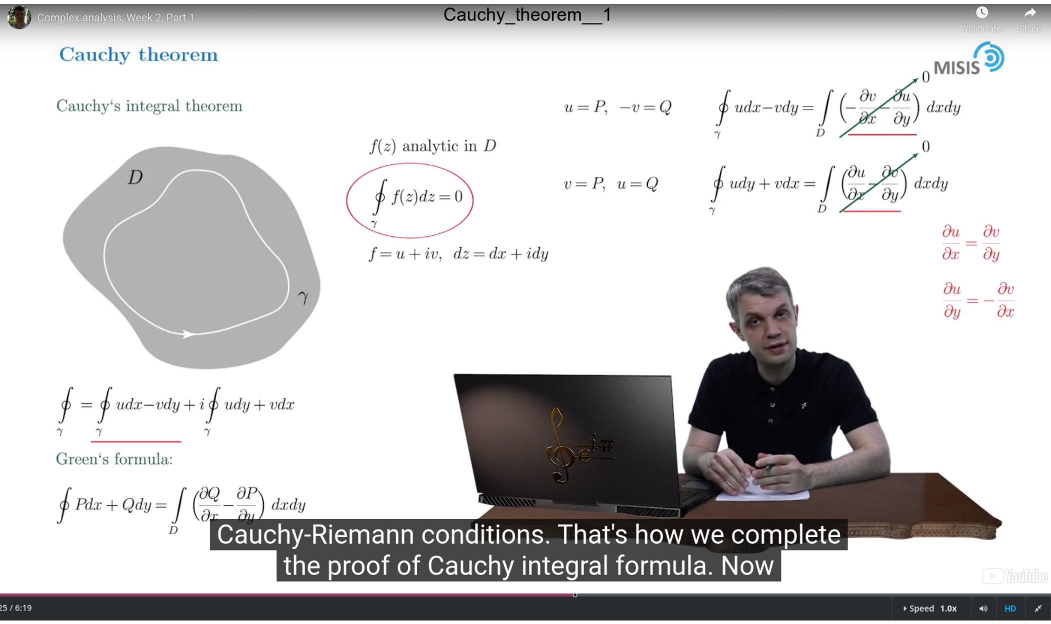
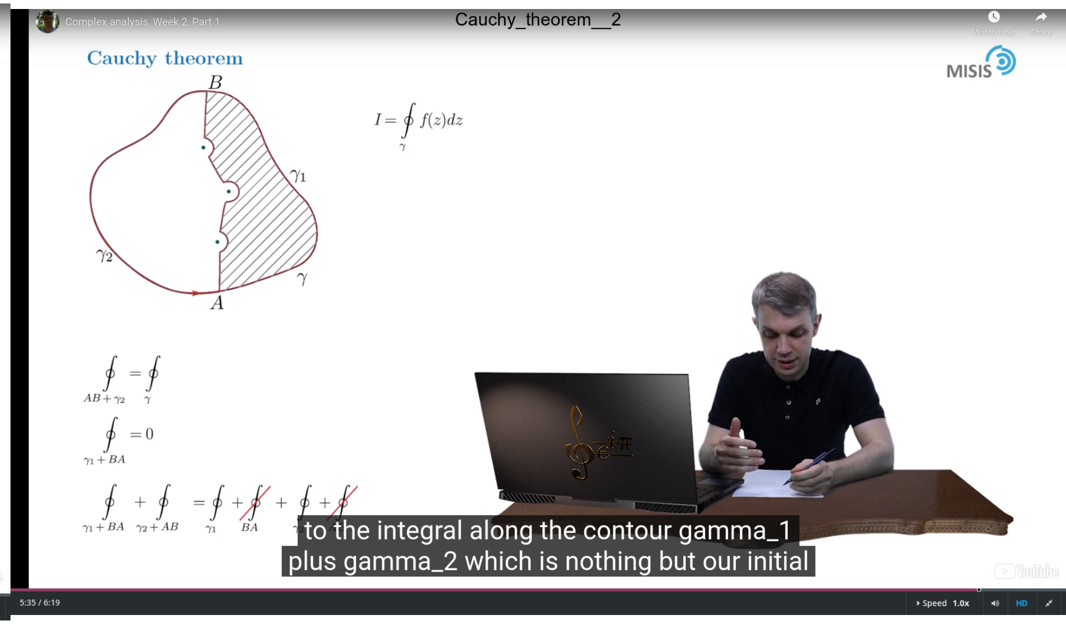
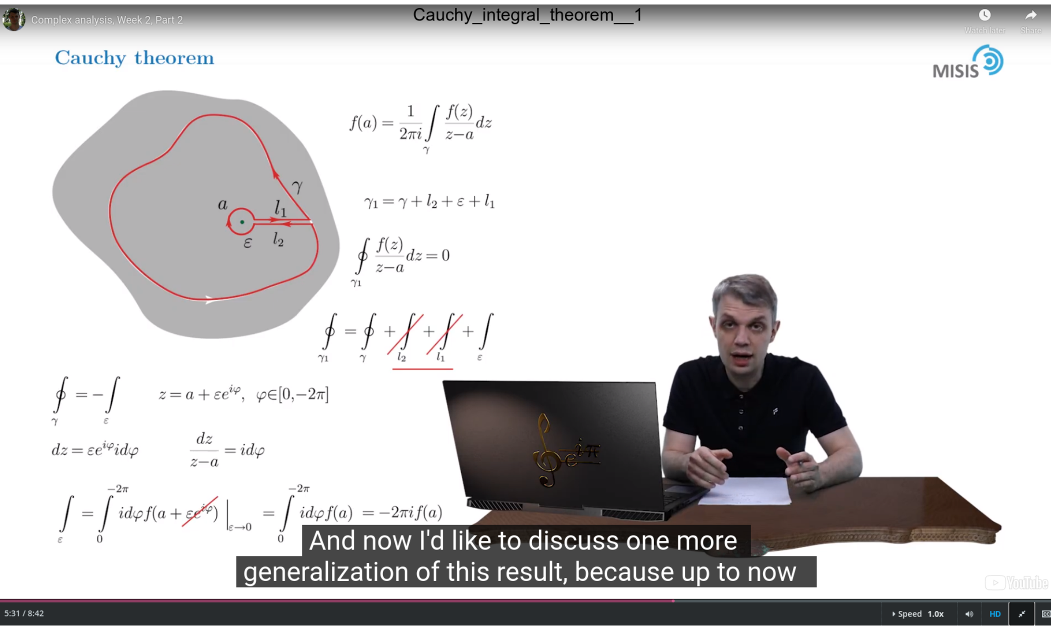
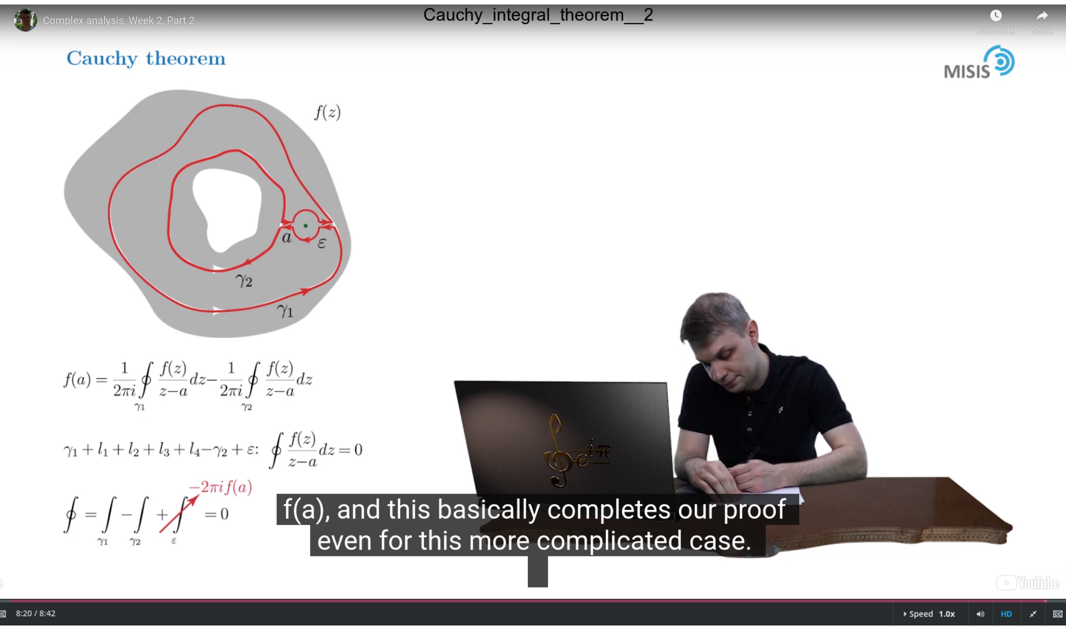
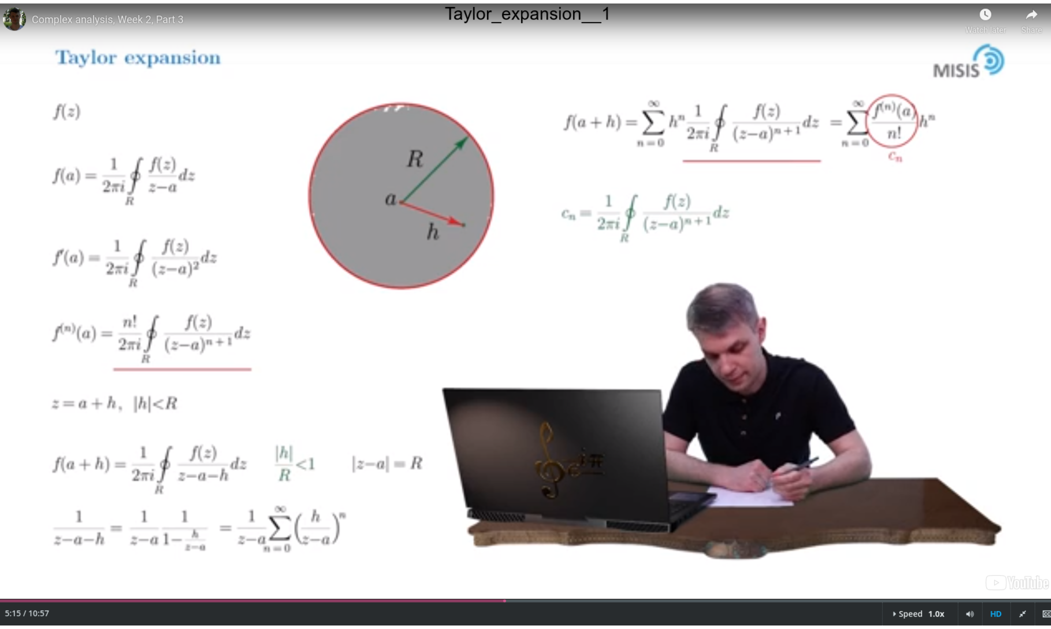

Complex analysis, Week 2, Part 1
Cauchy_theorem__1
Cauchy theorem
MISIS
Cauchy's integral theorem
$f(z)$ analytic in D
$\oint_\gamma f(z)dz = 0$
$f = u + iv, \ dz = dx + idy$
$u = P, \ -v = Q$
$v = P, \ u = Q$
$\oint_\gamma udx - vdy = \int_D \left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dxdy$
$\oint_\gamma udy + vdx = \int_D \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dxdy$
0
0
$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
D
γ
$\oint_\gamma = \oint_\gamma udx - vdy + i\oint_\gamma udy + vdx$
Green's formula:
$\oint Pdx + Qdy = \int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$
Cauchy-Riemann conditions. That's how we complete
the proof of Cauchy integral formula. Now
Speed 1.0x
HD

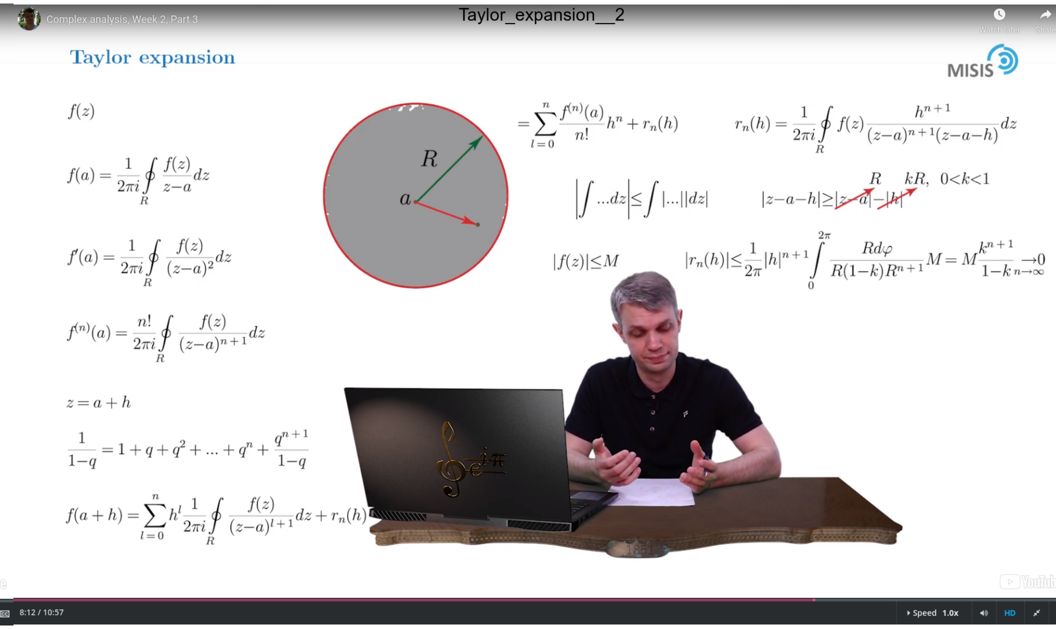

Complex analysis, Week 2, Part 1
Cauchy_theorem__2
Cauchy theorem
MISIS
B
A
γ_1
γ_2
γ
$I = \oint_\gamma f(z)dz$
$\oint_{AB+\gamma_2} = \oint_\gamma$
$\oint_{\gamma_1 + BA} = 0$
$\oint_{\gamma_1+BA} + \oint_{\gamma_2+AB} = \oint_{\gamma_1} + \cancel{\oint_{BA}} + \oint + \cancel{\oint}$
to the integral along the contour gamma_1
plus gamma_2 which is nothing but our initial
5:35 / 6:19
Speed 1.0x
HD


Complex analysis, Week 2, Part 2
Cauchy_integral_theorem__1
Cauchy theorem
MISIS
a
ε
l_1
l_2
γ
$f(a) = \frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z-a}dz$
$\gamma_1 = \gamma + l_2 + \varepsilon + l_1$
$\oint_{\gamma_1} \frac{f(z)}{z-a}dz = 0$
$\oint_{\gamma_1} = \oint_\gamma + \cancel{\int_{l_2}} + \cancel{\int_{l_1}} + \int_\varepsilon$
$\oint_\gamma = -\int_\varepsilon \qquad z = a + \varepsilon e^{i\varphi}, \ \varphi \in [0, -2\pi]$
$dz = \varepsilon e^{i\varphi} i d\varphi \qquad \frac{dz}{z-a} = id\varphi$
$\int_\varepsilon = \int_0^{-2\pi} id\varphi f(a + \cancel{\varepsilon e^{i\varphi}})\Big|_{\varepsilon\to 0} = \int_0^{-2\pi} id\varphi f(a) = -2\pi i f(a)$
And now I'd like to discuss one more
generalization of this result, because up to now
5:31 / 8:42
Speed 1.0x
HD

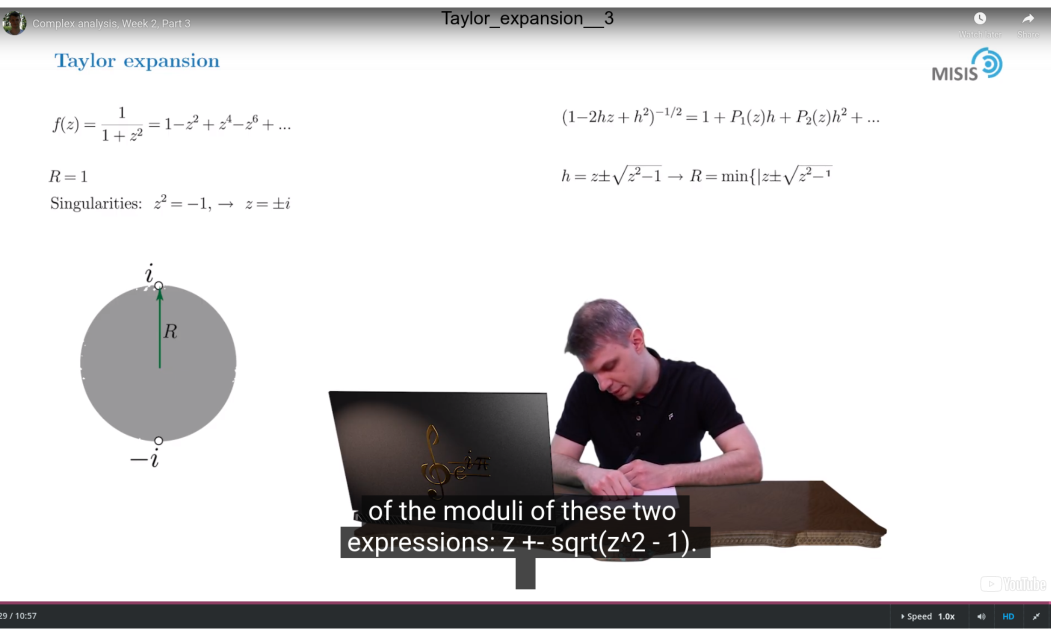

Complex analysis, Week 2, Part 2
Cauchy_integral_theorem__2
Cauchy theorem
MISIS
$f(z)$
a
ε
γ_2
γ_1
$f(a) = \frac{1}{2\pi i}\oint_{\gamma_1} \frac{f(z)}{z-a}dz - \frac{1}{2\pi i}\oint_{\gamma_2} \frac{f(z)}{z-a}dz$
$\gamma_1 + l_1 + l_2 + l_3 + l_4 - \gamma_2 + \varepsilon: \ \oint \frac{f(z)}{z-a}dz = 0$
$\oint = \int_{\gamma_1} - \int_{\gamma_2} + \int_\varepsilon = 0$
$-2\pi i f(a)$
f(a), and this basically completes our proof
even for this more complicated case.
8:20 / 8:42
Speed 1.0x
HD



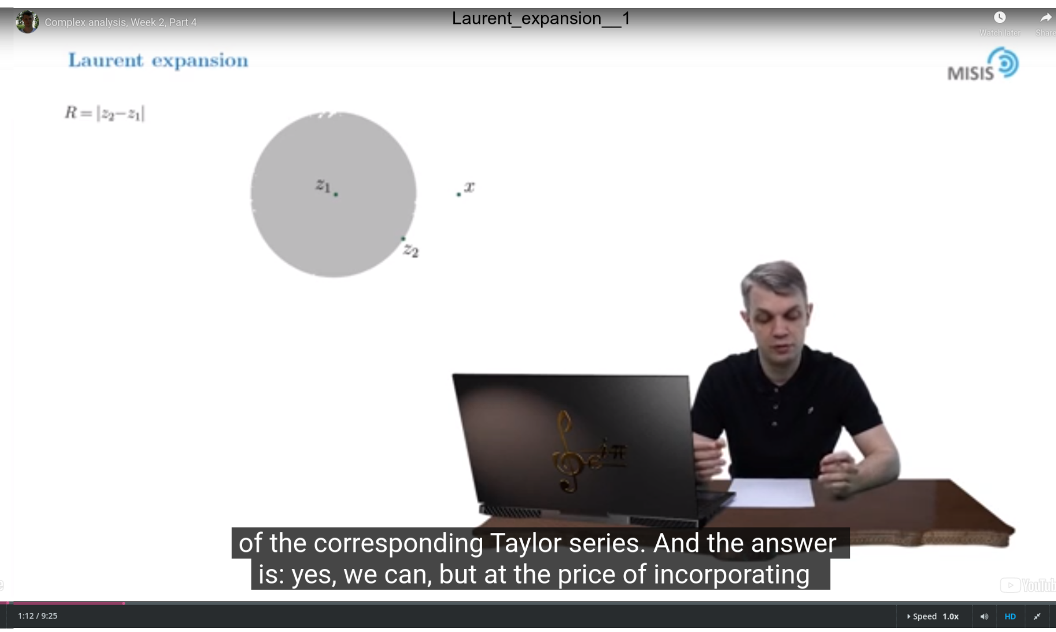
Taylor_expansion__1



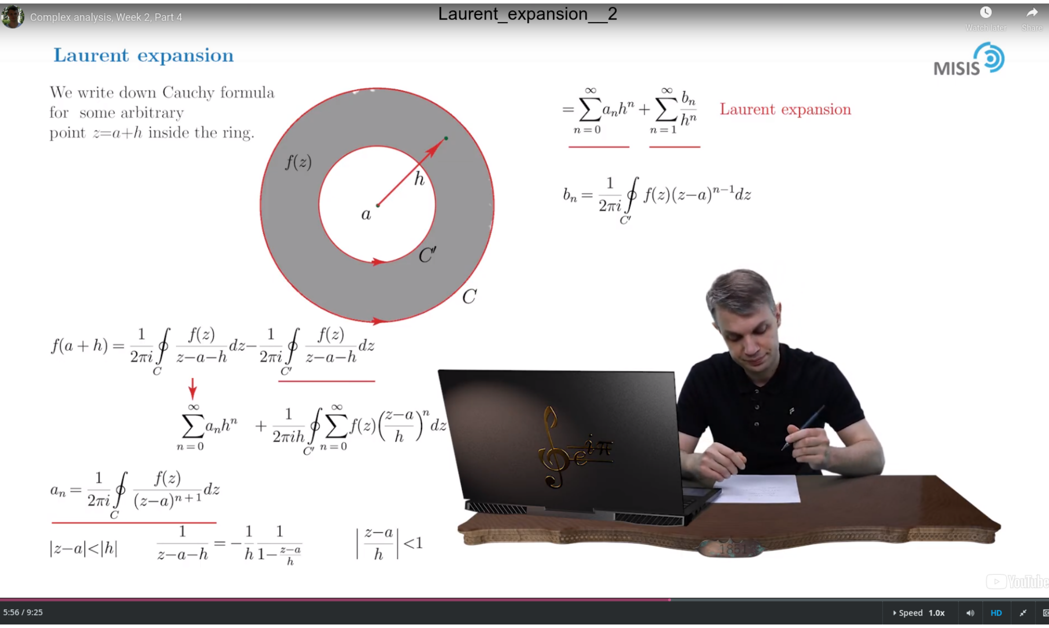
Taylor_expansion__2



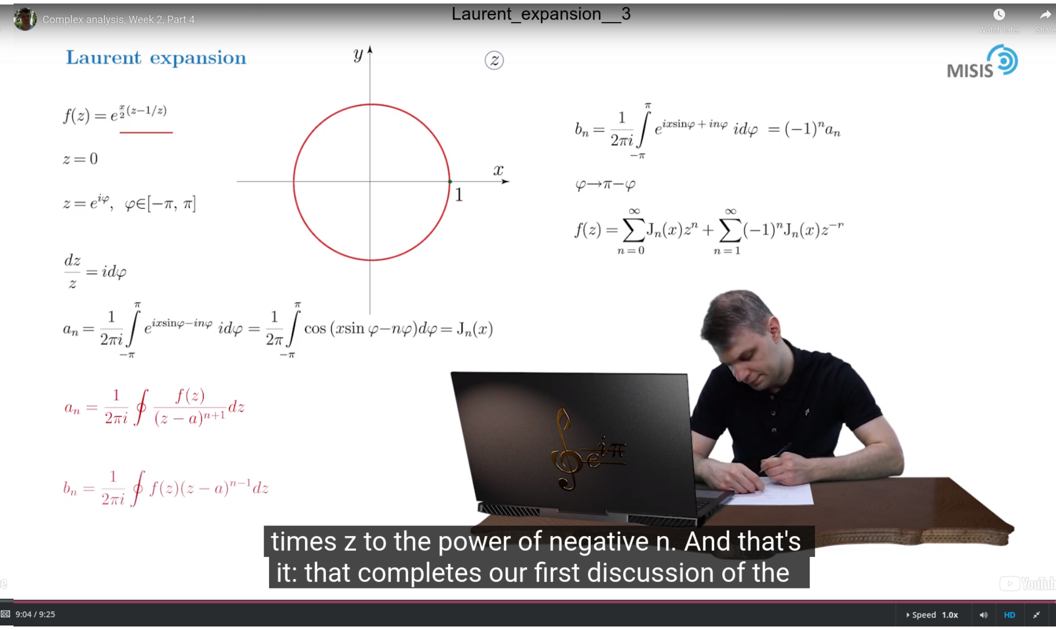
Taylor_expansion__3



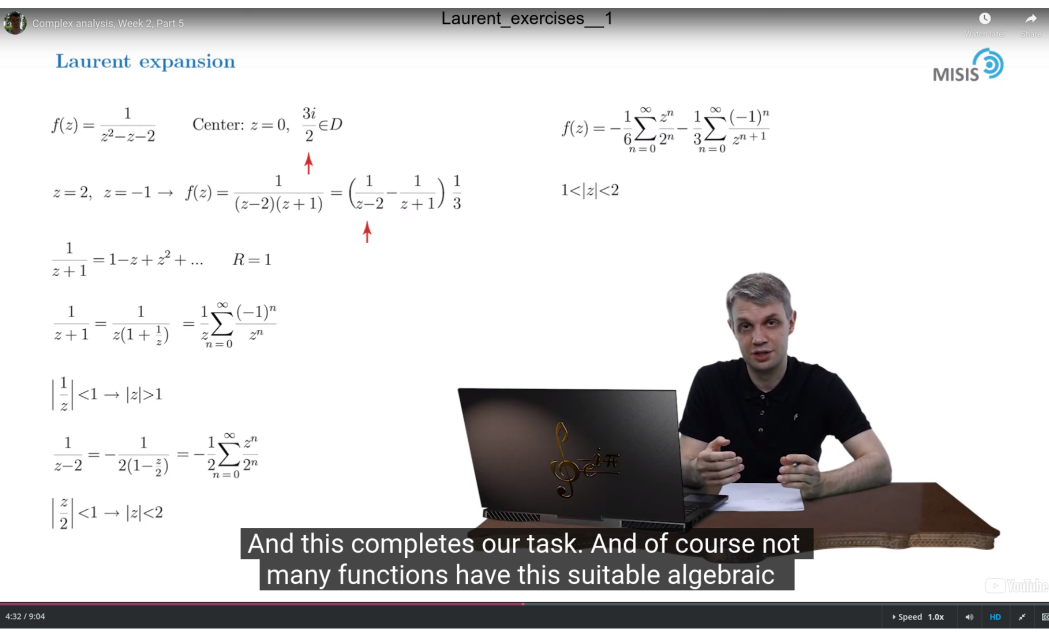
Laurent_expansion__1



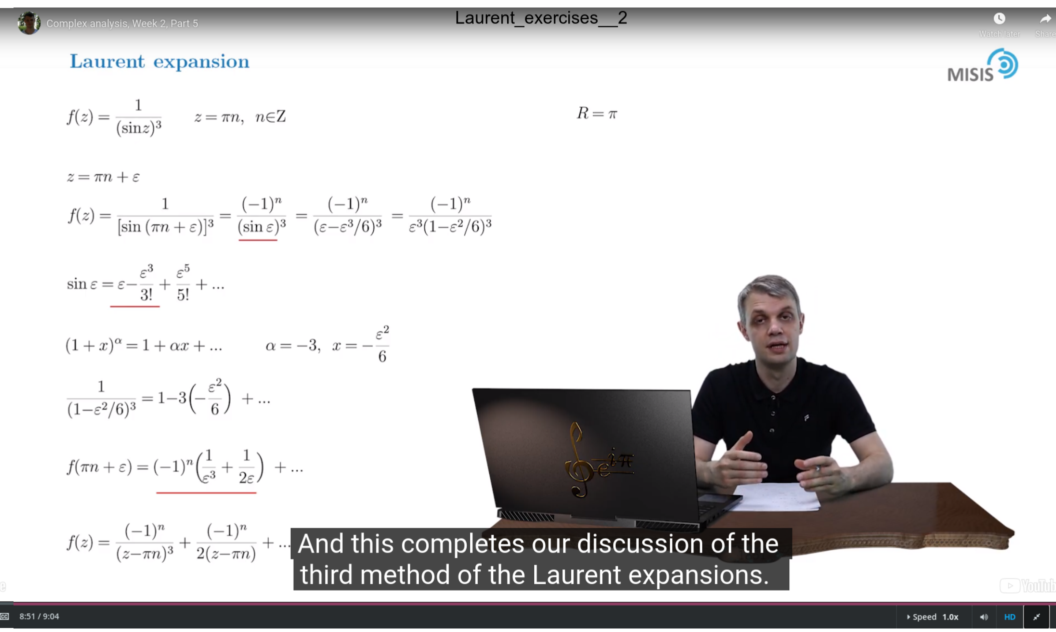
Complex analysis. Week 2. Part 4
Laurent_expansion__2
Laurent expansion
MISIS
We write down Cauchy formula for some arbitrary point $z = a + h$ inside the ring.
$f(z)$
a
h
C'
C
$= \sum_{n=0}^{\infty} a_n h^n + \sum_{n=1}^{\infty} \frac{b_n}{h^n}$
Laurent expansion
$b_n = \frac{1}{2\pi i} \oint_{C'} f(z)(z-a)^{n-1} dz$
$f(a+h) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a-h} dz - \frac{1}{2\pi i} \oint_{C'} \frac{f(z)}{z-a-h} dz$
$\sum_{n=0}^{\infty} a_n h^n \quad + \frac{1}{2\pi i h} \oint_{C'} \sum_{n=0}^{\infty} f(z) \left(\frac{z-a}{h}\right)^n dz$
$a_n = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz$
$|z-a| < |h| \qquad \frac{1}{z-a-h} = -\frac{1}{h} \frac{1}{1 - \frac{z-a}{h}} \qquad \left|\frac{z-a}{h}\right| < 1$
5:56 / 9:25
Speed 1.0x
HD



Complex analysis. Week 2. Part 4
Laurent_expansion__3
Laurent expansion
MISIS
y
z
x
1
$f(z) = e^{\frac{x}{2}(z - 1/z)}$
$z = 0$
$z = e^{i\varphi}, \ \varphi \in [-\pi, \pi]$
$\frac{dz}{z} = i d\varphi$
$a_n = \frac{1}{2\pi i} \int_{-\pi}^{\pi} e^{ix\sin\varphi - in\varphi} i d\varphi = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(x\sin\varphi - n\varphi) d\varphi = J_n(x)$
$a_n = \frac{1}{2\pi i} \oint \frac{f(z)}{(z-a)^{n+1}} dz$
$b_n = \frac{1}{2\pi i} \oint f(z)(z-a)^{n-1} dz$
$b_n = \frac{1}{2\pi i} \int_{-\pi}^{\pi} e^{ix\sin\varphi + in\varphi} i d\varphi = (-1)^n a_n$
$\varphi \to \pi - \varphi$
$f(z) = \sum_{n=0}^{\infty} J_n(x) z^n + \sum_{n=1}^{\infty} (-1)^n J_n(x) z^{-n}$
times z to the power of negative n. And that's
it: that completes our first discussion of the
9:04 / 9:25
Speed 1.0x
HD

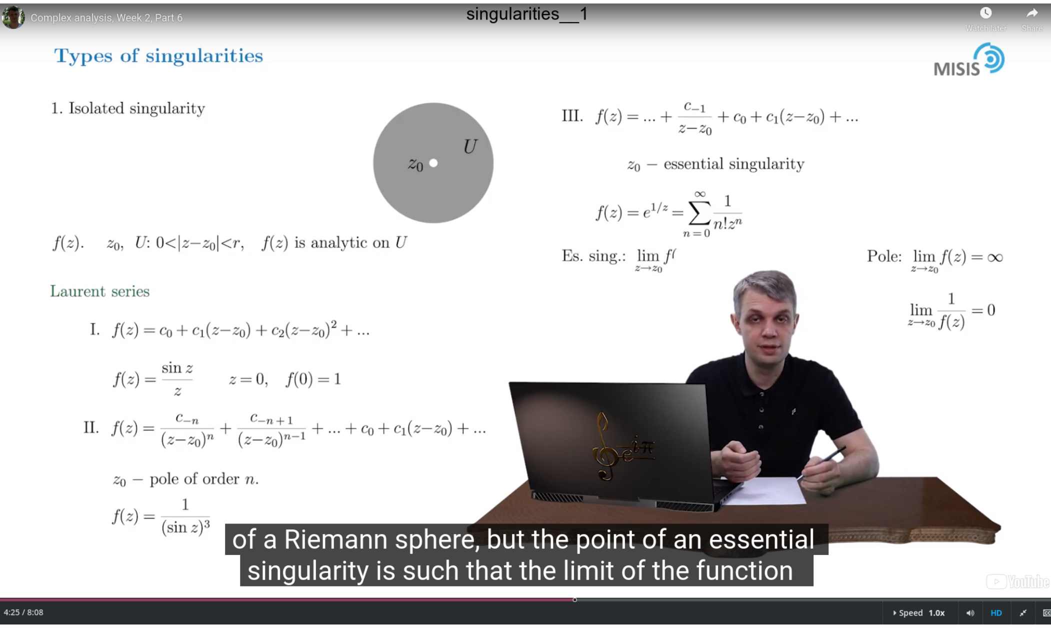


Complex analysis. Week 2. Part 5
Laurent_exercises__1
Laurent expansion
MISIS
$f(z) = \frac{1}{z^2 - z - 2}$
Center: $z = 0, \ \frac{3i}{2} \in D$
$z = 2, \ z = -1 \to f(z) = \frac{1}{(z-2)(z+1)} = \left(\frac{1}{z-2} - \frac{1}{z+1}\right) \frac{1}{3}$
$\frac{1}{z+1} = 1 - z + z^2 + \ldots \qquad R = 1$
$\frac{1}{z+1} = \frac{1}{z(1+\frac{1}{z})} = \frac{1}{z} \sum_{n=0}^{\infty} \frac{(-1)^n}{z^n}$
$\left|\frac{1}{z}\right| < 1 \to |z| > 1$
$\frac{1}{z-2} = -\frac{1}{2(1-\frac{z}{2})} = -\frac{1}{2} \sum_{n=0}^{\infty} \frac{z^n}{2^n}$
$\left|\frac{z}{2}\right| < 1 \to |z| < 2$
$f(z) = -\frac{1}{6} \sum_{n=0}^{\infty} \frac{z^n}{2^n} - \frac{1}{3} \sum_{n=0}^{\infty} \frac{(-1)^n}{z^{n+1}}$
$1 < |z| < 2$
And this completes our task. And of course not
many functions have this suitable algebraic
4:32 / 9:04
Speed 1.0x
HD

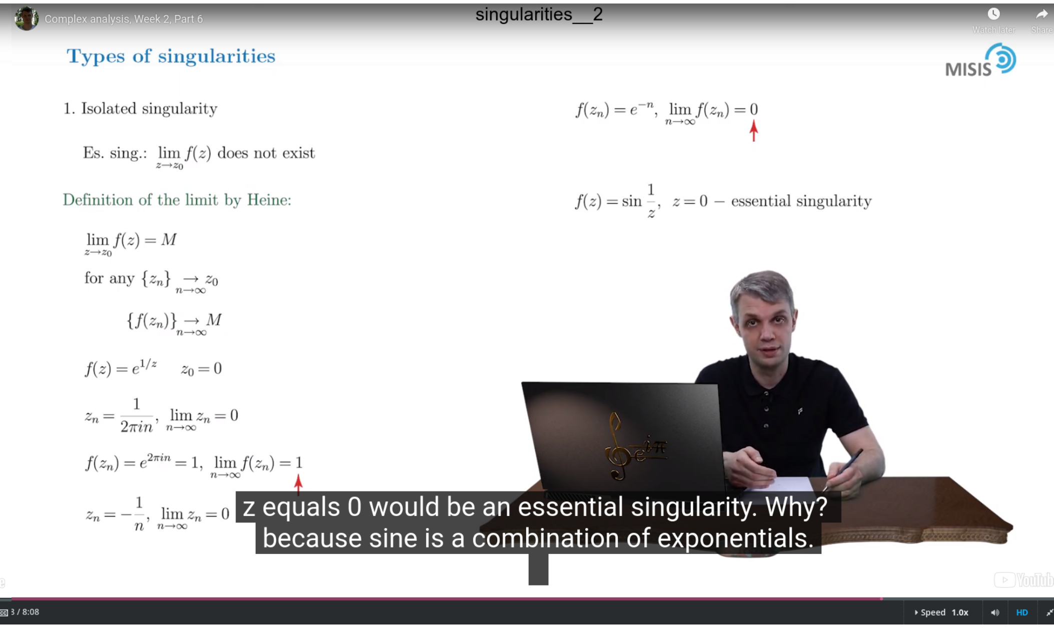


Complex analysis. Week 2. Part 5
Laurent_exercises__2
Laurent expansion
MISIS
$f(z) = \frac{1}{(\sin z)^3} \qquad z = \pi n, \ n \in Z$
$R = \pi$
$z = \pi n + \varepsilon$
$f(z) = \frac{1}{[\sin(\pi n + \varepsilon)]^3} = \frac{(-1)^n}{(\sin\varepsilon)^3} = \frac{(-1)^n}{(\varepsilon - \varepsilon^3/6)^3} = \frac{(-1)^n}{\varepsilon^3(1 - \varepsilon^2/6)^3}$
$\sin\varepsilon = \varepsilon - \frac{\varepsilon^3}{3!} + \frac{\varepsilon^5}{5!} + \ldots$
$(1+x)^\alpha = 1 + \alpha x + \ldots \qquad \alpha = -3, \ x = -\frac{\varepsilon^2}{6}$
$\frac{1}{(1-\varepsilon^2/6)^3} = 1 - 3\left(-\frac{\varepsilon^2}{6}\right) + \ldots$
$f(\pi n + \varepsilon) = (-1)^n \left(\frac{1}{\varepsilon^3} + \frac{1}{2\varepsilon}\right) + \ldots$
$f(z) = \frac{(-1)^n}{(z - \pi n)^3} + \frac{(-1)^n}{2(z - \pi n)} + \ldots$
And this completes our discussion of the
third method of the Laurent expansions.
8:51 / 9:04
Speed 1.0x
HD

singularities__1



singularities__2



singularities__3