1 Meshes with Curved Surfaces

The complicated shapes of each mesh zone create a challenge by having to solve the discretized equations for each unique mesh zone. We can avoid having to solve a unique set of equations for each mesh zone by transforming the mesh zone into the reference element. Each mesh zone will have a unique transformation but an identical set of equations to obtain the solution on the reference element.

1.1 Transformation

We set up the system of equations (Section **??**) on each individual mesh zone after we transform it to the reference element. After performing the following integrations, we map the solution back to the physical element. The bi-quadratic mapping from the reference element to the physical element, shown in Figure 1, has the following functional form

$$\begin{bmatrix} x(\rho,\kappa) \\ y(\rho,\kappa) \end{bmatrix} = \sum_{i=1}^{J_k}\sum_{j=1}^{J_k} \begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} N_i(\rho)N_j(\kappa) \tag{1}$$

where

$$N_l(\xi) = \begin{cases} (2\xi-1)(\xi-1), & l=1 \\ 4\xi(1-\xi), & l=2 \\ \xi(2\xi-1), & l=3 \end{cases} \tag{2}$$

are the quadratic basis functions that have support points at typical locations shown in the left image of Figure 1. The (x_{ij}, y_{ij}) coordinates are the locations of the support points in the physical element and are generally known. For example, the node (x_{12}, y_{12}) is the location on the physical zone that is mapped from $(\rho,\kappa) = (0, 0.5)$

on the reference element.

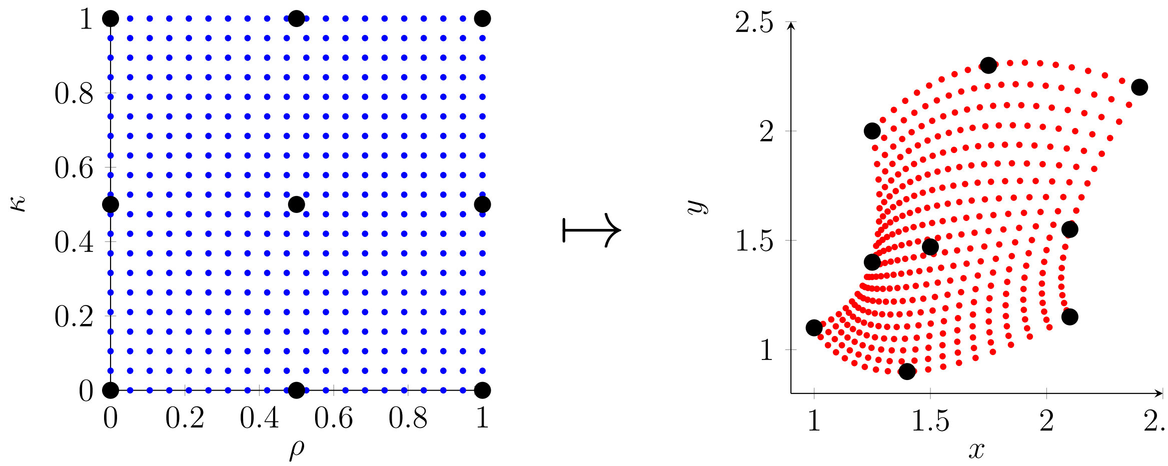

Figure 1: Example of mapping the reference element to a physical element. Switch these around to map the physical element to the reference element.

The determinant of the Jacobian of the transformation,

$$\det(J) = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} \\ \frac{\partial x}{\partial \kappa} & \frac{\partial y}{\partial \kappa} \end{vmatrix}, \tag{3}$$

is required for the volume integrations in the next section.