \omega = \boxed{} + \boxed{} \operatorname{Re} \omega$$

0	1	-1	$1/2$	$-1/2$	

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Solutions
It would be helpful if their could be a full solution available for the problems please. 4

Problem 1.4

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Problem 1.4

0.0/2.0 points (ungraded)

Do the following functions of $z = x + iy$ satisfy Cauchy-Riemann conditions?

Check all which satisfy.

- [ ] $w(z) = x^2 + y^2$
- [ ] $w(z) = x^2 - y^2 + 2ixy$
- [ ] $w(z) = \frac{1}{x+iy}$

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Problem 1.5

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Homework due Oct 24, 2020 20:00 EDT

Problem 1.5

0.0/4.0 points (graded)

Recover an analytic function $f\left(z=x+iy\right)$ satisfying the following equations.

1)$|f|=e^{r^2\cos 2\varphi}$ with $z=re^{i\varphi}$

Use i for complex unity, sqrt(#) for $\sqrt{\#}$, #^2 for $\#^2$ and e^(#) for the exponential function.

$f\left(z\right)=$

2)$\mathrm{Arg} f=xy$

Use i for complex unity, sqrt(#) for $\sqrt{\#}$, #^2 for $\#^2$ and e^(#) for the exponential function.

$f\left(z\right)=$

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Problem 1.6

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Problem 1.6

0.0/4.0 points (ungraded)
Find all harmonic functions $f(z = x + iy) = u(x, y) + iv(x, y)$ satisfying the requirements below. You need to find the general form of such $f(z)$ with two arbitrary constants a (complex) and b (real) ($f(z)$ should be zero at $a = b = 0$). Use i for complex unity, sqrt(#) for $\sqrt{\#}$, #^2 for $\#^2$, e^(#) for the exponential function and ln(#) for logarithmic function.

1)$u = \varphi(x^2 - y^2)$

.
$f(z) =$

2)$u = \varphi\left(\frac{y}{x}\right)$

.
$f(z) =$

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? Problem 1.6 Part 2) 4

? Where is Problem 5? 2
This problem is numbered 6, while the previous was numbered 4. Problem 5 where it is?

Problem 1.7

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Problem 1.7

0.0/2.0 points (ungraded)
Calculate the following integrals along the unit circle $\mathcal{C}$, centered at $z = 0$. Use i for complex unity and pi for π.

1) $\int_{\mathcal{C}} z dz.$

1) $\int_{\mathcal{C}} z^* dz.$

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Problem 1.7
I think the second integral should be numbered 2). And do we need to know whether path is clockwise or anticlockwise? 2

Problem 1.8

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Homework due Oct 24, 2020 20:00 EDT

Problem 1.8

0.0/4.0 points (graded)

Calculate the integral

$$\int_C \frac{ydx - xdy}{x^2 + y^2}$$

along the unit circle $\mathcal{C}$ -- counterclockwise -- in the complex plane $z = x + iy$, centered at different points. Use i for complex unity and pi for π.

1) Circle centered at $z = 0$.

2) Circle centered at $z = 2$

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Problem 1.9

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Homework due Oct 24, 2020 20:00 EDT

Problem 1.9

0.0/4.0 points (graded)

Consider a function of a natural number n defined by the following integral.

$$p\left(n\right) = \frac{1}{2\pi i}\int_{\mathcal{C}} dz z^{-1-n}\prod_{k=1}^{\infty}\frac{1}{1-z^{k}},$$

where $\mathcal{C}$ is a circle of a radius smaller than unity and show that $p\left(n\right)$ is natural number.

Evaluate p(1)

Evaluate p(4)

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Last dedicated problem in week 1

I got the answer by using residue theorem. However, I guess there might be other methods, since we are only at week 1! I tried to do th... 8

Problem 1.10

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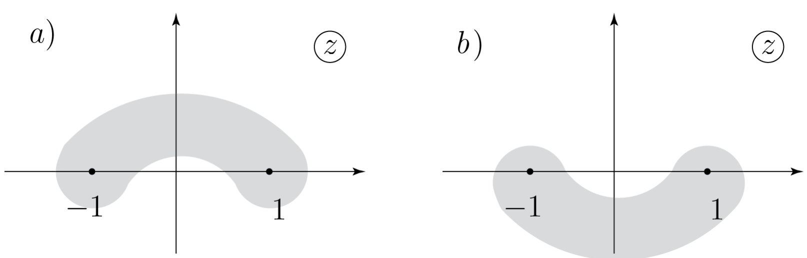
Problem 1.10

0.0/4.0 points (ungraded)

Consider the function $y(z)$ satisfying $y(1) = 0$ and $y'(z) = \frac{1}{2z}$ in the region $\mathcal{D}$. Evaluate $y(-1)$ for two cases of the region $\mathcal{D}$ shown on the Figure. Use i for complex unity and pi for π.



Regions (a) and (b).

Region (a)

Region (b)

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? Missing problems 1.8 and 1.9? 3
From Problem 1.7 to Problem 1.10? Where are the rest: 1.8 and 1.9?

How to tell which of the computed values corresponds to which of the two regions? 8