UNIVERSIDADE FEDERAL DO ABC

Tabela de Derivadas, Integrais e Identidades Trigonométricas

Derivadas

Regras de Derivação

- (cf(x))' = cf'(x)
- Derivada da Soma

$$(f(x) + g(x))' = f'(x) + g'(x)$$

• Derivada do Produto

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

• Derivada do Quociente

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

• Regra da Cadeia

$$(f(g(x))' = (f'(g(x))g'(x))$$

Funções Simples

- $\frac{d}{dx}c = 0$
- $\frac{d}{dx}x = 1$
- $\frac{d}{dx}cx = c$
- $\frac{d}{dx}x^c = cx^{c-1}$
- $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right) = -x^{-2} = -\frac{1}{x^2}$
- $\bullet \ \ \tfrac{d}{dx} \left(\tfrac{1}{x^c} \right) = \tfrac{d}{dx} \left(x^{-c} \right) = \tfrac{c}{x^{c+1}}$
- $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

Funções Exponenciais e Logarítmicas

- $\frac{\mathrm{d}}{\mathrm{d}x}e^{x} = e^{x}$
- $\frac{d}{dx} \ln(x) = \frac{1}{x}$
- $\frac{d}{dx}a^x = a^x \ln(a)$

Funções Trigonométricas

- $\frac{d}{dx} \operatorname{sen} x = \cos x$
- $\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\mathrm{sen} x$,
- $\frac{d}{dx}$ $tg x = sec^2 x$
- $\frac{d}{dx} \sec x = tg x \sec x$
- $\frac{d}{dx} \cot g x = -\csc^2 x$
- $\frac{d}{dx}$ cossec x = -cossec x cotg x

Funções Trigonométricas Inversas

- $\frac{d}{dx}$ arcsen $x = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} \arccos x = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}$ arctg $x = \frac{1}{1+x^2}$
- $\frac{d}{dx} \operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}$ arccotg $x = \frac{-1}{1+x^2}$
- $\frac{d}{dx}$ arccossec $x = \frac{-1}{|x|\sqrt{x^2-1}}$

Funções Hiperbólicas

- $\frac{d}{dx}$ senh $x = \cosh x = \frac{e^x + e^{-x}}{2}$
- $\frac{d}{dx} \cosh x = \operatorname{senh} x = \frac{e^x e^{-x}}{2}$
- $\frac{d}{dx} \operatorname{tgh} x = \operatorname{sech}^2 x$
- $\frac{d}{dx}$ sech x = -tgh x sech x
- $\frac{d}{dx}$ cotgh $x = -\operatorname{cossech}^2 x$

Funções Hiperbólicas Inversas

- $\frac{d}{dx}$ csch x = coth x cossech x
- $\frac{d}{dx}$ arcsenh $x = \frac{1}{\sqrt{x^2 + 1}}$
- $\frac{d}{dx}$ arccosh $x = \frac{1}{\sqrt{x^2 1}}$
- $\frac{d}{dx}$ arctgh $x = \frac{1}{1-x^2}$
- $\frac{d}{dx}$ arcsech $x = \frac{-1}{x\sqrt{1-x^2}}$
- $\frac{d}{dx}$ arccoth $x = \frac{1}{1-x^2}$
- $\frac{d}{dx}$ arccossech $x = \frac{-1}{|x|\sqrt{1+x^2}}$

Integrais

Regras de Integração

• $\int cf(x) dx = c \int f(x) dx$

• $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$

• $\int f'(x)g(x) dx = f(x)g(x) - \int f(x)g'(x) dx$

Funções Racionais

• $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ para $n \neq -1$

 $\bullet \int \frac{1}{x} \, \mathrm{d}x = \ln|x| + c$

• $\int \frac{du}{1+u^2} = \operatorname{arctg} u + c$

• $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan(x/a) + c$

Funções Logarítmicas

• $\int \ln x \, dx = x \ln x - x + c$

• $\int \log_{\alpha} x \, dx = x \log_{\alpha} x - \frac{x}{\ln \alpha} + c$

Funções Irracionais

• $\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + c$

• $\int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec} u + c$

• $\int \frac{du}{\sqrt{1+u^2}} = \operatorname{arcsenh} u + c$ $= \ln|u + \sqrt{u^2 + 1}| + c$

 $\oint \frac{du}{\sqrt{1 - u^2}} = \operatorname{arccosh} u + c$ $= \ln|u + \sqrt{u^2 - 1}| + c$

• $\int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{arcsech} |u| + c$

• $\int \frac{du}{u\sqrt{1+u^2}} = -\operatorname{arccosech} |u| + c$

• $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \frac{x}{a} + c$

 $\bullet \int \frac{-1}{\sqrt{a^2 - x^2}} \, dx = \arccos \frac{x}{a} + c$

Funções Trigonométricas

• $\int \cos x \, dx = \sin x + c$

• $\int \operatorname{sen} x \, \mathrm{d}x = -\cos x + c$

• $\int \operatorname{tg} x \, \mathrm{d}x = \ln|\operatorname{sec} x| + c$

• $\int \csc x \, dx = \ln|\csc x - \cot x| + c$

• $\int \sec x \, dx = \ln |\sec x + \operatorname{tg} x| + c$

• $\int \cot x \, dx = \ln|\sin x| + c$

• $\int \sec x \, \operatorname{tg} x \, \mathrm{d}x = \sec x + c$

• $\int \csc x \cot x \, dx = -\csc x + c$

• $\int \sec^2 x \, dx = \operatorname{tg} x + c$

• $\int \csc^2 x \, dx = -\cot x + c$

• $\int \operatorname{sen}^2 x \, \mathrm{d}x = \frac{1}{2} (x - \operatorname{sen} x \cos x) + c$

• $\int \cos^2 x \, dx = \frac{1}{2}(x + \sin x \cos x) + c$

Funções Hiperbólicas

• $\int \sinh x \, dx = \cosh x + c$

• $\int \cosh x \, dx = \sinh x + c$

• $\int tgh x dx = ln(cosh x) + c$

• $\int \operatorname{csch} x \, \mathrm{d}x = \ln \left| \operatorname{tgh} \frac{x}{2} \right| + c$

• $\int \operatorname{sech} x \, dx = \operatorname{arctg}(\sinh x) + c$

• $\int \coth x \, dx = \ln |\sinh x| + c$

Identidades Trigonométricas

1.
$$sen(90^{\circ} - \theta) = cos \theta$$

2.
$$cos(90^{\circ} - \theta) = sen \theta$$

3.
$$\frac{\sin \theta}{\cos \theta} = \operatorname{tg} \theta$$

4.
$$sen^2 \theta + cos^2 \theta = 1$$

$$5. \sec^2 \theta - tg^2 \theta = 1$$

6.
$$\csc^2 \theta - \cot^2 \theta = 1$$

7.
$$sen 2\theta = 2 sen \theta cos \theta$$

8.
$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$

9.
$$sen 2\theta = 2 sen \theta cos \theta$$

10.
$$sen(\alpha \pm \beta) = sen \alpha cos \beta \pm cos \alpha sen \beta$$

11.
$$cos(\alpha \pm \beta) = cos \alpha cos \beta \mp sen \alpha sen \beta$$

12.
$$tg(\alpha \pm \beta) = \frac{tg \alpha \pm tg \beta}{1 \mp tg \alpha tg \beta}$$

13.
$$\operatorname{sen} \alpha \pm \operatorname{sen} \beta = 2 \operatorname{sen} \frac{1}{2} (\alpha \pm \beta) \cos \frac{1}{2} (\alpha \pm \beta)$$

14.
$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2} (\alpha + \beta) \cos \frac{1}{2} (\alpha - \beta)$$

15.
$$\cos \alpha - \cos \beta = 2 \sin \frac{1}{2} (\alpha + \beta) \sin \frac{1}{2} (\alpha - \beta)$$