Stacks（2）
Stack Applications

College of Computer Science, CQU

Application 3
Evaluating RPN Expressions

- RPN?

  A notation for arithmetic expressions:

  operators are written after the operands.

- Infix notation: operators written between the operands
- Postfix notation (RPN): operators written after the operands
- Prefix notation : operators written before the operands
- Examples:

INFIX	RPN (POSTFIX)	PREFIX
A + B	A B +	+ A B
A * B + C	A B * C +	+ * A B C
A * (B + C)	A B C + *	* A + B C
A–(B –(C – D))	A B C D - - -	- A - B - C D
A – B – C – D	A B - C - D -	- - - A B C D

Application 3
Evaluating RPN Expressions

- "By hand": Underlining technique:

1. Scan the expression from left to right to find an operator.
2. Locate ("underline") the last two preceding operands and combine them using this operator.
3. Repeat until the end of the expression is reached.

- Example:

2 3 4 + 5 6 - - *

→ 2 3 4 + 5 6 - - *

→ 2 7 5 6 - - *

→ 2 7 5 6 - - *

→ 2 7 -1 - *

→ 2 7 -1 - *

→ 2 8 * → 2 8 * → 16

Application 3
Evaluating RPN Expressions

- STACK ALGORITHM

Receive:An RPN expression.

Return: A stack whose top element is the value of RPN expression(unless an error occurred).

1. Initialize an empty stack.
2. Repeat the following until the end of the expression is encountered:

a. Get next token (constant, variable, arithmetic operator) in the RPN expression.

b. If token is an operand, push it onto the stack.

If it is an operator, then

(i) Pop top two values from the stack.

If stack does not contain two items, error due to a malformed RPN Evaluation terminated

(ii) Apply the operator to these two values.

(iii) Push the resulting value back onto the stack.

3. When the end of expression encountered, its value is on top of the stack

(and, in fact, must be the only value in the stack).

Application 3
Evaluating RPN Expressions

- Unary minus causes problems

Example:

5 3 - -

→ 5 3 - -

→ 5 -3 - → 8

5 3 - -

→ 5 3 - -

→ 2 - → -2

Use a different symbol:

5 3 ~ -

5 3 - ~

Application 3
Evaluating RPN Expressions

- Example: 2 3 4 + 5 6 - - *

Push 2

Push 3

Push 4

Read +

    Pop 4, Pop 3, 3 + 4 = 7

    Push 7

Push 5

Push 6

Read –

    Pop 6, Pop 5, 5 - 6 = -1

    Push -1

Read –

    Pop -1, Pop 7, 7 - -1 = 8

    Push 8

Read *

    Pop 8, Pop 2, 2 * 8 = 16

Application 4
Converting Infix to RPN

1. Initialize an empty stack of operators.

2. While no error has occurred and end of infix expression not reached

a. Get next Token in the infix expression.

b. If Token is

(i) a left parenthesis: Push it onto the stack.

(ii) a right parenthesis:Pop and display stack elements until a left parenthesis is encountered,but do not display it. (Error if stack empty with no left parenthesis found.)

(iii) an operator:If stack empty or Token has higher priority than top stack element, push Token on stack.(Left parenthesis in stack has lower priority than operators)

Otherwise, pop and display the top stack element; then repeat the comparison of Token with new top stack item.

(iv) an operand: Display it.

3. When end of infix expression reached,pop and display stack items until stack is empty.

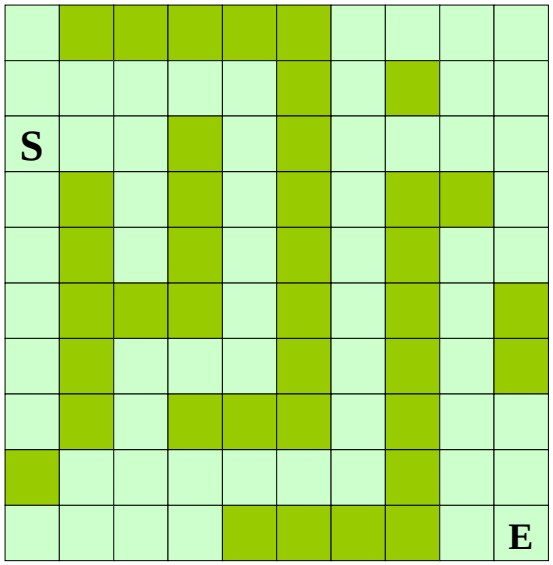
Application 4
Converting Infix to RPN

- **Example: ((A+B)*C)/(D-E)**

Push (	
Push (	
Display A	**A**
Push +	
Display B	**AB**
Read)	
Pop +, Display +, Pop (	**AB+**
Push *	
Display C	**AB+C**
Read)	
Pop *, Display *, Pop (	**AB+C* // stack now empty**
Push /	
Push (	
Display D	**AB+C*D**
Push –	
Display E	**AB+C*DE**
Read)	
Pop -, Display -, Pop (	**AB+C*DE-**
Pop /, Display /	**AB+C*DE-/**

Application 5
Maze

- A maze is a rectangular area with an entrance and an exit.
- The interior of a maze contains walls or obstacles that one cannot walk through.
- We view the maze as broken up into equal size squares, some of which are part of the walls and the others are part of the hallways. Thus they are "open".
- One of the open squares is designated as the Start position and another as the Exit position.



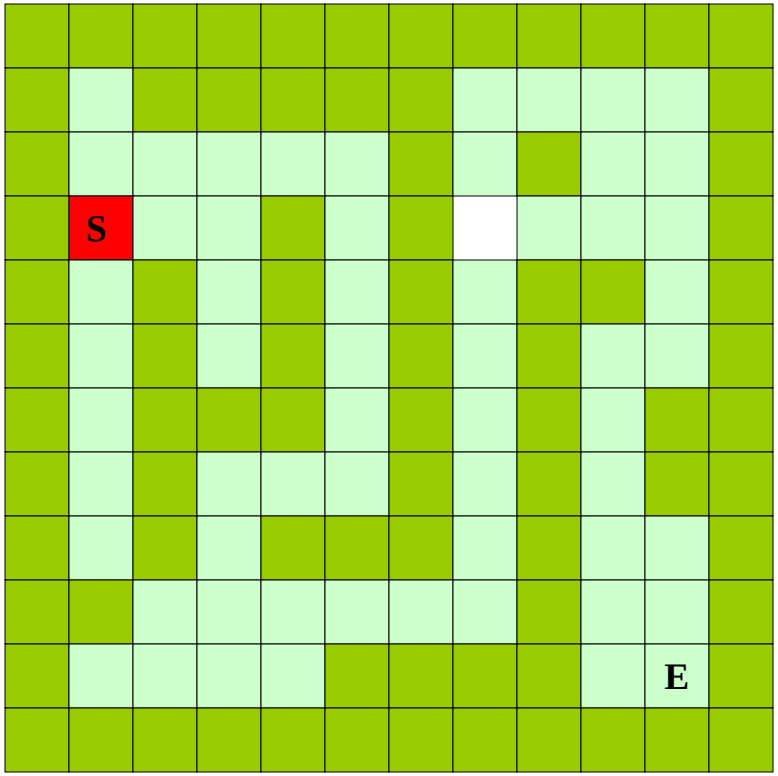
Maze M

Application 5
Maze

- **So we will use an enumerated type which will include constants OPEN and WALL.**
- **To reflect our search method for finding an exit path, we will also have a**

  third state: VISITED.
- **In traversing the maze, we can only move forward into an OPEN position .**
- **Thus a given position has 4 possible neighboring positions in the directions east, south, west and north.**

Searching the Example Maze

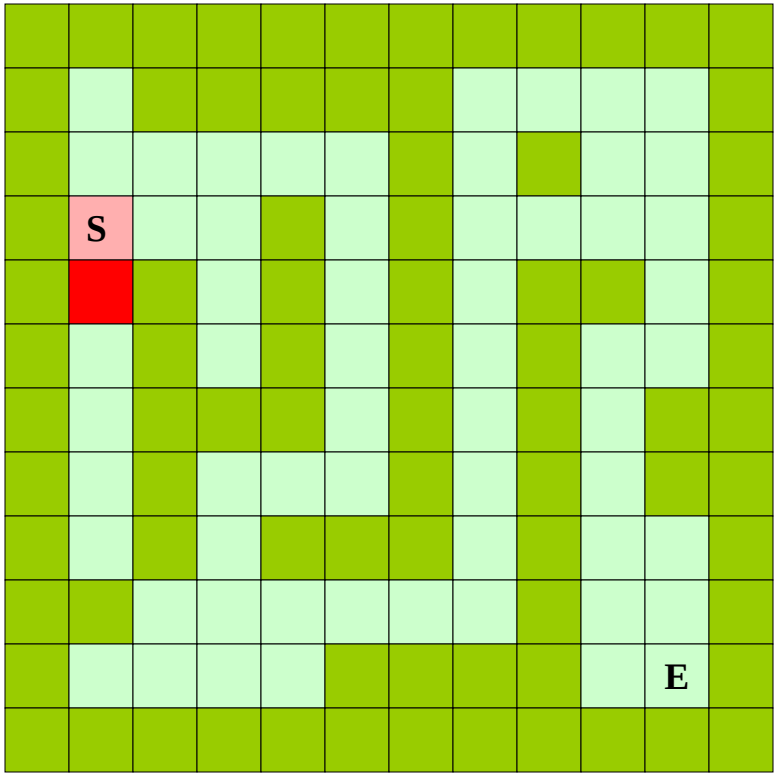


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze



Current position

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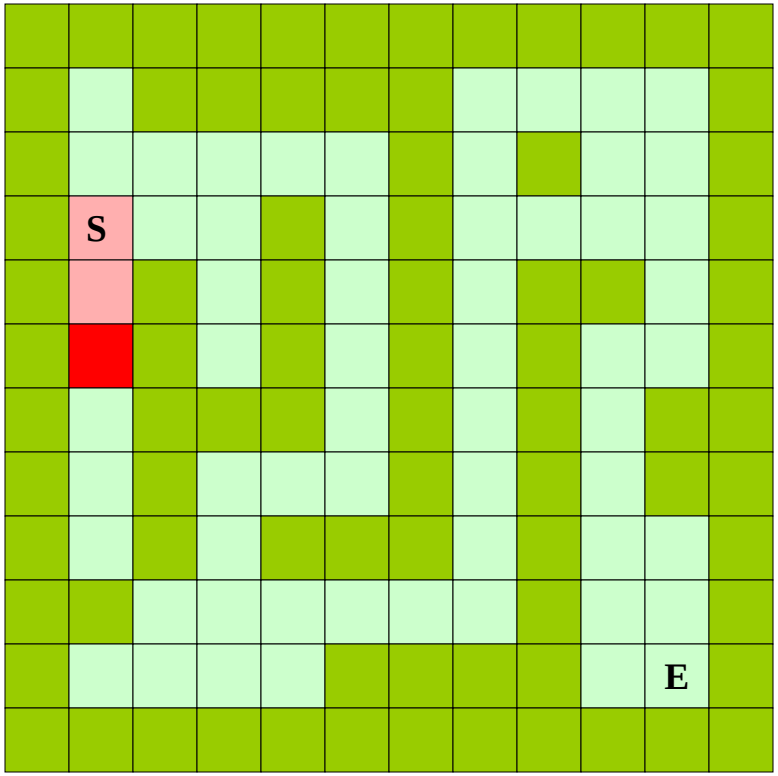
Visited and Done (not on current path)

Searching the Example Maze



Current position

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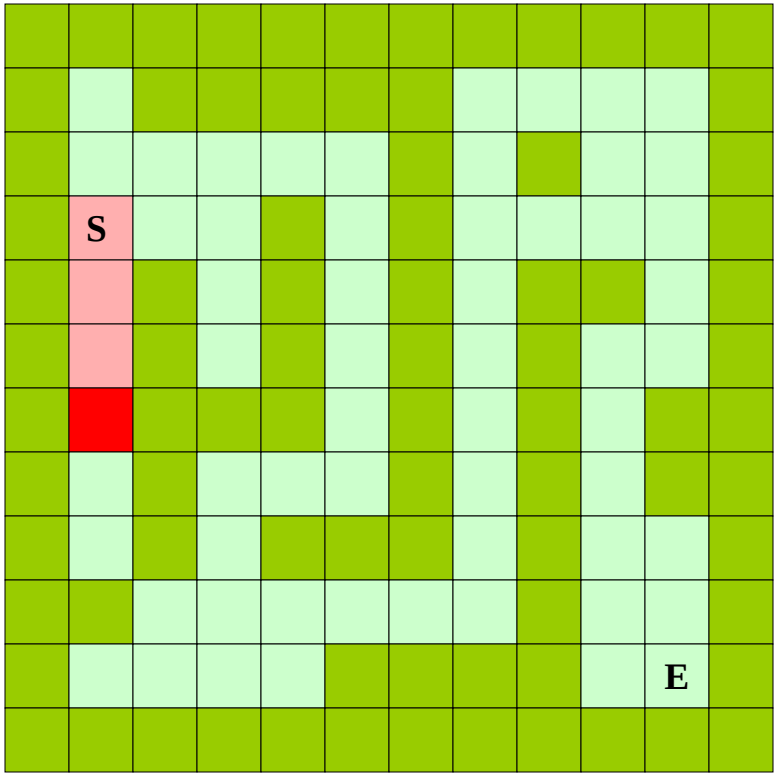
Visited and Done (not on current path)

Searching the Example Maze



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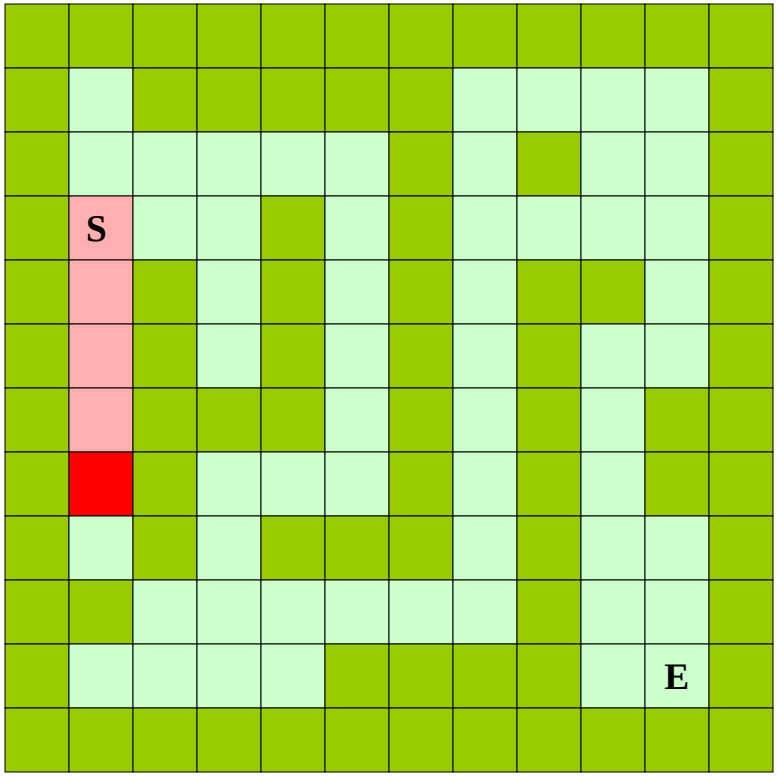
Searching the Example Maze



Current position

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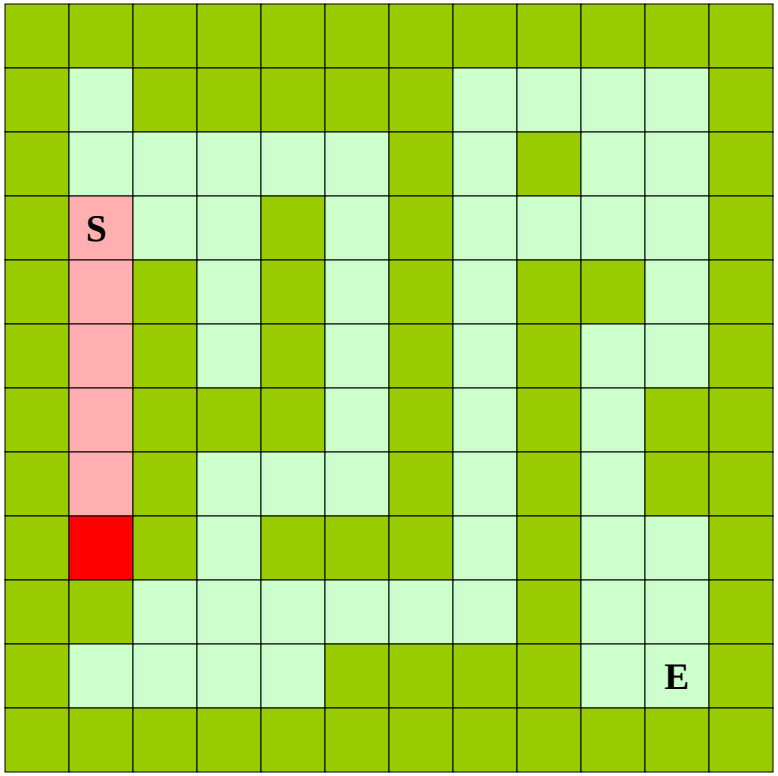
Searching the Example Maze



Current position

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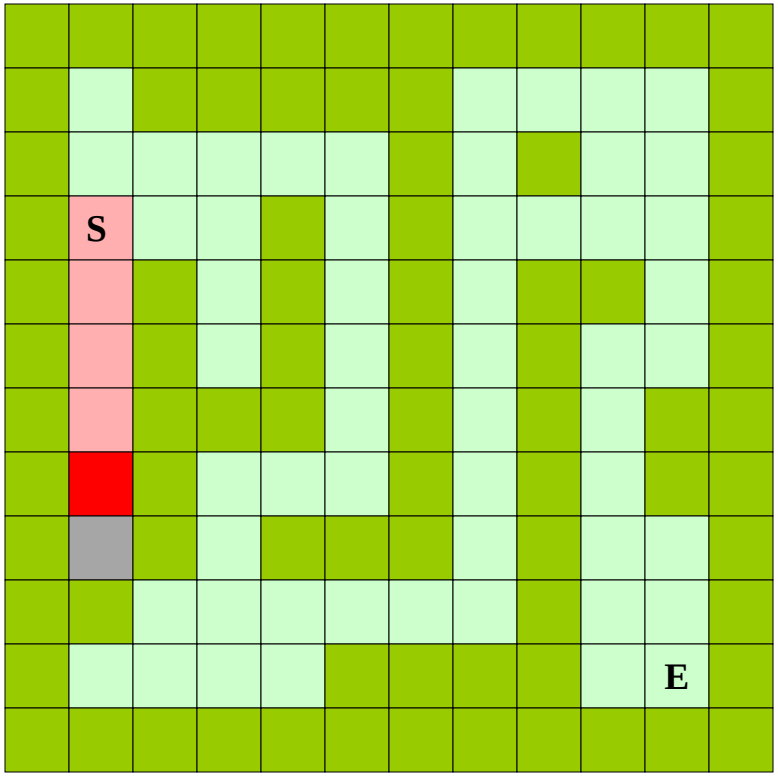
Visited and Done (not on current path)

Searching the Example Maze



Current position

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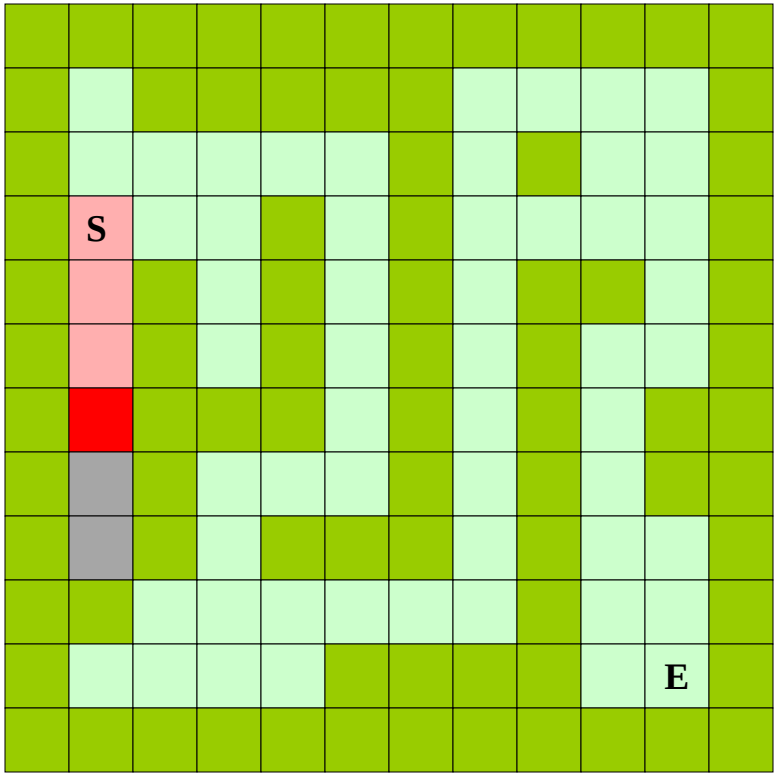
Visited and Done (not on current path)

Searching the Example Maze



Current position

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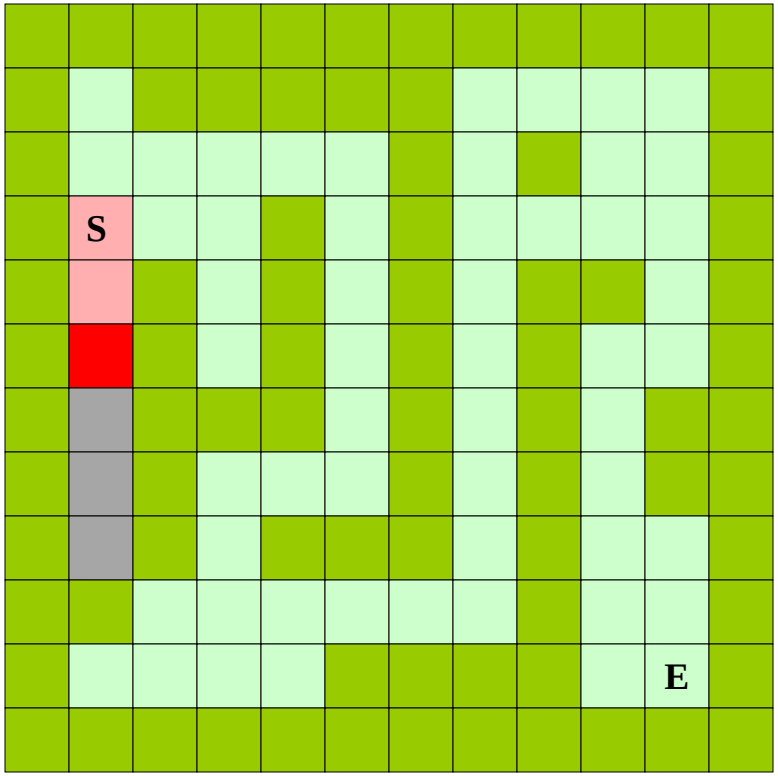
Visited and Done (not on current path)

Searching the Example Maze



Current position

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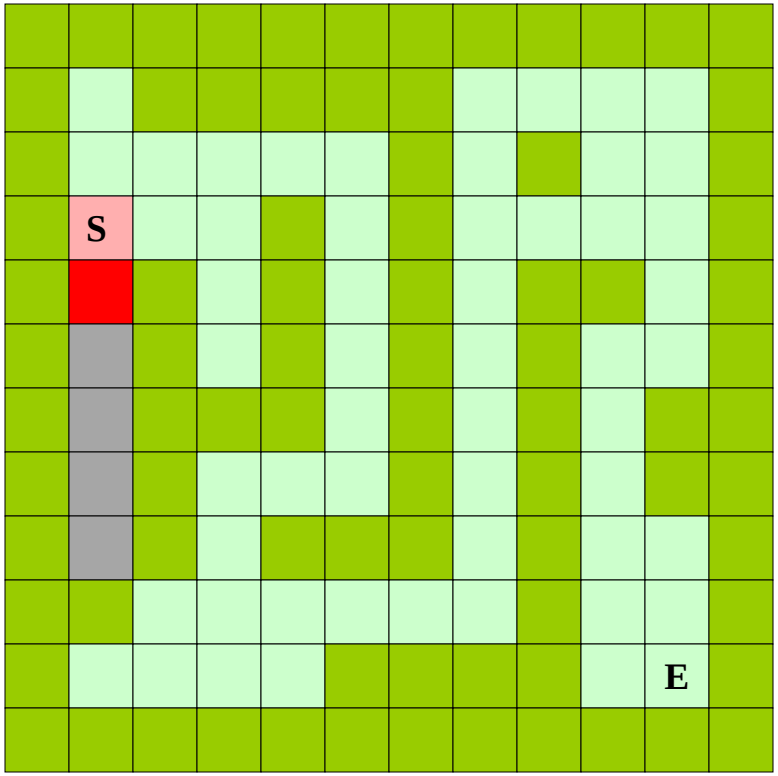
Visited and Done (not on current path)

Searching the Example Maze

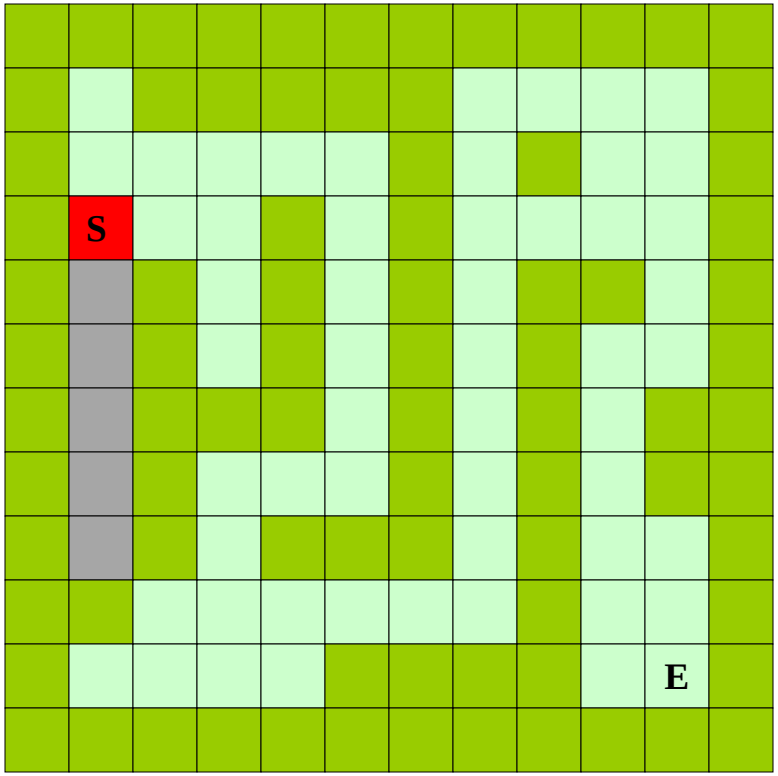


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze



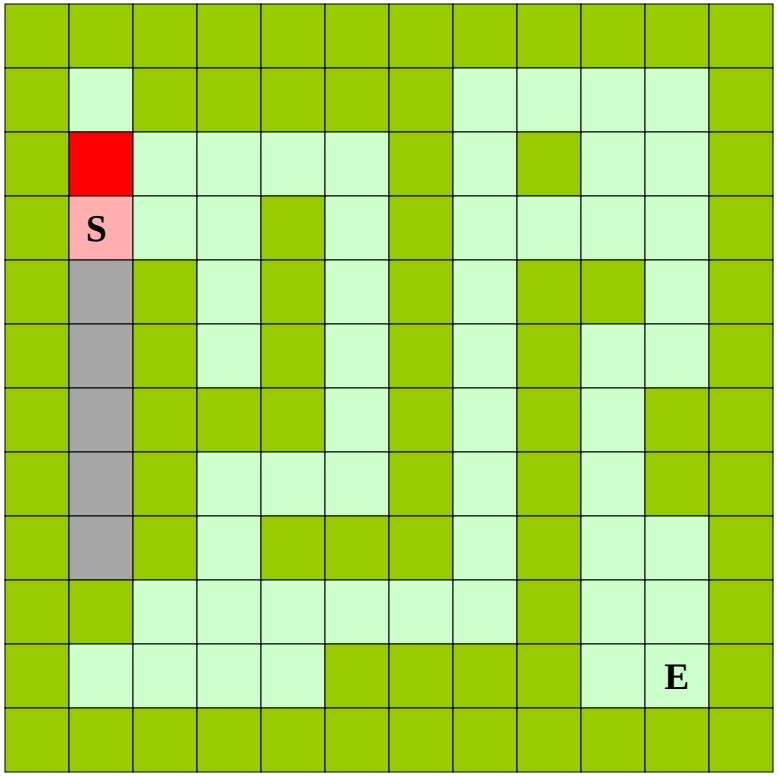
Arrows indicate the current path

Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze

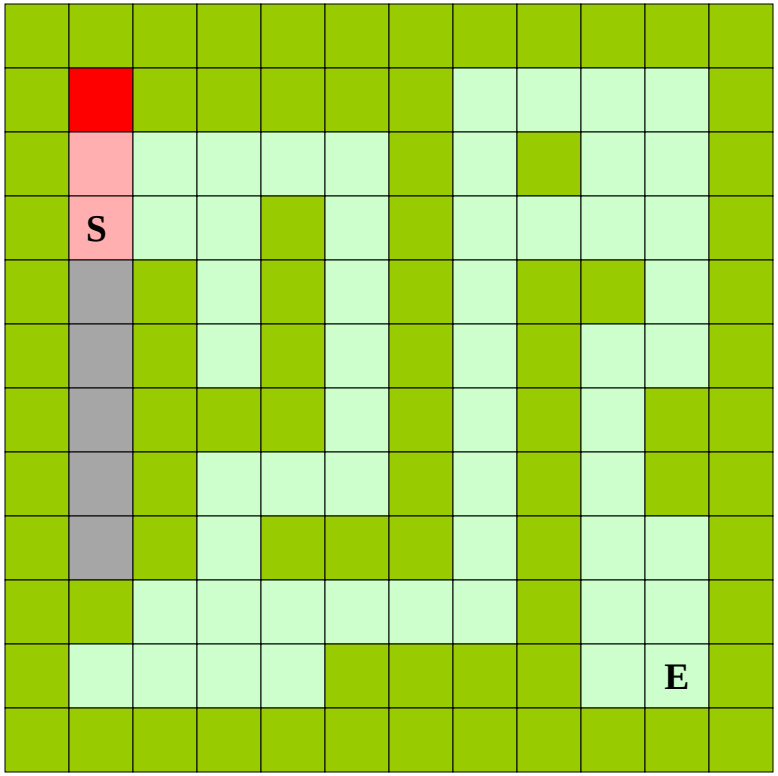


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze



Current position

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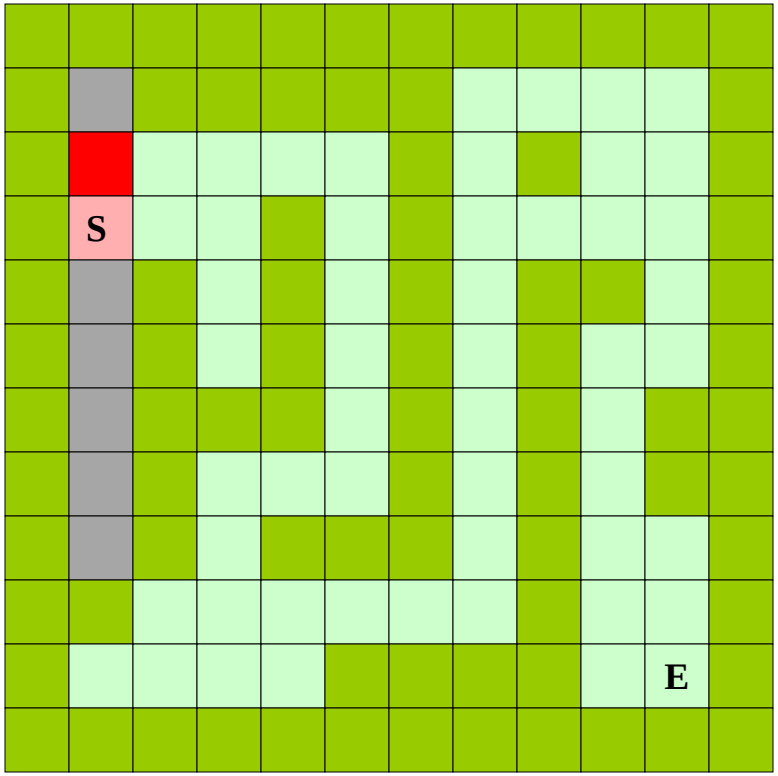
Searching the Example Maze



Current position

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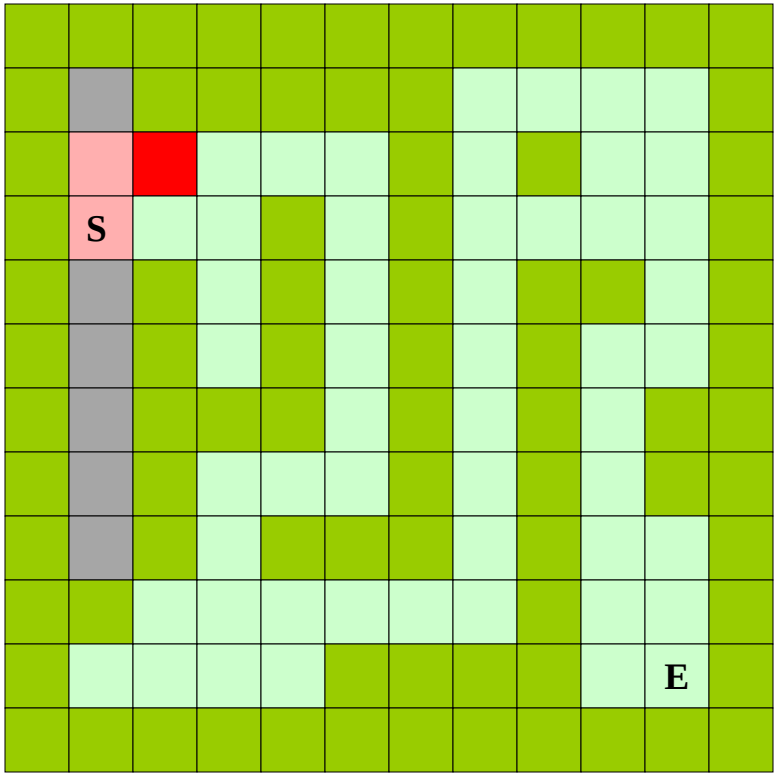
Visited and Done (not on current path)

Searching the Example Maze

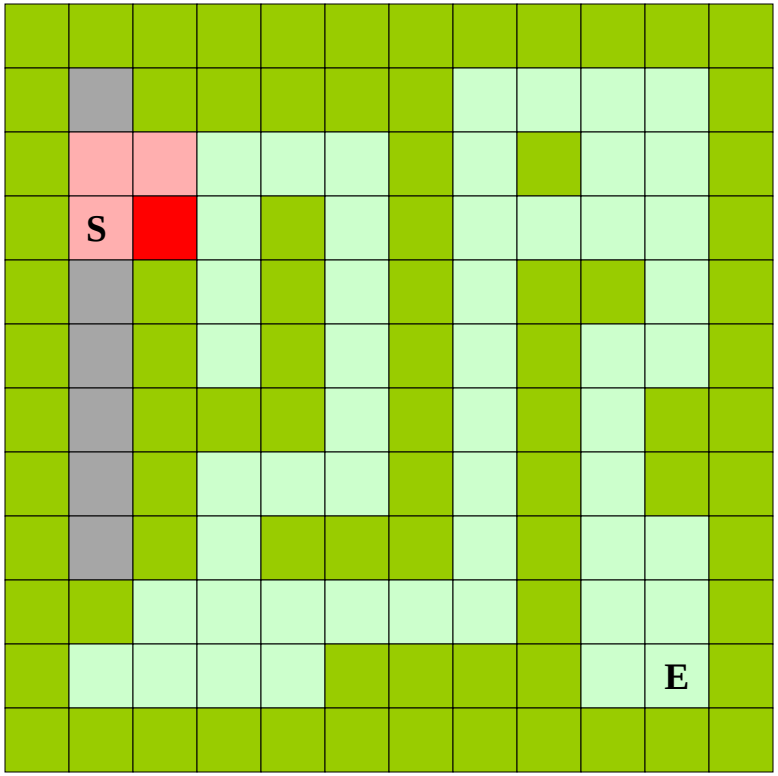


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze

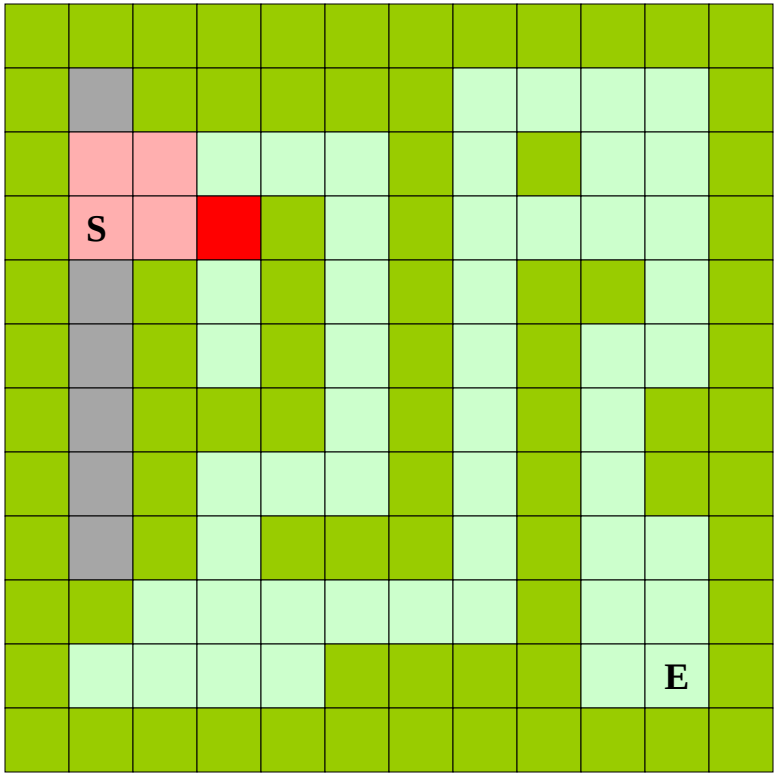


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze

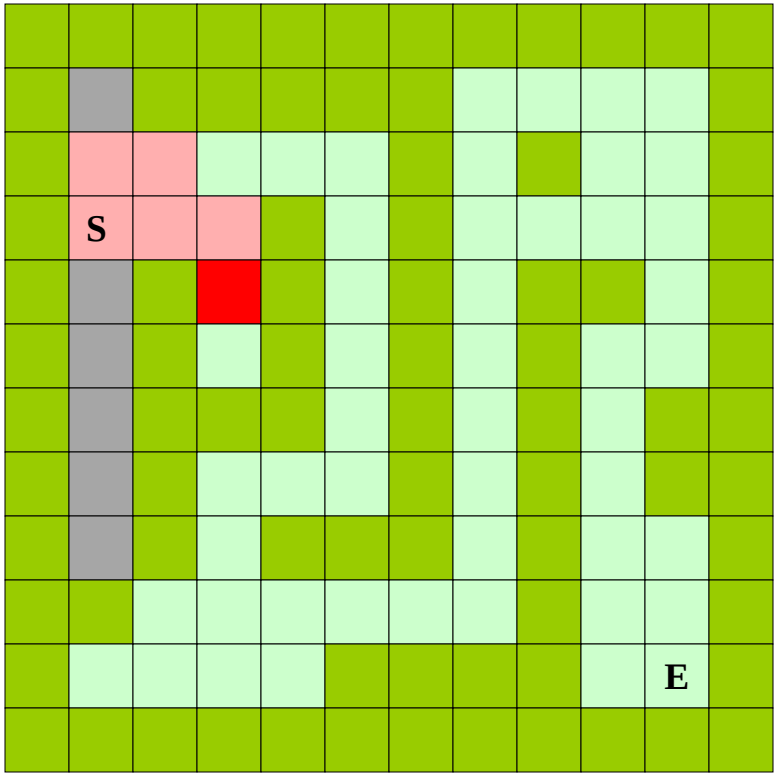


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze



Current position

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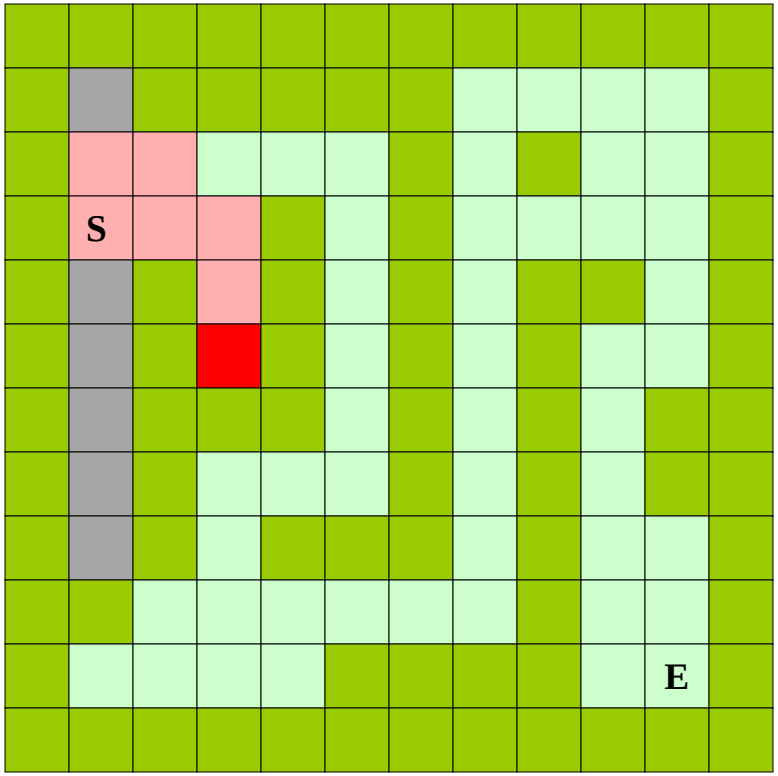
Searching the Example Maze

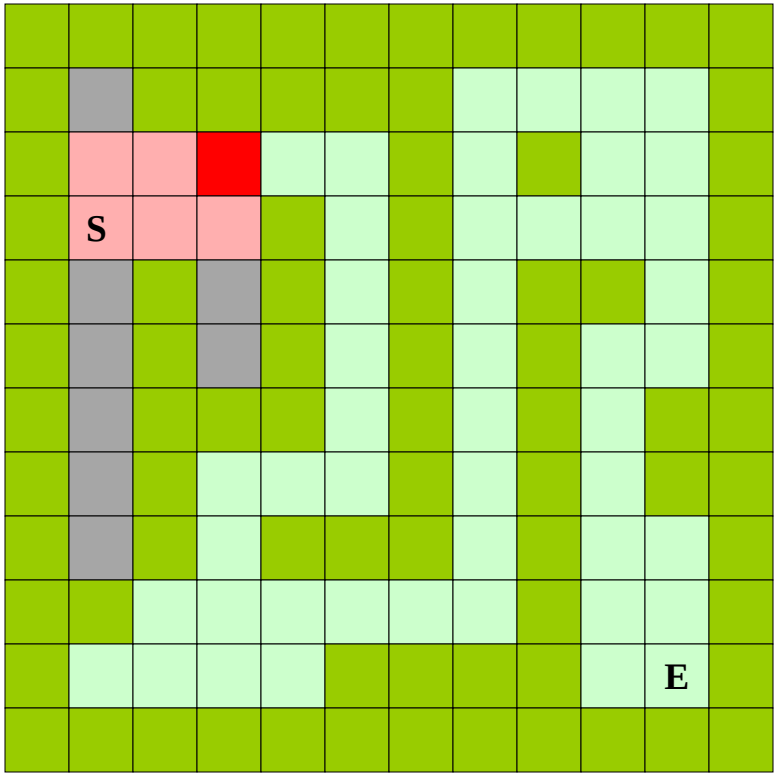


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze



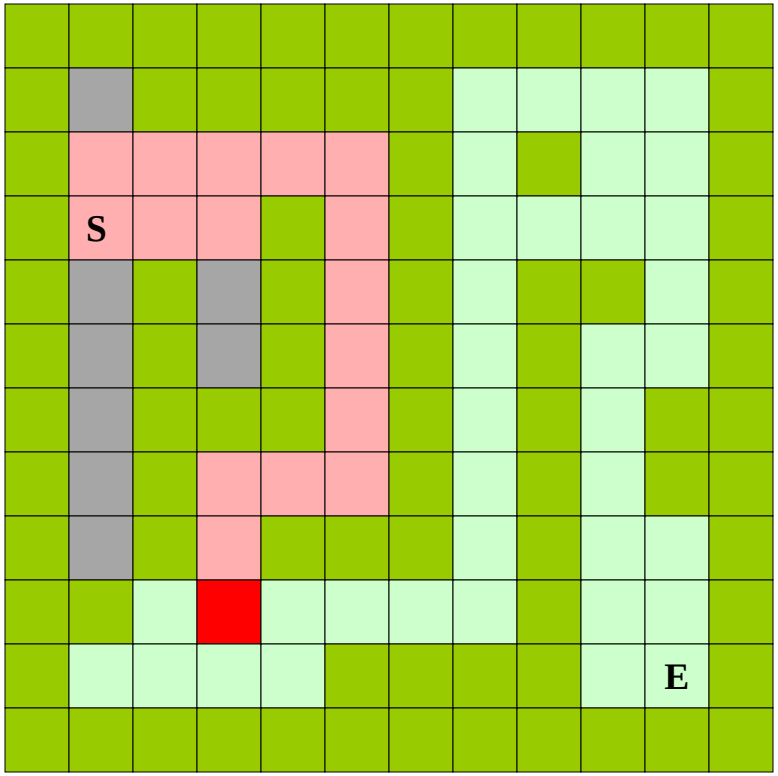
Three steps combined

Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze



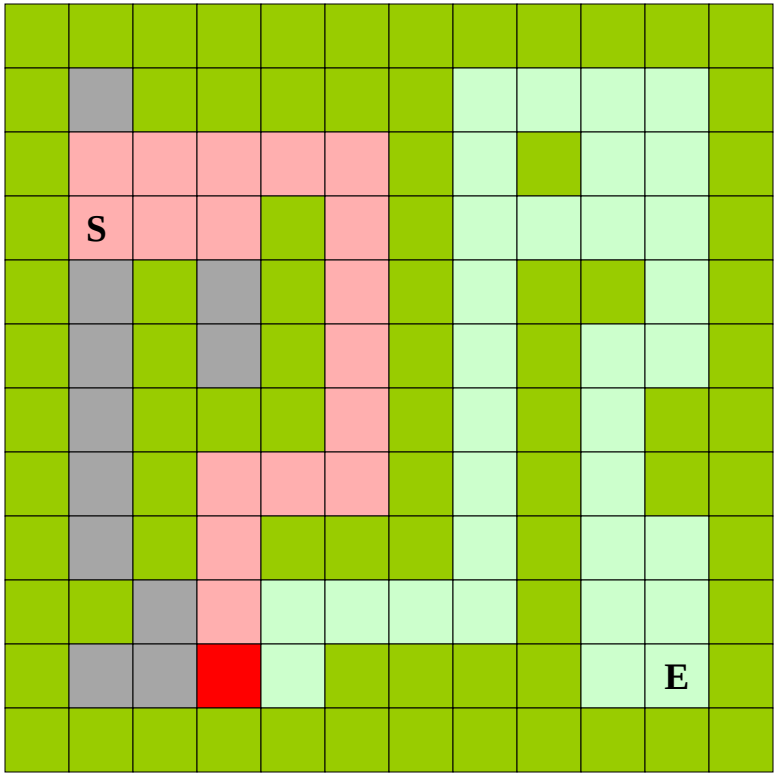
Several steps combined

Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze

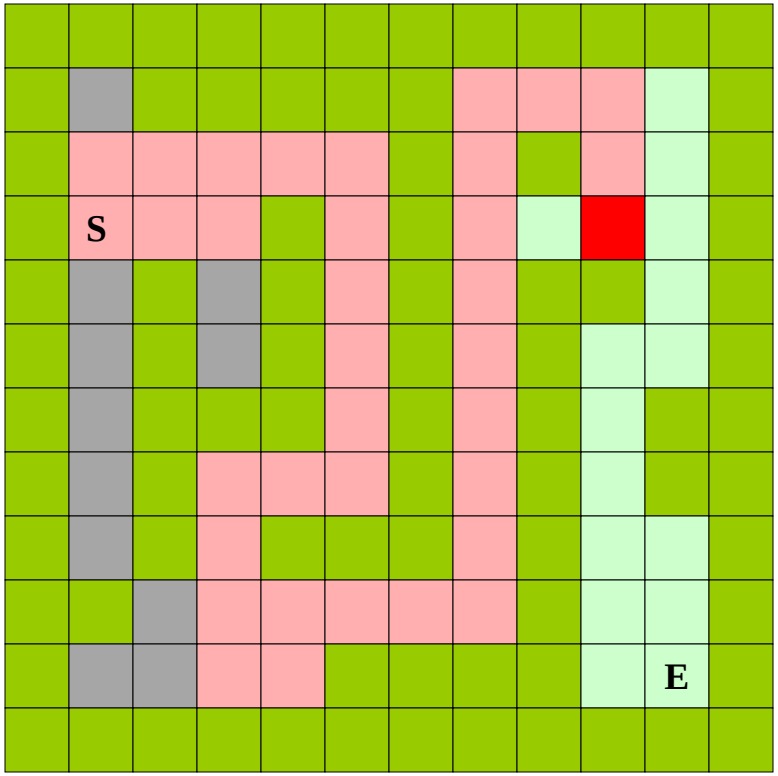


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze

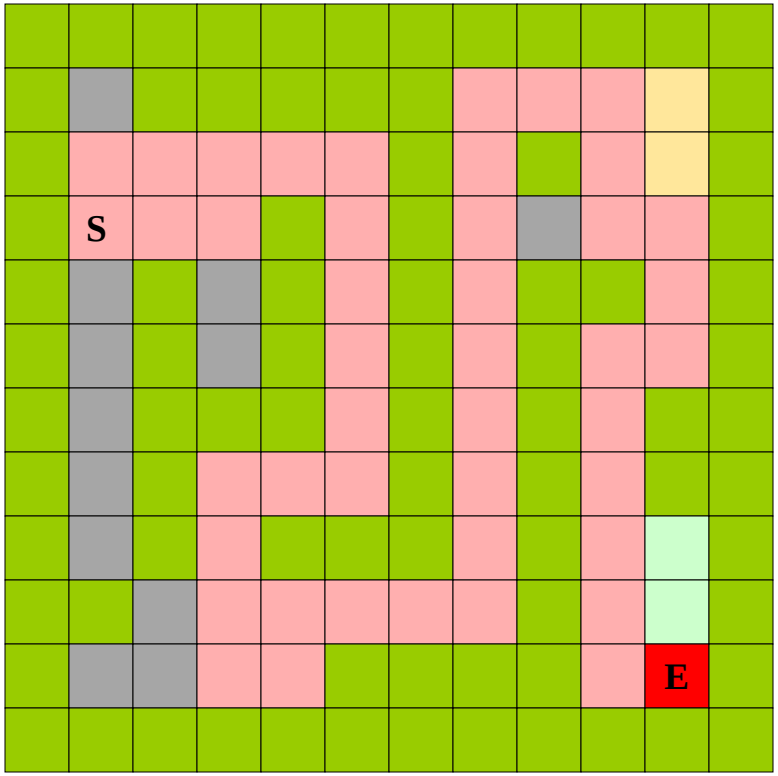


Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Searching the Example Maze



Current position

Visited and pending (on current path)

Visited and Done (not on current path)

Application 5
Maze：Search Method

- First we mark the Start position as VISITED and make it the current position.
- When exploring from a current position P, we look for an open neighbor in a fixed pattern: east, south, west and north.
- When an OPEN neighbor Q is found, we mark it as visited.
- In this case we say that Q was entered from P and that P is "pending": we have not finished exploring from P.
- We first check to see if Q is the Exit position.
- If so, the search concludes successfully. In this case, we print out the path from the Start to the Exit.
- If not, we continue exploring from position Q.

Application 5

Maze : Search Method

- If all four neighbors of Q are either WALL or VISITED, we return to the position P from which Q was first visited and continue exploring from P.
- That is, we "back out" from Q to P. This is the only way we can move to a previously visited position.
- Once we back out from Q, we will never return. Also, Q will not be on the path.
- If we return to the Start position and find no OPEN neighbors, the search concludes unsuccessfully: there is no path from the Start to the Exit.

Application 5
Maze

- We now consider how to carry out the algorithm described previously.
- In particular, how do we keep track of the position from which the current position was entered?
- Consider this: Suppose we go from S to P, then P to Q, then Q to R and R to T, then at position T find no OPEN neighbors.
- Then we back out to R. Suppose R has no OPEN neighbors. Then we should back out to Q.
- If Q has no OPEN neighbors, we back out to P and then to S.
- So the Path progresses like this:
- S
- S P
- S P Q
- S P Q R
- S P Q R T
- S P Q R
- S P Q
- S P
- S

So we will maintain a stack of Positions.
When we first visit a Position, we push it on the sta
To back out from a Position, we pop the top of the s
and continue exploring from there.

Application 6
Towers of Hanoi

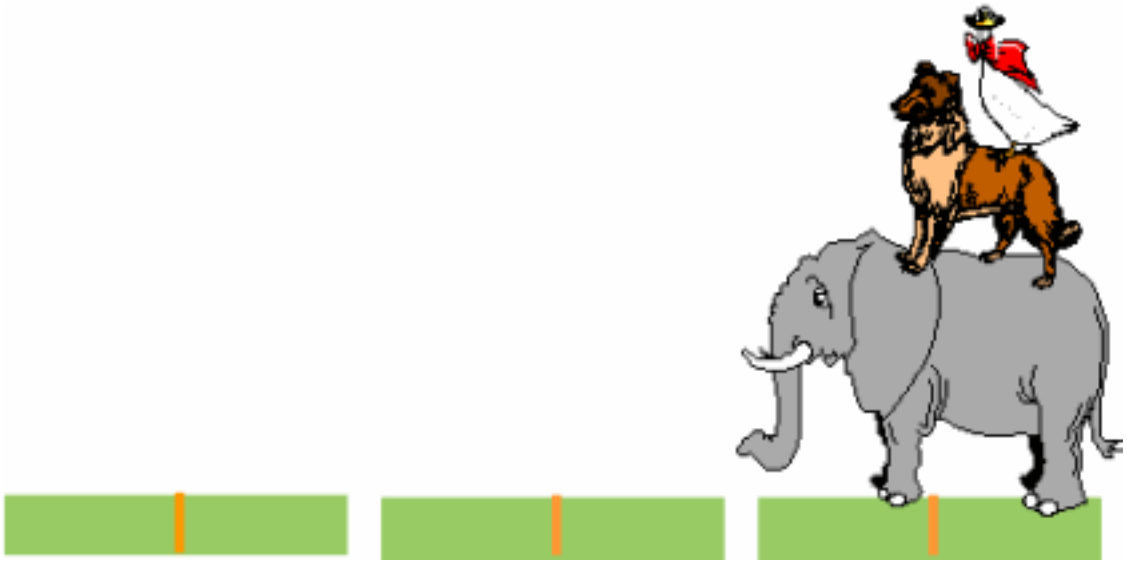
- A Legend ：The Towers of Hanoi

In the great temple of Brahma in Benares, on a brass plate under the dome that marks the center of the world, there are 64 disks of pure gold that the priests carry one at a time between these diamond needles according to Brahma's immutable law: No disk may be placed on a smaller disk. In the begging of the world all 64 disks formed the Tower of Brahma on one needle. Now, however, the process of transfer of the tower from one needle to another is in mid course. When the last disk is finally in place, once again forming the Tower of Brahma but on a different needle, then will come the end of the world and all will turn to dust.

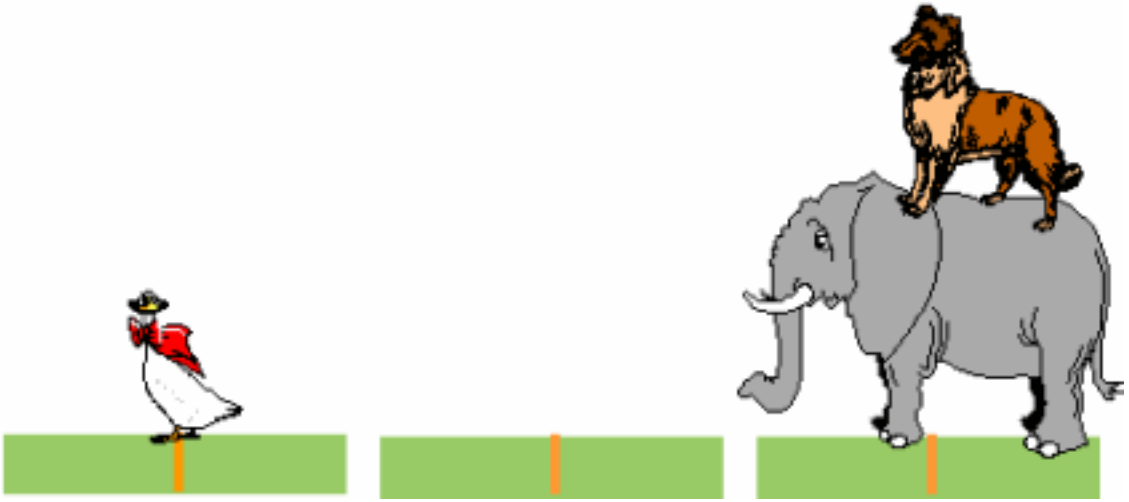
Application 6
The Towers of Hanoi

- GIVEN: three poles
- a set of discs on the first pole, discs of different sizes, the smallest discs at the top
- GOAL: move all the discs from the left pole to the right one.
- CONDITIONS: only one disc may be moved at a time.
- A disc can be placed either on an empty pole or on top of a larger disc.

Application 6
The Towers of Hanoi



Application 6
The Towers of Hanoi



Application 6
The Towers of Hanoi

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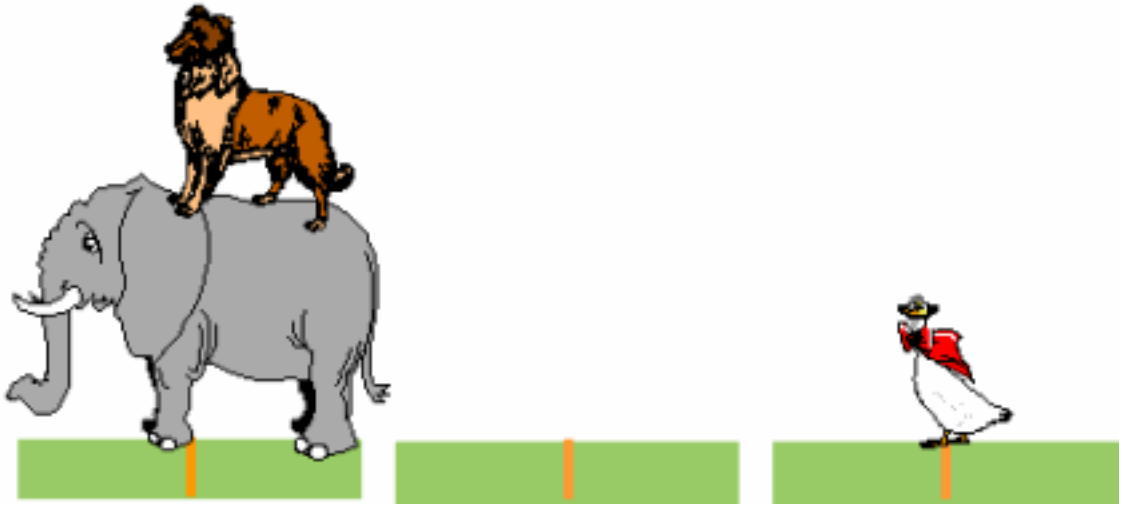
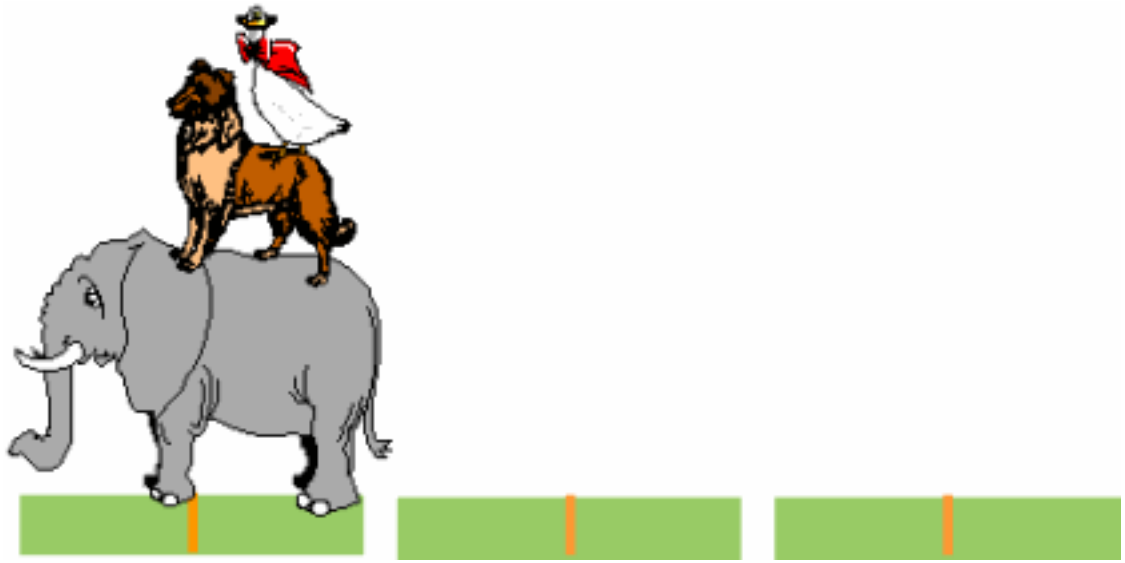
The Towers of Hanoi



Application 6
The Towers of Hanoi



Application 6
Towers of Hanoi – Recursive Solution

```
void TOH(int n, Pole start, Pole goal, Pole temp) {
  if (n == 0) return;              // Base case
  TOH(n-1, start, temp, goal); // Recursive call: n-1 rings
  move(start, goal);               // Move bottom disk to goal
  TOH(n-1, temp, goal, start); // Recursive call: n-1 rings
}
```

Application 6
Towers of Hanoi – Recursive Solution

```
void hanoi (int discs,Stack fromPole, Stack toPole, Stack aux) {
    Discs d;
    if( discs >= 1) {
            hanoi(discs-1, fromPole, aux, toPole);
            d = fromPole.pop();
            toPole.push(d);
            hanoi(discs-1,aux, toPole, fromPole);
    }
```

Is the End of the World Approaching?

- Problem complexity 2^n
- 64 gold discs
- Given 1 move a second

→ 600,000,000,000 years until the end of the world ☺

Summary

- ADT stack operations have a last-in, first-out (LIFO) behavior
- Stack applications
- A strong relationship exists between recursion and stacks

Reading Materials

- 《数据结构（ C 语言版）》，严蔚敏，吴伟民编著，清华大学出版社，1997年第1版,P50-58

-END-