A. (Video 1.2)

B. Loci of the complex equations (Video 1.2)

Let us consider another example. Here we have to describe the complex plane set defined by the inequality:

$$|z+3| < |z+4i|. \tag{1}$$

The simplest way to approach this kind of problem is to establish the loci of the equation, defining the boundary of the desired set:

$$|z+3| = |z+4i|. \tag{2}$$

As in the previous example, let us interpret the complex numbers $z+3$ and $z+4i$ as vectors pointing from $A=-3$ and $B=-4i$ to z, correspondingly. The lengths of the vectors $z+3$ and $z+4i$ coincide if and only if z belongs to a line l, perpendicular to AB and crossing this segment in a middle. The vector AB is described by components $(3,-4)$ and the normal to this vector has components $(1,3/4)$. The line l intersects AB at the point $(x_0,y_0)=(-3/2,-2)$. Hence, l has the following equation:

$$y = \frac{3}{4}(x-x_0)+y_0 = \frac{3}{4}\left(x+\frac{3}{2}\right)-2 = \frac{3}{4}x-\frac{7}{8}. \tag{3}$$

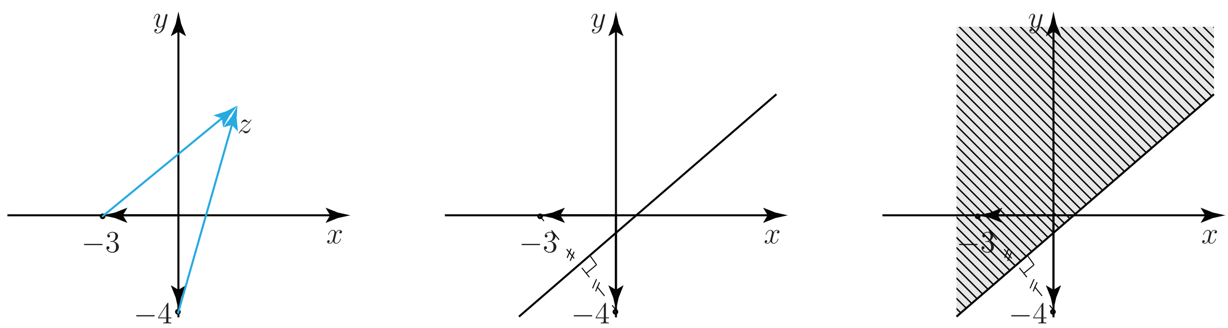

FIG. 1: Graphical solution to the equation $|z+3| = |z+4i|$

Again, this problem admits an explicit analytic solution:

$$|z+3| = \sqrt{(x+3)^2+y^2} = |z+4i| = \sqrt{x^2+(y+4)^2} \Rightarrow (x+1)^2+y^2 = 1$$

Squaring both parts of the equation, we establish the same result Eq. (3).

After determining the domain, we can easily establish that the region we are looking for lies to the right of the boundary.