Measuring Persistence of Macroeconomic Time Series: Evidence from the Jordá-Schularick-Taylor Macrohistory Database*

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Abstract

This paper examines the persistence properties of 9 macroeconomic time series, that is, real GDP, aggregate inflation, long-short interest rate spread, real exchange rate, real housing prices, consumption-income ratio, investment-income ratio, public debt-to-GDP ratio and the total loan-to-GDP ratio, for 17 advanced economies. This is done by examining appropriately defined measures of persistence in the econometric framework of generalized local-to-unity model (cf. Dou and Müller (2019)), and using the Jordá-Schularick-Taylor Macrohistory Database. I find that allowing for the generality in modelling long-range dependence can substantially alter quantitative statements about the persistence of macroeconomic time series. In practice, I recommend using a measure of the half-life, as proposed by Dou and Müller (2019), to gauge the persistence of macroeconomic time series.

Keywords: Persistence; Half-life; Generalized Local-to-Unity Models

JEL Codes: C22; E01; E30; E32

*PRELIMINARY AND INCOMPLETE, please do not cite or circulate without permission. The idea of this empirical paper came out during discussions with Mark Watson, whom I thank for many valuable suggestions and continued support. I thank Mikkel Plagborg-Møller for introducing me the Jordá-Schularick-Taylor Macrohistory Database. I also thank Paul Ho and Ulrich Müller for helpful discussions.

1 Introduction

This paper empirically investigates a basic and important feature of business cycle fluctuations: The persistence of macroeconomic time series. By "persistent" I mean the low-frequency variability is a large, sometimes dominant, source of the overall temporal variability in a time series. This low-frequency variability or persistence has been an important and ongoing empirical issue in macroeconomics. Nelson and Plosser (1982) sparked the debate in macroeconomics by arguing that stochastic variations due to persistent real factors were an essential element of any model of macroeconomic fluctuations. The interest in persistence of deviations from Purchasing Power Parity (PPP) arose naturally since Rogoff (1996). Moreover, some important relations among macroeconomic variables are expected to hold over long horizons, such as the so-called balanced growth relations, which refer to consumption-income and investment-income ratios. A common examination of these relations focuses on the persistence of their residuals.

Standard measures of persistence include the magnitude of the largest autoregressive roots in the individual series and in the residual from the long-run relation (see, for instance, Stock and Watson (1999)), and the so-called half-life under various definitions (see, for instance, Andrews and Chen (1994), Murray and Papell (2002, 2005) or Rossi (2005)). These two types of persistence measures essentially impose the assumption that the long-range persistence patterns of the underlying time series are of the AR(1) type. Moreover, methods for estimation of and inference about these persistence measures rely on the so-called local-to-unity (LTU) asymptotic framework (cf. Bobkoski (1983), Cavanagh (1985), Chan and Wei (1987), Phillips (1987)). Partly, this is because of the following two observations: First, for many macroeconomic time series tests for an autoregressive unit root are often inconclusive, or rejections are not exceedingly significant; Second, it is impossible to perfectly discriminate between a LTU process and a unit root process, even in large samples.

But it is not clear that the particular AR(1) form of long-range dependence should adequately model the persistence properties of macroeconomic time series. In fact, the LTU model is not the only persistence model with the feature as mentioned above, even with attention restricted to stationary models. On the other hand, Dou and Müller (2019) recently propose a more flexible asymptotic framework called the generalized local-to-unity model, GLTU(p), for the modelling of persistent time series. They show that the GLTU(p) class is a nearly unrestricted starting pointing for approximating stationary forms of persistence that are not entirely distinct from the unit root model in large samples. They also suggest a limited-information framework for likelihood based inference with the GLTU(p) model.

The purpose of this paper is to examine and document the persistence of macroeconomic time series from a compiled long-run cross-country dataset, the Jordá-Schularick-Taylor Macrohistory Database (cf. Jordà, Schularick, and Taylor (2017)), in the GLTU(p) model. To that end, I apply Dou and Müller's limited-information likelihood framework to conduct Bayesian inference about appropriately defined persistence measures, and compare them with those standard measures as mentioned above. As a byproduct, this paper also provides empirical researchers a user's guide to the econometric framework of Dou and Müller (2019), and examines the numerical stability of their framework with real data.

It is found that allowing for the generality in modelling long-range dependence can substantially alter quantitative statements about the persistence of macroeconomic time series. As a guide for practitioners, I recommend using a measure of half-life, as proposed by Dou and Müller (2019), in conjunction with the GLTU(p) model to gauge the persistence of macroeconomic time series. The suggested procedure is numerically stable throughout exercises conducted in this paper.

The remainder of the paper is organized as follows: Section 2 introduces the data-generating process (DGP) considered and the measures of persistence used. Section 3 describes the method to construct point estimates and credible intervals for the persistence measures. Section 4 explains the data used and necessary transformations taken. Section 5 discusses the empirical results, and Section 6 concludes.

2 Models and Measures of Persistence

Let the DGP for the observed time series, y_t, be

$$y_t = d_t + u_t,\ t = 1, \dots, T \tag{1}$$

where d_t is a deterministic component that can be modelled as a constant $d_t = \mu$ or as a constant plus linear trend $d_t = \mu_0 + \mu_1 t$. This paper focuses on measuring the persistence of the stochastic component u_t, which is modelled as a triangular array process in the GLTU(p) class

$$(1 - \rho_{T,1}L)(1 - \rho_{T,2}L) \cdots (1 - \rho_{T,p}L)u_{T,t} = (1 - \gamma_{T,1}L) \cdots (1 - \gamma_{T,(p-1)}L)\epsilon_t, \tag{2}$$

where $\rho_{T,j} = 1 - c_j/T$ and $\gamma_{T,j} = 1 - g_j/T$. I follow Dou and Müller (2019) to make the following assumptions about the building blocks of $u_{T,t}$.

Condition 2.1 *(i) The innovations* $\{\epsilon_t\}_{t=-\infty}^{\infty}$ *are mean-zero covariance stationary with absolutely summable autocovariances and satisfy* $T^{-1/2}\sum_{t=1}^{\lfloor \cdot T\rfloor}\epsilon_t \Rightarrow W(\cdot)$, *where* $W(\cdot)$ *is a Wiener process of variance* ω^2.

(ii) The parameters $\{c_j\}_{j=1}^{p}$ *and* $\{g_j\}_{j=1}^{p-1}$ *do not depend on* T *and have positive real parts. They can be complex valued, but if they are, then they appear in conjugate pairs, so that the polynomials* $a(z)=\prod_{j=1}^{p}(c_j+z)=z^p+\sum_{j=1}^{p}a_j z^{p-j}$ *and* $b(z)=\prod_{j=1}^{p-1}(g_j+z)=z^{p-1}+\sum_{j=0}^{p-2}b_j z^j$ *have real coefficients.*

(iii) For all T, *the process* $\{u_{T,t}\}_{t=-\infty}^{\infty}$ *is covariance stationary.*

It is shown by Dou and Müller (2019) that under Condition 2.1, the GLTU(p) model satisfies

$$T^{-1/2}u_{T,\lfloor \cdot T\rfloor} \Rightarrow J_p(\cdot), \tag{3}$$

where J_p is a mean-zero stationary continuous time Gaussian ARMA($p, p-1$) process with parameters $\{c_j\}_{j=1}^{p}$, $\{g_j\}_{j=1}^{p-1}$ and ω^2 of Condition 2.1 (ii) (denoted by CARMA($p, p-1$) process in the following). Following Brockwell (2001), the process J_p can be written as a scalar *observation*

$$J_p(s) = \mathbf{b}'\mathbf{X}(s) \tag{4}$$

of the $p \times 1$ *state* process $\mathbf{X}$ with

$$\mathbf{X}(s) = e^{\mathbf{A}s}\mathbf{X}(0) + \int_0^s e^{\mathbf{A}(s-r)}\mathbf{e}dW(r) \tag{5}$$

where $\mathbf{X}(0) \sim \mathcal{N}(0, \mathbf{\Sigma})$ is independent of the scalar Wiener process W of variance ω^2,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_p & -a_{p-1} & -a_{p-2} & \cdots & -a_1 \end{pmatrix}, \mathbf{e} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{p-2} \\ 1 \end{pmatrix}$$

and the coefficients a_j and b_j are defined in Condition 2.1 (ii). The covariance matrix of $\mathbf{X}(0)$, and hence $\mathbf{X}(s)$, is given by

$$\mathbf{\Sigma} = E[\mathbf{X}(0)\mathbf{X}(0)'] = \omega^2 \int_{-\infty}^{0} e^{-\mathbf{A}r}\mathbf{e}\mathbf{e}'e^{-\mathbf{A}'r}dr, \tag{6}$$

the autocovariance function of J_p is $\gamma_p(r) = E[J_p(s)J_p(s+r)] = \mathbf{b}'e^{\mathbf{A}|r|}\mathbf{\Sigma}\mathbf{b}$.

Now I discuss two measures of persistence as proposed by Dou and Müller (2019). The first measure is the sum of autoregressive coefficients in the AR(∞) representation, which in the GLTU(p) model (2) equals to

$$\alpha_T = 1 - d_T/T \approx 1 - T^{-1}a(0)/b(0) = 1 - T^{-1}\prod_{j=1}^{p} c_j / \prod_{j=1}^{p-1} g_j \tag{7}$$

for large T. This can be seen directly from the augmented Dickey-Fuller representation of (2), assuming uncorrelated innovations $\{\epsilon_t\}_{t=-\infty}^{\infty}$. Note that for $p = 1$, this measure reduces to the largest autoregressive root in the LTU model, so it is a generalization of that standard measure by accounting for more flexible patterns of low-frequency variability. It is also noted that short-run dynamics are not taken into account in this persistence measure (and are whitened asymptotically, as implied by (7)), so α_T captures only long-run persistence in large samples.

The second measure used in this paper is the half-life defined in terms of the following thought experiment: Given the model parameters, suppose we learn that the value of the stationary process $u_{T,t}$ at the time $t = 0$ is one unconditional standard deviation above its mean, but we don't observe any other values of $u_{T,t}$. What is the smallest horizon τ such that the best linear predictor of $u_{T,t}$ given $u_{T,0}$ is within 1/2 unconditional standard deviations of its mean, for all $t \geq \tau$? Specifically, assuming that ϵ_t has more than two moments, (3) implies that

$$T^{-1}E[u_{T,0}u_{T,\lfloor sT \rfloor}] \to E[J_p(0)J_p(s)] = \mathbf{b}' e^{\mathbf{A}|s|}\mathbf{\Sigma}\mathbf{b}.$$

Because the best linear predictor of $u_{T,t}$ given $u_{T,0}$ is proportional to the correlation between $u_{T,t}$ and $u_{T,0}$, the large sample approximation to τ is

$$\tau_T \approx T \inf_r \{r : \frac{\mathbf{b}' e^{\mathbf{A}|s|}\mathbf{\Sigma}\mathbf{b}}{\mathbf{b}'\mathbf{\Sigma}\mathbf{b}} \leq 1/2 \text{ for all } s \geq r\}. \tag{8}$$

Note that this definition differs from the traditional ones that are based on the impulse response of the Wold innovation to $u_{T,t}$. As mentioned by Dou and Müller (2019), it is often not obvious what the structural interpretation of Wold innovations to $u_{T,t}$ would be in applications, for example, when y_t is the log of real exchange rate and is modelled as a GLTU(p) process. As a result, impulse responses as well as their induced measures of the half-life in those contexts are not meaningful. The definition (8) of the half-life, however, does not rely on any structural models. It is also noted that, for $p = 1$, the definition (8) is equivalent to the (asymptotic) half-life of the impulse response by ignoring short-run dynamics in ϵ_t, $(\ln 2)T/c_1$, where c_1 is the LTU parameter. But these two definitions differ for $p > 2$: The impulse response function of J_p is equal to $\mathbf{1}[s \geq 0]\mathbf{b}'e^{\mathbf{A}s}\mathbf{e}$ (cf. (4) and (5)), while the autocovariance function is $\mathbf{b}'e^{\mathbf{A}|s|}\mathbf{\Sigma}\mathbf{b}$.

In Section 5, I report persistence measures using the above two definitions under p equals to 1 to 5, which nest the two standard measures in the LTU model. I then compare them and illustrate empirical relevance of the GLTU(p) class beyond the LTU model.

3 Econometric Method

After introducing the model and discussing two measures of persistence, I address the issue of how to conduct inference. A natural place to start would be the methods that are known to produce valid confidence intervals for standard measures of persistence in the LTU model. Given that all of the aforementioned standard measures are monotone functions of the parameter c_1 in the LTU model, those methods amount to constructing valid confidence intervals for c_1 that cannot be consistently estimated (as implied by the fact that the LTU model cannot be perfectly discriminated from the unit root model).[1]

The extensions of those methods are, however, not obvious in the GLTU(p) model, especially for constructing valid confidence intervals about the two persistence measures introduced in Section 2. After all, for $p > 1$, the parameters $\{c_j\}_{j=1}^{p}$, $\{g_j\}_{j=1}^{p-1}$ are allowed to take complex values as in Condition 2.1 (ii), so no natural ordering exists between two different sets of parameters. Moreover, the two persistence measures are not necessarily monotone functions of $\{c_j\}_{j=1}^{p}$, $\{g_j\}_{j=1}^{p-1}$ for $p > 1$. It is thus not clear what the end points of the confidences sets for $\{c_j\}_{j=1}^{p}$ and $\{g_j\}_{j=1}^{p-1}$, should they be achievable, imply for confidence intervals of α_T and τ_T.

To make progress, I borrow from Dou and Müller (2019) the limited-information likelihood framework and conduct Bayesian inference about the two persistence measures α_T and τ_T. First of all, by the continuous mapping theorem, an immediate implication of (2) and (3) is that for any fixed integer $N \geq 1$,

$$\{T^{-1/2}(y_{T,\lfloor jT/N \rfloor} - d_{\lfloor jT/N \rfloor})\}_{j=1}^{N} \Rightarrow \{J_p(j/N)\}_{j=1}^{N}. \tag{9}$$

It follows that the likelihood of the discretely sampled CARMA($p, p-1$) process J_p provides an asymptotically justified limited-information likelihood of the GLTU(p) model. The choice of N specifies the extent of "limited-information" and reflects a classic efficiency vs. robustness trade-off: A small N utilizes less observations but renders more flexibility in modelling weak dependence

[1]See, for instance, Mikusheva (2007) for an overview of uniform inference in the LTU model.

structure of $\epsilon_{T,t}$, as long as Condition 2.1 holds and asymptotics kick in; A large N takes (9) more seriously in finite samples but reduces the robustness of the resulting inference. For the empirical examinations in this paper, I follow Dou and Müller (2019) to set N to 50.

The evaluation of the likelihood of a discretely sampled CARMA$(p, p-1)$ process is, however, not straightforward. Partly, this is because that for $p > 1$, the parameters $\{c_j\}_{j=1}^p$, $\{g_j\}_{j=1}^{p-1}$ are allowed to take complex values, so is the companion matrix $\mathbf{A}$, and more importantly the *state* process $\mathbf{X}$ in (5). What is more, the evaluations of the stationary covariance matrix Σ in (6) are not obvious with matrix exponentials involved. Even for cases when the parameters $\{c_j\}_{j=1}^p$ are distinct, more careful treatments need to be taken, needless to mention that there is no good reason to rule out potentially identical parameters a priori.[2]

To avoid all these difficulties, Dou and Müller (2019) suggests to leverage on the limit result (9), together with a novel state space representation of the original GLTU(p) formulation

$$y_{T,t} = \mathbf{b}'\mathbf{Z}_{T,t} + d_t \tag{10}$$

$$\mathbf{Z}_{T,t} = (\mathbf{I} + \mathbf{A}/T)\mathbf{Z}_{T,t-1} + \mathbf{e}\epsilon_t \tag{11}$$

where $\mathbf{Z}_{T,t} \in \mathbb{R}^p$. In particular, for fixed $\{c_j\}_{j=1}^p$, $\{g_j\}_{j=1}^{p-1}$ and thus $\rho_{T_0,j} = 1 - c_j/T_0,\ j = 1, \ldots, p$ and $\gamma_{T_0,j} = 1 - g_k/T_0,\ k = 1, \ldots, p-1$, consider the discrete time *Gaussian* ARMA$(p, p-1)$ process

$$(1 - \rho_{T_0,1}L) \cdots (1 - \rho_{T_0,p}L)(y^0_{T_0,t} - d_t) = (1 - \gamma_{T_0,1}L) \cdots (1 - \gamma_{T_0,(p-1)}L)\epsilon^0_t \tag{12}$$

for $t = 1, \ldots, T_0$, where T_0 is sufficiently large and $\epsilon^0_t \sim iid\mathcal{N}(0, \omega^2)$. Two implications are immediate. First, as Condition 2.1 holds for (12), similar to (9),

$$\{T_0^{-1/2}(y^0_{T_0,\lfloor jT_0/N \rfloor} - d_{\lfloor jT_0/N \rfloor})\}_{j=1}^N \Rightarrow \{J_p(j/N)\}_{j=1}^N. \tag{13}$$

As a result, the likelihood of $\{T_0^{-1/2}(y^0_{T_0,\lfloor jT_0/N \rfloor} - d_{\lfloor jT_0/N \rfloor})\}_{j=1}^N$ approximates the likelihood of $\{J_p(j/N)\}_{j=1}^N$ arbitrarily well (pointwisely) as $T_0 \to \infty$. (This is mainly due to the joint Gaussianity of $\{y^0_{T_0,t}\}_{t=1}^{T_0}$, as well as the convergence of the first two moments implied by (13).) Second, by (10) and (11), $y^0_{T_0,t}$ has the state space representation

$$y^0_{T_0,t} = \mathbf{b}'\mathbf{Z}^0_{T_0,t} + d_t \tag{14}$$

$$\mathbf{Z}^0_{T_0,t} = \mathbf{\Phi}_{T_0}\mathbf{Z}^0_{T_0,t-1} + \mathbf{e}\epsilon^0_t \tag{15}$$

[2]See Jones (1981) for cases where $\{c_j\}_{j=1}^p$ are distinct and a rotation strategy can be taken to avoid the complication. Also, see Dini and Mandic (2012) for a general discussion of a robust scheme called *widely linear complex Kalman filter* for signal extraction in complex valued systems.

where $\mathbf{\Phi}_{T_0} = \mathbf{I}_p + \mathbf{A}/T_0$ and $\mathbf{\Omega}^0_{T_0} = E[\mathbf{Z}^0_{T_0,0}(\mathbf{Z}^0_{T_0,0})']$ satisfies $\operatorname{vec}\mathbf{\Omega}^0_{T_0} = \omega^2(\mathbf{I}_{p^2} - \mathbf{\Phi}_{T_0} \otimes \mathbf{\Phi}_{T_0})^{-1}\operatorname{vec}(\mathbf{ee}')$. In contrast to the state space representation (4) and (5) of J_p, the state variable $Z^0_{T_0,t}$ is real valued and the computation of the covariance matrix $\Omega^0_{T_0}$ is straightforward.[3] A direct application of the conventional Kalman filter gives rise to an asymptotically justified approximation to the limited-information likelihood for the parameters ω^2, $\{c_j\}_{j=1}^p$ and $\{g_j\}_{j=1}^{p-1}$ of the GLTU(p), and those in d_t. I follow Dou and Müller (2019) to use $T_0 = 1000$ throughout the empirical examinations in this paper.

In actual implementations of the above scheme, two extra things are noteworthy. First, to avoid dealing with parameter spaces of complex numbers, it is helpful to follow Jones (1981) to reparametrize the parameters $\{c_j\}_{j=1}^p$, $\{g_j\}_{j=1}^{p-1}$ of Condition 2.1 (ii) such that $a(z)$ and $b(z)$ are products of quadratic factors (and a linear factor if p is odd), where each quadratic factor collapses a potentially conjugate pair of roots into a quadratic polynomial with positive coefficients. Specifically, for each conjugate pair c_j and c_k (or a selected pair of real parameters without replacement if no conjugate pairs appear), consider h_j and h_k such that $h_j = \sqrt{c_j c_k}$ and $h_k = (c_j + c_k)/2$. In this way, $(z + c_j)(z + c_k) = z^2 + 2h_k z + h_j^2$, and Condition 2.1 (ii) ensures strictly positiveness of the real valued parameters h_j and h_k without further constraints. Second, we are applying Kalman filter to a discrete system with state (15) and observations $y^0_{T_0,\lfloor jT_0/N \rfloor} = y_{T,\lfloor jT/N \rfloor}$, $j = 1, \ldots, N$, and with all other observations of $y^0_{T_0,t}$ treated as missing. This also works even if the original data say $y_{T,\lfloor jT/N \rfloor}$ is unavailable for some j, because the Kalman filter can easily incorporate missing observations.

Now I begin describing the Bayesian inference procedure for the two persistence measures α_T and τ_T, based on the above limited-information likelihood framework. First of all, note that a useful byproduct of the above framework is an asymptotically justified and computationally easy approximation to the persistence measure

$$\tau_T \approx T \inf_r \{ r : \frac{\mathbf{b}'\mathbf{\Phi}_{T_0'}^{\lfloor sT_0' \rfloor}\mathbf{\Omega}^0_{T_0'}\mathbf{b}}{\mathbf{b}'\mathbf{\Omega}^0_{T_0'}\mathbf{b}} \leq 1/2 \text{ for all } s \geq r \}, \tag{16}$$

where T_0' is sufficiently large and can be differently chosen from the T_0 in the limited-information likelihood approximation. Dou and Müller (2019) claims that $T_0' = 1000$ generates numerically stable results so I will stick to that in this paper.

[3] In fact, to obtain more precise computations, consider the eigenvalue decomposition of $\mathbf{\Phi}_{T_0} = \mathbf{Q}_{T_0}\mathbf{\Lambda}_{\mathbf{T_0}}\mathbf{Q}_{T_0}^{-1}$, then $(\mathbf{I}_{p^2} - \mathbf{\Phi}_{T_0} \otimes \mathbf{\Phi}_{T_0})^{-1}\operatorname{vec}(\mathbf{ee}') = (\mathbf{Q}_{T_0} \otimes \mathbf{Q}_{T_0})(\mathbf{I}_p \otimes \mathbf{I}_p - \mathbf{\Lambda}_{T_0} \otimes \mathbf{\Lambda}_{T_0})^{-1}(\mathbf{Q}_{T_0}^{-1} \otimes \mathbf{Q}_{T_0}^{-1})\operatorname{vec}(\mathbf{ee}') = (\mathbf{Q}_{T_0} \otimes \mathbf{Q}_{T_0})(\mathbf{I}_p \otimes \mathbf{I}_p - \mathbf{\Lambda}_{T_0} \otimes \mathbf{\Lambda}_{T_0})^{-1}\operatorname{vec}\left(\mathbf{Q}_{T_0}^{-1}\mathbf{ee}'\mathbf{Q}_{T_0}^{-1}\right)$. I am grateful to Ulrich Müller for teaching me this trick.

Second, for a formal (limited-information) Bayesian inference, one has to specify prior distributions for the parameters in d_t, ω^2, and the collection of reparametrized $\{c_j\}_{j=1}^p$ and $\{g_j\}_{j=1}^{p-1}$ as the vector $\mathbf{h} = (h_1^c, \ldots, h_p^c, h_1^g, \ldots, h_{p-1}^g)' \in \mathbb{R}^p \times \mathbb{R}^{p-1}$, where for p odd, $c_p = 2h_p^c$, and for p even, $g_{p-1} = 2h_{p-1}^g$. Standard improper prior is used for ω^2 such that the resulting conditional posterior distribution is inverse-Gamma. Throughout empirical exercises in this paper, d_t is assumed to be a constant μ with a standard improper prior imposed such that the resulting conditional posterior distribution is normal.[4] There is nearly no reference for the prior on $\mathbf{h}$ in the literature so I follow Dou and Müller (2019) to impose an implicit prior (independent of p) on $\mathbf{h}$. Specifically, the prior on $\mathbf{h}$ is assumed to be proportional to $g(\tau_T(\mathbf{h}))$, where the function $g : \mathbb{R} \mapsto [0, \infty)$ is such that the implied prior distribution of $\tau_T(\mathbf{h})$ is uniform on the interval $[3, 50]$. (In practice, g is computed by generating many draws from a prior with $g = g_0$ for some initial guess g_0, and g is then given by $\mathbf{1}[3 \leq \tau_T(\mathbf{h}) \leq 50] g_0(\tau_T(\mathbf{h}))$ divided by a kernel estimate of the resulting density of $\tau_T(\mathbf{h})$.)

Lastly, in actual implementations, each component of $\mathbf{h}$ is restricted to be in the interval $(0, 40]$, where 40 corresponds to a very fast mean reversion in the LTU model with parameter 80. With the above specified prior distributions, I obtain the posterior using a Gibbs sampler: The μ and ω^2 blocks are sampled with above known distributions, respectively;[5] The $\mathbf{h}$ block is sampled with a random walk Metropolis-Hastings algorithm by making use of (independent) mean-zero normal distributions as jump proposals for each element of $\mathbf{h}$. The jump size is chosen differently for each empirical examination. (For convergence of the resulting MCMC chain, it requires trials-and-errors to obtain a suitable jump size. In practice, I run multiple chains of 8000 draws with different jump sizes, discard their first 4000 as burn-in draws, and then choose the smallest jump size such that the induced acceptance rate in the Metropolis-Hastings step is between 0.2 and 0.5. After that, I generate 100000 draws using the selected jump size and discard the first 50000 draws.)

[4] In the case that $d_t = \mu_0 + \mu_1 t$, standard improper priors on $(\mu_0, \mu_1)'$ lead to a bivariate normal conditional posterior distribution. But normality breaks down when d_t is nonlinear in parameters.

[5] For given ω^2 and GLTU parameters, analytic expressions for $\mathrm{E}\left[\mu | \omega^2, \mathbf{h}, \{y_{T,t}\}_{t=1}^T\right]$ and $\mathrm{Var}\left[\mu | \omega^2, \mathbf{h}, \{y_{T,t}\}_{t=1}^T\right]$ can be obtained. In the case that d_t is a time trend, element by element, one can numerically identify the mean vector and covariance matrix of $(\mu_0, \mu_1)'$. Details are omitted for brevity. A even cleaner approach is to augment the state variables in (15) by including μ (constant case) or $(\mu_0, \mu_1)'$ (trend case). The outputs in the last step of that Kalman filter are the objects of interest. I thank Mikkel Plagborg-Møller for this observation.

4 Data Description

The data used in this paper are from the Release 4 of the Jordá-Schularick-Taylor Macrohistory Database in May 2019. It is a compiled long-run macro-financial dataset that covers 17 advanced economies since 1870 on an annual basis. In this paper, I examined 9 macroeconomic time series for all the 17 economies: Real GDP, aggregate inflation, spreads between long-term and short-term interest rates, real exchange rate (local currency/USD), real housing prices, consumption-income ratio, investment-income ratio, public debt-to-GDP ratio and the total loan-to-GDP ratio (loan to non-financial private sector). Table 1 gives a detailed per variable and per country overview of the coverage for the 13 original data series as needed in constructing the above 9 variables.[6] Precise descriptions of and sources of each data series per country appear in the JST Dataset R4 documentation.

The 9 series were subject to a preliminary screen for outliers and then transformed. Specifically, aggregate inflation rates during the hyperinflation period of 1915 to 1924 were excluded for Germany; aggregate inflation rates during the (post) World War II period of 1945 to 1948 were excluded for Japan; aggregate inflation rates during the World War I period of 1917 to 1918 were excluded for Finland. After these, I follow Section 4 in Müller and Watson (2008) to transform the 9 time series and focus on measuring the persistence of the remaining stationary component. The transformations used depends on the series: Linearly detrended logarithms of real GDP and real housing prices are used,[7] while logarithms of the remaining variables or indicated ratios are used.[8] Moreover, since the econometric method elaborated in Section 3 can incorporate missing observations, I do not apply special treatments to the unavailable data points as summarized in Table 1. Also, for asymptotic arguments outlined in Section 3 to be sensible, I use $T = 147$, the full time span of the Jordá-Schularick-Taylor Macrohistory dataset, for all series under consideration.

[6]Table 1 updates part of Table 1 in Jordà, Schularick, and Taylor (2017) with an extended sample and with corrections for previous reporting errors.

[7]For real DGP and real housing prices, I should have executed the implementations under the assumption that $d_t = \mu_0 + \mu_1 t$, and treated both μ_0 and μ_1 as unknowns. But I was not aware of footnote 5 and overclaimed the difficulty under $d_t = \mu_0 + \mu_1 t$ in an earlier version. The results following the suggestions in footnote 5 would be updated in near future.

[8]The Hodrick-Prescott filter were used to detrend logarithms of all data series in Jordà, Schularick, and Taylor (2017), so their reported persistence (first-order autocorrelation) is not directly comparable to those in this chapter.

Table 1: Available samples per variable and per country

Country	R. GDP	N.GDP	PPP GDP	Population	CPI	L.t. rate	S.t. rate	Ex. rate	H. Prices	Pub. debt	Bnk lend.	Con.	Inv.
Australia	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-1944	1870-2016	1870-2016	1870-2016	1870-1945	1870-2016	1870-1946
							1948-2016				1948-2016		1949-2016
Belgium	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-1912	1870-1914	1870-2016	1919-2016	1870-1913	1885-1913	1913-2016	1900-1913
						1920-2016	1920-2016			1920-1939	1920-1940		1920-1939
										1946-1979	1950-2016		1941
										1982-2016			1943
										1982-2016			1946-2016
Canada	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1934-2016	1870-2016	1921-1949	1870-2016	1870-2016	1871-2016	1870-2016
									1956-2016				
Switzerland	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1880-2016	1870-2016	1870-2016	1901-2016	1880-2016	1870-2016	1870-2016	1870-1913
													1948-2016
Germany	1870-2016	1870-1944	1870-2016	1870-2016	1870-2016	1870-1921	1870-1922	1870-1944	1870-1922	1871-1913	1870-1920	1870-2016	1870-1913
		1946-2016				1924-1943	1924-1944	1946-2016	1924-1938	1927-1943	1924-1940		1920-1939
						1948-2016	1950-2016		1962-2016	1950-2016	1946-2016		1948-2016
Denmark	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1875-2016	1870-2016	1875-2016	1880-1946	1870-2016	1870-2016	1870-1914
										1953-1956			1922-2016
										1960-1996			
										1998-2016			
Spain	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-1936	1870-2016	1870-2016	1971-2016	1880-1935	1900-1935	1870-2016	1870-2016
						1940-2016				1940-2016	1946-2016		

Continued on next page

Table 1 – Continued from previous page

Country	R. GDP	N.GDP	PPP GDP	Population	CPI	L.t. rate	S.t. rate	Ex. rate	H. Prices	Pub. debt	Bnk lend.	Con.	Inv.
Finland	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-1938	1870-2016	1870-2016	1905-2016	1914-2016	1870-2016	1870-2016	1870-2016
						1948-2016							
France	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-1914	1870-2016	1870-2016	1880-1913	1900-1938	1870-2016	1870-1918
							1922-2016			1920-1938	1946-2016		1820-1944
										1949-2016			1946-2016
UK	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1899-1938	1870-2016	1880-2016	1870-2016	1870-2016
									1946-2016				
Italy	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-1871	1870-2016	1970-2016	1870-2016	1870-2016	1870-2016	1870-2016
							1885-1914						
							1922-2016						
Japan	1870-2016	1875-1944	1870-2016	1870-2016	1870-2016	1870-2016	1879-1938	1870	1913-1930	1875-1944	1874-2016	1874-2016	1885-1944
		1946-2016					1957-2016	1873-2016	1936-2016	1946-2016			1946-2016
Netherlands	1870-2016	1870-1913	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-1939	1900-2016	1870-2016	1870-1913
		1921-1939								1946-2016			1921-1939
		1945-2016											1948-2016
Norway	1870-2016	1870-1939	1870-2016	1870-2016	1870-2016	1870-2016	1870-1965	1870-2016	1870-2016	1880-1939	1870-2016	1870-2016	1870-1939
		1946-2016					1967-2016			1947-2016			1946-2016
Portugal	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1880-2016	1870-2016	1988-2016	1870-2016	1870-1903	1910-2016	1953-2016
											1920-2016		
Sweden	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1875-2016	1870-2016	1871-2016	1870-2016	1870-2016
USA	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1870-2016	1890-2016	1870-2016	1880-2016	1870-2016	1870-2016

5 Empirical Results

Table 2 reports the 5th, 50th and 95th percentiles of α_T for $p = 1$, which as mentioned earlier, corresponds to to percentiles of the largest autoregressive root in the LTU model. In general, the magnitudes of the reported α_T for the U.S. turn out to be large, even relative to those computed using U.S. quarterly data (cf. Stock and Watson (1999)). This can be attributed to the fact that a uniform prior distribution for τ_T over $[3, 50]$ implicitly induces a lower bound of approximately 0.77 for α_T under $T = 147$. It would also be interesting to see how the computed α_T vary as the resolution of "limited-information" lens (controlled by N) changes. I leave this investigation and the scrutiny of prior sensitivity for future studies.

In line with the consensus of the literature, aggregate inflation has a very high degree of persistence for almost all economies under consideration, except for Japan. However, in contrast to what was documented in earlier literature (see, for instance, Stock and Watson (1999)), the long-short interest rate spread as well displays high persistence, especially for France, Finland, Denmark, Belgium and Canada. What's more, high persistence remains for putative cointegration error correction terms involving consumption, income, and investment, for almost all economies in the sample. This does not line up perfectly with the simple balanced growth predictions. For the U.S., the large increase in the consumption—income ratio over the 1985—2004 period were pointed out as an apparent explanation for the high persistence of the consumption–income relationship (see, for instance, Section 4.3 in Müller and Watson (2008)).

Tables 3 and 6 report the 5th, 50th and 95th percentiles of α_T, for $p = 2$ and 5 respectively. Similar tables of α_T for $p = 3, 4$ are collected in Appendix 6. Comparing these tables with Table 2, two things are noteworthy: First, the 90% posterior intervals for α_T get wider in general as more generality of modelling is allowed for each series (p gets larger). This is not surprising given a fixed sample size $T = 147$ and the non-negligible number of missing data points for several series, as documented in Table 1. Second, the 5th percentiles of α_T are uninformative (downcoded by 0) for many series under consideration for $p = 5$. This reflects a numerical drawback of using α_T as a persistence measure in the GLTU(p) model for a relatively large p: For a given sample size T, α_T as in (7) would fall below 0 as long as the term $\prod_{j=1}^{p-1} g_j$ is an order of magnitude smaller than $\prod_{j=1}^{p} c_j$. But with no further restrictions on the parameter space, this cannot be avoided numerically.

Table 4 shows the 5th, 50th and 95th percentiles of τ_T for $p = 1$, which is equivalent to standard measures of the half-life in the LTU model without considering short-run dynamics. The persistence of (log) real exchange rate is of particular interest given a large literature on examining deviations from PPP. For $p = 1$, the reported τ_T for that variable is more or less in line with the exiting literature (see, for instance, Rossi (2005)), with few exceptions as Sweden, Japan, Finland and Belgium. But, as can be seen from Table 5, much more mass has been put in posteriors on longer half-lives for almost all countries in the GLTU(2) model. This accords Dou and Müller's (2019) finding, which is based on a longer sample of the US/UK real exchange rate only.

Surprisingly, as shown in Tables 5, 9, 11 and 7, for most of the macroeconomic series under consideration, percentiles of τ_T remain numerically similar for $p = 2, 3, 4, 5$. But they are substantially different from when $p = 1$. This empirical observation further substantiates the motivation of this chapter and Dou and Müller (2019): AR(1) form of long-range dependence may not adequately model the persistence properties of a generic macroeconomic time series. That being said, the GLTU(p) model with a relatively large p may also lead to numerical instabilities, like the 5th percentiles of α_T for $p = 5$ as in Table 6. But our empirical results suggest that τ_T is reasonably stable, even for $p = 5$.

6 Concluding Remarks

This chapter examines the persistence of macroeconomic time series based on the Jordá-Schularick-Taylor Macrohistory Database. It is found that quantitative statements about the persistence of macroeconomic time series can be substantially altered, when more general long-range dependence modelling other than the usual AR(1) form is allowed. In practice, I recommend computing the persistence measure τ_T as in (16) within the econometric framework of Dou and Müller (2019).

There is still much work to do on persistence measuring. First of all, as is well known, the limiting result (3) would continue to hold as long as the time variation in variances of the driving disturbances $\{u_{T,t}\}_{t=-\infty}^{\infty}$ is a stationary short-memory process. But this is often not the case for macroeconomic time series, that is, there seems to be much low-frequency variability in the second moment (cf. Müller and Watson (2008)). This can be arguably relevant for the empirical exercises conducted in this chapter, because the Jordá-Schularick-Taylor macrohistory sample spans over a long enough period on an annual basis. To make progress along this direction, one can consider a version of the GLTU(p) model with low-frequency heteroskedastic driving disturbances in their natural moving average representations, as Müller and Watson (2008) did for many alternative models of low-frequency variability. It seems that the half-life measure used in this chapter and the

Table 2: The 5th, 50th and 95th percentiles of α_T for $p = 1$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	0.77	0.93	0.90	0.94	0.97	0.97	0.96	0.93	0.94	0.96	0.80	0.95	0.91	0.94	0.84	0.95	0.91
	0.83	0.97	0.96	0.98	0.98	0.98	0.98	0.97	0.97	0.98	0.90	0.98	0.97	0.98	0.93	0.98	0.96
	0.94	0.99	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.99	0.99	0.99	0.98	0.99	0.98
Int. rate spr.	0.77	0.77	0.82	0.79	0.83	0.98	0.77	0.77	0.85	0.82	0.82	0.98	0.98	0.98	0.98	0.98	0.83
	0.81	0.83	0.87	0.89	0.84	0.98	0.81	0.81	0.89	0.87	0.87	0.98	0.98	0.98	0.98	0.98	0.88
	0.92	0.93	0.89	0.98	0.88	0.98	0.90	0.91	0.92	0.89	0.89	0.98	0.98	0.98	0.98	0.98	0.90
Inflation	0.96	0.96	0.93	0.98	0.96	0.98	0.95	0.95	0.87	0.98	0.94	0.98	0.97	0.93	0.95	0.91	0.96
	0.98	0.97	0.94	0.98	0.98	0.98	0.96	0.96	0.88	0.98	0.96	0.98	0.98	0.96	0.96	0.92	0.98
	0.98	0.98	0.95	0.98	0.98	0.98	0.97	0.97	0.93	0.98	0.96	0.98	0.98	0.97	0.98	0.93	0.98
Real ex. rate	N.A.	0.82	0.89	0.98	0.83	0.84	0.84	0.84	0.94	0.98	0.82	0.82	0.98	0.86	0.83	0.98	0.84
	N.A.	0.87	0.92	0.98	0.88	0.88	0.88	0.88	0.98	0.98	0.87	0.87	0.98	0.90	0.88	0.98	0.88
	N.A.	0.89	0.93	0.98	0.90	0.90	0.90	0.90	0.98	0.98	0.89	0.89	0.98	0.92	0.89	0.98	0.90
Real housing	0.91	0.93	0.89	0.89	0.87	0.86	0.91	0.89	0.89	0.98	0.84	0.94	0.89	0.94	0.91	0.87	0.90
	0.93	0.95	0.92	0.92	0.91	0.89	0.93	0.92	0.92	0.98	0.88	0.95	0.92	0.96	0.93	0.91	0.93
	0.94	0.96	0.93	0.93	0.92	0.92	0.94	0.93	0.93	0.98	0.90	0.96	0.93	0.96	0.94	0.92	0.94
Debt-GDP	0.96	0.94	0.92	0.95	0.91	0.97	0.85	0.86	0.89	0.95	0.86	0.93	0.87	0.89	0.95	0.92	0.94
	0.98	0.97	0.94	0.98	0.93	0.98	0.89	0.90	0.92	0.98	0.90	0.95	0.91	0.92	0.98	0.94	0.97
	0.99	0.99	0.95	0.99	0.95	0.99	0.92	0.92	0.93	0.99	0.92	0.96	0.92	0.93	0.99	0.95	0.99
Real I-GDP	0.78	0.93	0.90	0.90	0.94	0.95	0.88	0.93	0.87	0.93	0.89	0.86	0.90	0.86	0.80	0.87	0.84
	0.84	0.97	0.93	0.96	0.96	0.98	0.91	0.95	0.91	0.97	0.92	0.90	0.97	0.89	0.89	0.91	0.93
	0.95	0.99	0.94	0.99	0.96	0.98	0.92	0.96	0.92	0.99	0.93	0.92	1.00	0.92	0.98	0.92	0.98
Loan-GDP	0.89	0.94	0.94	0.97	0.91	0.96	0.90	0.92	0.98	0.97	0.84	0.89	0.98	0.98	0.96	0.89	0.94
	0.92	0.96	0.95	0.98	0.93	0.98	0.93	0.94	0.98	0.98	0.88	0.92	0.98	0.98	0.98	0.92	0.95
	0.93	0.96	0.96	0.99	0.94	0.98	0.94	0.95	0.98	0.99	0.90	0.93	0.98	0.98	0.99	0.93	0.96
Real C-I	0.90	0.84	0.84	0.97	0.87	0.87	0.98	0.98	0.90	0.95	0.98	0.85	0.89	0.90	0.83	0.89	0.90
	0.93	0.88	0.88	0.98	0.91	0.91	0.98	0.98	0.93	0.98	0.98	0.89	0.91	0.93	0.93	0.92	0.93
	0.94	0.90	0.90	0.99	0.92	0.92	0.98	0.98	0.94	0.99	0.98	0.92	0.93	0.94	0.98	0.93	0.94

Note: The first row is 3-letter ISO country code.

Table 3: The 5th, 50th and 95th percentiles of τ_T for $p = 1$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	3.1	10.0	7.3	12.0	20.6	22.7	19.2	9.6	10.9	18.3	3.5	12.8	7.7	11.1	4.3	13.8	7.5
	4.1	25.4	18.8	28.6	37.8	39.2	37.2	23.7	26.6	36.3	6.7	29.3	20.2	28.4	9.9	30.9	18.1
	10.8	47.2	46.1	47.4	48.8	48.8	48.7	46.8	47.3	48.5	29.0	47.5	46.5	47.2	39.7	47.8	45.8
Int. rate spr.	3.0	3.1	3.9	3.4	4.1	33.3	3.0	3.0	4.7	3.9	3.9	33.3	33.3	33.3	33.3	33.3	4.2
	3.7	4.0	5.2	6.1	4.4	40.6	3.7	3.7	6.4	5.2	5.2	40.6	33.3	40.6	40.6	40.6	5.6
	8.6	10.1	6.4	27.9	5.7	40.7	7.2	8.0	8.2	6.4	6.4	40.7	33.4	40.7	40.7	40.7	6.6
Inflation	17.3	15.4	9.6	33.3	19.6	33.3	13.5	14.2	5.2	33.3	12.0	33.3	24.7	10.0	12.7	7.6	16.5
	28.3	20.3	10.9	33.3	33.3	33.3	18.7	16.6	5.7	33.3	15.5	33.3	33.3	15.8	17.4	9.0	33.3
	33.4	33.4	13.3	33.4	33.4	33.4	24.9	19.8	9.4	33.4	17.9	33.4	33.4	23.1	31.2	10.2	33.4
Real ex. rate	N.A.	3.9	6.2	33.3	4.2	4.2	4.2	4.2	12.4	33.3	3.9	3.9	33.3	4.8	4.1	33.3	4.4
	N.A.	5.2	8.2	40.6	5.7	5.7	5.7	5.7	33.3	40.6	5.2	5.2	40.6	6.6	5.6	40.6	5.8
	N.A.	6.4	9.4	40.7	6.6	6.9	6.9	6.9	33.4	40.7	6.4	6.4	40.7	8.2	6.6	40.7	7.1
Real housing	7.6	10.1	6.0	6.3	5.4	4.8	7.6	6.2	6.3	33.3	4.2	10.8	6.1	11.2	7.4	5.4	7.1
	9.5	13.8	8.2	8.4	7.4	6.6	9.5	8.2	8.3	40.6	5.7	14.4	8.2	15.8	9.4	7.5	9.3
	11.7	17.3	9.3	9.8	9.0	8.2	12.0	9.3	9.7	40.7	6.9	16.7	9.3	18.4	11.7	9.1	11.7
Debt-GDP	16.2	12.2	8.7	13.4	7.6	22.5	4.8	5.0	6.3	12.8	4.8	10.0	5.4	6.3	13.0	8.6	10.7
	34.0	27.4	11.3	31.1	9.8	38.9	6.5	6.8	8.4	29.3	6.6	13.6	7.5	8.3	30.4	11.7	26.1
	48.4	47.4	13.8	48.1	13.1	49.0	8.2	8.3	9.7	47.8	8.2	15.8	9.0	9.6	48.3	13.7	47.0
Real I-GDP	3.1	10.3	7.1	7.1	12.5	14.2	5.6	10.5	5.4	9.8	6.2	4.8	7.1	4.8	3.5	5.3	4.2
	4.4	24.8	9.4	18.3	15.7	33.3	7.5	14.8	7.4	24.3	8.3	6.6	20.8	6.6	6.5	7.5	9.6
	13.7	47.1	11.7	46.6	19.1	33.4	9.0	17.3	9.0	47.1	9.7	8.2	$+\infty$	8.2	28.4	9.0	37.9
Loan-GDP	6.0	11.1	10.8	21.3	7.4	15.6	7.1	8.6	33.3	22.5	4.4	6.2	33.3	33.3	18.4	6.0	10.8
	8.2	15.6	14.9	39.6	9.4	33.3	9.4	11.3	40.6	38.9	5.8	8.2	33.3	33.3	37.0	8.2	14.2
	9.3	18.7	17.8	48.9	11.7	33.4	11.7	13.7	40.7	48.8	7.1	9.6	33.4	33.4	48.7	9.3	16.5
Real C-I	7.1	4.2	4.2	20.3	5.4	5.4	33.3	33.3	7.1	15.3	33.3	4.8	6.0	7.1	4.1	6.1	7.1
	9.3	5.7	5.7	37.5	7.5	7.5	33.3	40.6	9.3	33.6	40.6	6.5	8.1	9.4	9.6	8.2	9.3
	11.7	6.9	6.9	48.8	9.0	9.0	33.4	40.7	11.7	48.3	40.7	8.2	9.3	11.7	39.3	9.3	11.7

Note: The first row is 3-letter ISO country code.

Table 4: The 5th, 50th and 95th percentiles of α_T for $p = 2$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	0.08	0.93	0.10	0.90	0.93	0.95	0.96	0.91	0.88	0.94	0.80	0.94	0.89	0.85	0.64	0.94	0.90
	0.86	0.97	0.94	0.97	0.98	0.99	0.98	0.96	0.97	0.98	0.92	0.98	0.96	0.96	0.92	0.98	0.96
	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.99	0.98	0.99	0.98	0.99	0.98
Int. rate spr.	0.87	0.78	0.83	0.88	0.89	0.79	0.83	0.77	0.88	0.85	0.83	0.83	0.83	0.86	0.86	0.85	0.84
	0.96	0.94	0.95	0.95	0.96	0.95	0.95	0.94	0.95	0.95	0.95	0.95	0.96	0.95	0.95	0.95	0.95
	0.97	0.97	0.97	0.97	0.98	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.98	0.97	0.97	0.97	0.97
Inflation	0.93	0.94	0.93	0.94	0.94	0.95	0.94	0.92	0.93	0.96	0.89	0.94	0.95	0.93	0.93	0.94	0.93
	0.96	0.96	0.96	0.96	0.97	0.97	0.96	0.96	0.96	0.98	0.96	0.97	0.96	0.96	0.96	0.96	0.96
	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.98	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.98
Real ex. rate	N.A.	0.71	0.93	0.78	0.81	0.87	0.82	0.85	0.96	0.81	0.72	0.80	0.80	0.88	0.82	0.82	0.86
	N.A.	0.91	0.97	0.93	0.94	0.95	0.93	0.95	0.98	0.95	0.91	0.94	0.94	0.95	0.95	0.95	0.95
	N.A.	0.97	0.98	0.97	0.97	0.97	0.98	0.97	0.99	0.97	0.97	0.97	0.97	0.98	0.97	0.97	0.98
Real housing	0.94	0.95	0.92	0.94	0.84	0.00	0.94	0.91	0.92	0.78	0.85	0.96	0.91	0.96	0.94	0.92	0.94
	0.97	0.98	0.97	0.97	0.94	0.12	0.98	0.97	0.97	0.91	0.95	0.98	0.96	0.98	0.97	0.96	0.97
	0.99	0.99	0.98	0.99	0.98	0.72	0.99	0.98	0.99	0.97	0.97	0.99	0.99	0.99	0.99	0.98	0.98
Debt-GDP	0.96	0.87	0.95	0.94	0.94	0.96	0.87	0.86	0.93	0.93	0.86	0.95	0.91	0.92	0.94	0.95	0.92
	0.98	0.97	0.98	0.98	0.98	0.99	0.95	0.94	0.97	0.97	0.95	0.98	0.96	0.96	0.98	0.98	0.97
	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.99	0.99	0.98	0.99	0.98	0.99	0.99	0.99	0.99
Real I-GDP	0.83	0.93	0.94	0.93	0.96	0.96	0.91	0.95	0.92	0.93	0.92	0.90	0.92	0.87	0.81	0.92	0.86
	0.95	0.97	0.97	0.96	0.97	0.98	0.96	0.97	0.97	0.97	0.97	0.96	0.96	0.95	0.93	0.96	0.95
	0.97	0.98	0.98	0.98	0.98	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.98	0.98
Loan-GDP	0.91	0.96	0.96	0.97	0.94	0.97	0.94	0.94	0.87	0.97	0.84	0.93	0.97	0.98	0.96	0.91	0.96
	0.96	0.98	0.99	0.99	0.97	0.98	0.98	0.97	0.95	0.99	0.94	0.97	0.99	0.99	0.98	0.96	0.98
	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.97	0.99	0.98	0.98	0.99	1.00	0.99	0.98	0.99
Real C-I	0.94	0.86	0.87	0.96	0.93	0.89	0.98	0.87	0.93	0.95	0.76	0.92	0.91	0.94	0.81	0.92	0.94
	0.97	0.95	0.95	0.98	0.96	0.96	0.99	0.95	0.97	0.98	0.93	0.96	0.96	0.97	0.92	0.96	0.97
	0.98	0.97	0.97	0.99	0.97	0.98	0.99	0.97	0.99	0.99	0.97	0.97	0.98	0.98	0.98	0.98	0.99

Note: The first row is 3-letter ISO country code.

Table 5: The 5th, 50th and 95th percentiles of τ_T for $p = 2$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	3.1	9.3	5.9	8.4	15.3	15.4	14.7	7.5	8.4	11.8	3.8	9.8	6.9	7.1	4.3	10.6	7.1
	4.9	26.5	11.8	18.1	29.3	28.2	30.9	16.5	18.2	21.9	8.4	22.9	19.1	14.8	8.1	23.5	18.4
	40.7	47.3	42.6	44.8	47.9	47.5	48.2	44.4	44.7	45.6	42.0	46.7	46.0	44.1	39.4	46.3	45.3
Int. rate spr.	3.8	3.2	3.5	4.9	6.2	3.2	3.5	3.2	5.3	3.8	3.5	3.4	4.3	3.7	3.5	3.8	4.0
	29.0	12.5	20.4	25.6	33.5	25.3	20.9	10.1	23.1	21.6	21.0	23.5	20.4	26.8	25.1	24.5	19.4
	48.7	46.2	48.1	47.9	48.8	48.2	47.9	46.6	47.8	47.8	47.5	48.1	47.5	48.7	48.5	48.4	47.0
Inflation	8.5	11.3	6.3	12.5	13.3	21.3	11.2	6.5	6.5	25.6	4.4	12.8	15.0	7.1	6.5	5.9	11.6
	34.3	35.7	33.8	38.8	36.9	40.1	37.0	33.7	33.7	43.7	33.1	36.2	37.6	34.1	33.5	35.9	36.3
	49.0	49.0	48.8	49.2	49.2	49.2	49.1	49.1	49.0	49.5	49.0	49.1	49.2	49.0	49.1	49.1	49.1
Real ex. rate	N.A.	3.2	11.2	3.4	3.7	4.9	4.0	4.4	20.3	3.4	3.1	3.4	3.4	5.6	3.7	3.5	4.9
	N.A.	7.2	31.3	11.8	13.7	22.8	12.2	18.5	38.5	18.2	6.2	14.7	16.3	18.5	17.8	19.0	18.2
	N.A.	45.1	48.1	46.7	45.3	47.0	44.5	46.7	49.2	47.9	44.1	46.6	47.2	46.5	47.0	47.2	46.0
Real housing	11.8	14.0	8.7	9.4	5.7	4.7	10.1	8.4	8.5	3.2	4.4	10.0	6.6	16.9	10.7	9.4	13.1
	27.8	30.4	24.8	25.1	11.5	30.6	25.6	22.6	20.0	7.8	18.1	19.1	14.6	34.3	25.9	29.4	32.3
	48.2	48.5	47.9	47.9	41.5	40.0	47.3	46.6	46.0	44.8	46.3	44.7	44.2	48.8	47.5	48.7	48.5
Debt-GDP	12.5	9.0	11.2	9.8	10.4	15.3	5.0	5.0	9.3	10.1	5.6	12.2	7.6	6.9	9.1	12.3	7.9
	26.9	17.8	24.5	24.1	24.1	29.7	15.7	10.6	23.8	23.4	15.4	25.9	23.1	13.1	21.3	29.7	17.8
	47.2	44.1	47.0	46.7	47.3	47.9	45.6	41.7	47.6	46.9	45.4	47.8	47.2	41.6	45.7	48.5	44.7
Real I-GDP	3.8	10.9	12.6	11.3	20.1	20.0	8.4	16.2	9.4	10.0	9.4	6.6	10.0	5.4	3.8	8.7	4.7
	20.4	29.3	30.4	32.5	38.2	37.8	26.3	34.8	29.0	28.2	25.9	24.0	31.8	19.4	9.8	30.9	16.6
	46.9	47.8	48.7	48.2	49.0	49.1	48.1	49.0	48.4	48.1	47.9	47.6	48.5	47.2	44.2	48.7	45.9
Loan-GDP	7.2	16.0	13.7	14.0	10.4	17.6	11.8	10.6	5.1	16.0	4.4	9.7	17.8	15.4	15.1	7.5	16.9
	17.8	33.2	31.6	30.9	25.1	35.3	30.6	24.4	25.0	34.3	13.1	26.6	35.6	31.9	32.5	19.6	34.8
	45.9	48.8	48.2	48.1	47.5	49.0	48.5	47.2	48.1	48.4	45.3	48.1	49.0	48.2	48.1	46.2	49.0
Real C-I	15.0	4.6	4.9	20.7	13.8	6.2	21.8	4.3	9.0	14.0	3.2	8.8	7.3	16.5	4.1	8.7	9.8
	34.0	18.4	23.4	38.4	35.6	15.3	37.2	26.5	20.1	32.8	8.8	32.8	17.9	35.6	8.2	23.1	25.4
	49.0	47.0	47.9	49.0	49.1	45.0	49.1	48.5	45.9	48.4	47.8	48.8	45.3	49.0	42.0	46.9	47.8

Note: The first row is 3-letter ISO country code.

Table 6: The 5th, 50th and 95th percentiles of α_T for $p = 5$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.96	0.00	0.85	0.97	0.98	0.97	0.87	0.74	0.92	0.69	0.92	0.82	0.53	0.24	0.94	0.67
	0.96	0.98	0.95	0.99	0.99	0.99	0.99	0.99	0.98	0.99	0.97	0.99	0.98	0.98	0.95	0.99	0.98
Int. rate spr.	0.88	0.00	0.00	0.00	0.42	0.00	0.11	0.00	0.00	0.00	0.31	0.79	0.00	0.62	0.54	0.56	0.00
	0.94	0.92	0.93	0.93	0.96	0.93	0.91	0.93	0.94	0.91	0.93	0.92	0.86	0.93	0.92	0.93	0.91
	0.96	0.97	0.97	0.97	0.98	0.98	0.96	0.98	0.98	0.97	0.97	0.96	0.98	0.97	0.97	0.96	0.97
Inflation	0.90	0.88	0.91	0.90	0.88	0.67	0.92	0.87	0.91	0.91	0.88	0.87	0.62	0.91	0.89	0.90	0.89
	0.96	0.94	0.95	0.95	0.96	0.97	0.95	0.95	0.95	0.97	0.95	0.96	0.95	0.95	0.94	0.94	0.96
	0.98	0.98	0.99	0.97	0.98	0.99	0.98	0.99	0.99	0.99	0.98	0.99	0.98	0.97	0.99	0.97	0.98
Real ex. rate	N.A.	0.00	0.47	0.00	0.00	0.00	0.00	0.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.66	0.00
	N.A.	0.74	0.96	0.90	0.88	0.92	0.89	0.91	0.97	0.93	0.89	0.92	0.90	0.93	0.93	0.94	0.91
	N.A.	0.97	0.98	0.97	0.97	0.97	0.97	0.98	0.99	0.97	0.98	0.97	0.97	0.98	0.98	0.97	0.97
Real housing	0.00	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.97	0.97	0.96	0.96	0.73	0.00	0.95	0.94	0.96	0.89	0.91	0.96	0.96	0.97	0.96	0.95	0.96
	0.99	0.99	0.98	0.99	0.97	0.48	0.99	0.98	0.99	0.98	0.98	0.99	0.99	0.99	0.99	0.98	0.98
Debt-GDP	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.00	0.00	0.00	0.00	0.00	0.55	0.00	0.00	0.00	0.00
	0.97	0.70	0.96	0.97	0.96	0.97	0.95	0.87	0.95	0.93	0.90	0.96	0.96	0.96	0.97	0.97	0.92
	0.99	0.98	0.99	0.99	0.99	0.99	0.98	0.98	0.99	0.99	0.98	0.99	0.98	0.99	0.99	0.99	0.99
Real I-GDP	0.00	0.00	0.57	0.00	0.00	0.93	0.00	0.77	0.56	0.00	0.00	0.00	0.66	0.00	0.00	0.89	0.00
	0.91	0.96	0.96	0.95	0.97	0.98	0.95	0.97	0.96	0.96	0.96	0.95	0.94	0.94	0.86	0.96	0.93
	0.97	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.98	0.98	0.97	0.98
Loan-GDP	0.00	0.61	0.00	0.00	0.00	0.61	0.00	0.00	0.60	0.41	0.00	0.00	0.26	0.00	0.00	0.00	0.54
	0.95	0.98	0.98	0.98	0.96	0.98	0.97	0.96	0.94	0.98	0.85	0.95	0.99	0.99	0.98	0.96	0.97
	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.97	0.99	0.98	0.98	0.99	1.00	0.99	0.98	0.99
Real C-I	0.00	0.44	0.00	0.00	0.89	0.00	0.00	0.64	0.00	0.44	0.00	0.79	0.00	0.66	0.00	0.00	0.00
	0.96	0.94	0.93	0.97	0.95	0.93	0.98	0.93	0.96	0.97	0.93	0.93	0.92	0.96	0.85	0.95	0.97
	0.98	0.97	0.97	0.99	0.97	0.99	0.99	0.97	0.99	0.99	0.98	0.96	0.98	0.98	0.98	0.98	0.99

Notes: The first row is 3-letter ISO country code. α_T is downcoded by 0.

Table 7: The 5th, 50th and 95th percentiles of τ_T for $p = 5$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	3.1	8.4	5.0	7.3	13.7	14.6	14.7	6.6	7.1	9.7	3.5	8.2	5.6	6.0	3.7	8.7	5.6
	3.8	24.7	7.6	12.1	24.4	26.5	27.8	10.4	11.3	15.3	6.0	15.3	11.0	8.5	6.0	16.6	10.0
	26.9	46.3	17.6	36.8	45.9	46.0	46.6	36.5	35.6	39.0	34.3	42.0	40.6	29.7	17.3	41.5	42.3
Int. rate spr.	6.6	3.2	3.5	4.4	8.5	3.2	4.0	3.2	5.3	4.1	3.7	6.9	3.8	4.7	5.0	6.3	3.8
	33.7	8.7	19.4	21.9	35.3	26.9	20.1	19.0	20.6	17.1	25.1	26.3	8.8	27.8	23.2	25.9	18.8
	48.8	45.1	46.2	47.0	48.8	47.5	46.2	46.7	46.3	44.5	47.0	47.0	44.4	47.8	46.9	47.5	44.2
Inflation	4.7	15.4	5.9	17.8	15.6	23.4	20.6	18.1	5.1	21.8	8.7	10.0	11.0	14.7	12.3	18.4	8.8
	35.3	37.8	34.7	39.0	37.8	40.9	39.4	36.9	34.1	40.9	34.7	35.9	36.0	36.3	36.0	38.4	34.0
	49.0	49.1	48.8	49.4	49.1	49.4	49.2	49.0	48.7	49.5	48.8	49.0	49.1	49.1	48.8	49.1	48.7
Real ex. rate	N.A.	3.1	12.3	3.2	3.4	4.9	3.7	3.8	29.7	3.4	3.1	3.5	3.4	5.1	3.5	3.5	4.4
	N.A.	4.9	27.5	8.8	10.7	19.7	9.0	14.8	41.7	21.3	5.1	20.6	15.7	16.2	18.4	30.1	14.8
	N.A.	37.6	47.0	44.2	42.5	46.3	40.4	44.2	49.4	46.7	43.2	45.0	42.8	44.0	45.7	47.6	42.5
Real housing	11.3	14.6	8.2	8.1	5.1	4.0	8.5	7.2	7.8	3.2	4.0	9.0	6.6	16.2	10.3	10.0	12.9
	27.2	29.3	27.2	21.0	8.4	30.9	18.4	16.3	17.5	7.6	12.3	15.3	21.5	31.3	22.8	28.4	25.7
	47.0	47.6	46.9	46.8	24.7	39.0	44.7	43.5	44.0	42.2	43.5	41.3	46.0	48.2	45.9	47.8	47.5
Debt-GDP	10.9	6.8	10.1	9.0	9.4	13.7	4.6	4.9	8.2	8.8	5.1	11.3	7.1	6.9	8.4	10.6	6.9
	22.8	10.9	21.6	18.8	19.0	24.8	13.7	8.5	16.5	16.2	10.4	19.7	21.5	12.2	17.1	26.2	12.6
	45.1	30.1	44.5	45.7	44.5	45.9	45.7	38.1	42.2	40.9	40.3	43.7	45.9	44.2	44.2	46.7	40.9
Real I-GDP	3.7	10.0	13.8	12.2	18.8	19.4	7.5	16.2	9.6	9.1	8.7	5.9	13.8	5.0	3.7	7.4	4.3
	17.9	26.6	30.0	28.8	36.9	37.0	23.7	32.2	28.1	28.4	23.2	26.8	30.1	16.8	7.5	34.0	12.6
	45.6	46.6	47.6	47.3	48.5	49.0	46.7	48.4	47.3	46.7	46.3	47.5	47.6	45.6	38.5	48.4	44.8
Loan-GDP	6.9	15.1	12.2	12.8	10.0	15.7	10.7	9.7	11.6	14.1	3.8	10.0	17.3	14.7	14.0	6.8	16.3
	13.4	30.7	29.8	27.2	21.6	34.0	26.3	19.6	30.9	33.4	8.5	26.2	34.0	29.7	26.9	20.1	30.0
	43.4	47.6	48.1	46.9	44.8	48.4	46.5	43.8	48.1	48.7	39.5	46.5	48.4	48.1	46.6	46.6	47.3
Real C-I	17.8	4.3	4.4	21.2	20.9	5.9	22.8	8.7	7.9	16.8	3.2	15.7	6.5	20.0	3.8	8.1	9.0
	34.7	25.6	18.8	36.9	38.1	11.8	34.1	27.2	16.6	33.5	9.7	35.0	13.4	37.0	6.6	19.0	31.0
	48.5	46.7	46.0	48.5	49.0	40.6	48.1	47.3	44.7	48.4	45.3	48.8	40.1	49.0	39.2	44.1	47.0

Note: The first row is 3-letter ISO country code.

limited-information likelihood framework can as well be adapted to that new model. I leave this for future research.

Second, it is natural to ask how to choose p for a given series in the GLTU(p) class. In a Bayesian framework, this sort of model comparison can be investigated by using Bayes factors. But the computations are not straightforward given the implicit prior I used for $\mathbf{h}$. Alternatively, a formal statistical test needs to be developed in order to discriminate between different GLTU(p) models, which is nontrivial due to presence of (at least one) nuisance parameters that cannot be consistently estimated.

Appendix A More Tables of α_T and τ_T under GLTU(p)

In this section, I list the tables for the 5th, 50th and 95th percentiles of α_T and τ_T, for $p = 3, 4$, respectively. A few findings stand out from these tables to supplement those stated in Section 5. First, regardless of the choice of country and how the persistence properties are modelled, aggregate inflations always display high degree of persistence. Second, as p gets larger, the persistence measure α_T becomes numerically unstable, which creates a seemingly heavy tail in the posterior distribution of α_T for some variables after downcoding. But this is not the case for τ_T, at least for the five p's I considered. Of course, many further analysis can be done to provide more economic insights out of these tables. But I do not pursue that in this paper.

Table 8: The 5th, 50th and 95th percentiles of α_T for $p = 3$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	0.00	0.78	0.00	0.00	0.86	0.48	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.71	0.96	0.65	0.93	0.98	0.98	0.98	0.93	0.94	0.96	0.88	0.97	0.94	0.80	0.72	0.97	0.94
	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.97	0.99	0.98	0.99	0.97	0.99	0.98
Int. rate spr.	0.88	0.22	0.81	0.84	0.87	0.00	0.83	0.42	0.84	0.81	0.84	0.86	0.00	0.84	0.84	0.85	0.81
	0.94	0.93	0.93	0.94	0.96	0.95	0.92	0.94	0.94	0.91	0.94	0.93	0.91	0.94	0.93	0.93	0.92
	0.96	0.97	0.97	0.97	0.98	0.98	0.96	0.97	0.97	0.96	0.96	0.96	0.98	0.96	0.96	0.96	0.97
Inflation	0.92	0.92	0.93	0.93	0.95	0.96	0.92	0.92	0.93	0.94	0.90	0.93	0.92	0.92	0.90	0.91	0.92
	0.95	0.95	0.95	0.95	0.97	0.98	0.96	0.95	0.95	0.97	0.95	0.96	0.95	0.95	0.95	0.95	0.96
	0.97	0.97	0.97	0.97	0.98	0.99	0.98	0.97	0.99	0.99	0.97	0.98	0.97	0.97	0.97	0.97	0.98
Real ex. rate	N.A.	0.00	0.90	0.00	0.00	0.82	0.00	0.33	0.95	0.79	0.00	0.76	0.62	0.39	0.44	0.78	0.51
	N.A.	0.87	0.96	0.91	0.91	0.93	0.91	0.93	0.97	0.93	0.90	0.93	0.91	0.94	0.93	0.94	0.93
	N.A.	0.97	0.98	0.97	0.97	0.97	0.97	0.97	0.99	0.96	0.98	0.96	0.97	0.98	0.97	0.97	0.97
Real housing	0.48	0.91	0.70	0.55	0.00	0.00	0.00	0.00	0.39	0.20	0.60	0.00	0.34	0.80	0.70	0.88	0.87
	0.97	0.97	0.96	0.97	0.88	0.00	0.96	0.95	0.96	0.90	0.93	0.97	0.96	0.97	0.97	0.95	0.96
	0.99	0.99	0.98	0.99	0.98	0.59	0.99	0.98	0.99	0.97	0.97	0.99	0.99	0.99	0.99	0.98	0.98
Debt-GDP	0.42	0.00	0.48	0.65	0.45	0.20	0.80	0.00	0.36	0.00	0.33	0.67	0.85	0.38	0.39	0.55	0.00
	0.98	0.93	0.97	0.97	0.97	0.98	0.95	0.92	0.96	0.96	0.93	0.97	0.96	0.96	0.97	0.97	0.96
	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.99	0.99	0.98	0.99	0.98	0.99	0.99	0.99	0.99
Real I-GDP	0.82	0.90	0.92	0.89	0.78	0.95	0.82	0.92	0.89	0.79	0.87	0.47	0.89	0.59	0.00	0.90	0.54
	0.93	0.96	0.97	0.95	0.97	0.98	0.95	0.97	0.96	0.96	0.96	0.95	0.95	0.94	0.89	0.96	0.94
	0.96	0.98	0.98	0.97	0.98	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.98	0.97	0.98	0.98
Loan-GDP	0.19	0.74	0.68	0.65	0.78	0.63	0.89	0.71	0.86	0.84	0.00	0.86	0.78	0.74	0.93	0.64	0.94
	0.96	0.98	0.98	0.98	0.97	0.98	0.97	0.97	0.94	0.98	0.91	0.96	0.99	0.99	0.98	0.96	0.97
	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.97	0.99	0.98	0.98	0.99	1.00	0.99	0.98	0.99
Real C-I	0.86	0.71	0.82	0.35	0.91	0.00	0.00	0.86	0.00	0.78	0.00	0.89	0.00	0.93	0.00	0.81	0.52
	0.96	0.94	0.94	0.97	0.95	0.94	0.98	0.93	0.96	0.97	0.92	0.94	0.94	0.96	0.89	0.96	0.97
	0.98	0.97	0.97	0.99	0.97	0.98	0.99	0.96	0.99	0.99	0.97	0.96	0.98	0.98	0.98	0.98	0.99

Notes: The first row is 3-letter ISO country code. α_T is downcoded by 0.

Table 9: The 5th, 50th and 95th percentiles of τ_T for $p = 3$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	3.1	9.3	5.3	7.6	15.0	15.1	14.4	6.8	7.5	10.7	3.5	8.8	6.0	6.3	3.8	9.6	6.5
	4.0	25.3	8.4	13.2	27.5	28.5	29.4	12.1	13.2	16.9	6.8	19.4	14.3	9.4	6.2	20.0	15.1
	35.9	46.7	29.7	39.8	47.0	47.2	47.5	40.7	40.7	43.1	40.3	45.6	44.7	35.7	27.6	45.4	44.0
Int. rate spr.	4.4	3.2	3.5	5.0	6.9	3.2	4.3	3.2	5.1	4.0	3.7	5.6	3.7	4.4	4.9	5.1	4.0
	32.2	10.4	19.7	24.1	33.8	26.3	22.1	20.1	21.8	17.3	25.6	26.5	11.2	27.2	24.0	25.6	19.8
	48.2	46.5	46.6	47.3	48.7	47.6	46.5	47.3	46.7	46.0	47.3	47.6	45.7	47.6	46.9	47.2	46.0
Inflation	14.6	16.5	12.9	20.6	21.2	21.9	11.5	14.0	5.3	25.3	6.5	14.7	13.7	8.5	12.8	19.1	11.0
	36.9	37.3	37.9	40.0	39.8	40.1	37.5	37.2	37.5	43.4	34.7	37.6	37.5	37.2	35.3	38.8	35.1
	48.8	49.1	49.1	49.4	49.1	49.4	49.0	49.0	49.0	49.7	48.7	49.0	49.1	48.8	48.7	49.1	48.8
Real ex. rate	N.A.	3.1	11.6	3.2	3.5	4.9	3.7	4.0	24.7	3.5	3.1	3.4	3.4	5.3	3.5	3.4	4.6
	N.A.	5.3	29.5	9.3	13.2	20.1	9.7	15.6	39.0	21.8	5.3	21.6	18.2	17.9	16.8	25.9	16.3
	N.A.	42.6	47.6	45.0	44.5	46.2	44.2	45.7	49.1	46.5	46.2	46.3	45.6	45.7	46.2	48.2	45.7
Real housing	11.8	14.6	8.2	8.7	5.1	4.3	9.0	7.5	7.9	3.2	3.8	9.7	6.8	16.8	10.6	10.4	13.5
	27.2	30.9	25.0	23.7	8.8	31.3	20.6	17.6	17.3	7.8	13.8	16.5	15.4	33.1	24.0	29.4	28.4
	47.0	47.3	46.9	46.7	34.4	38.8	46.2	45.3	45.1	44.0	45.4	42.9	45.1	48.1	46.7	47.5	47.5
Debt-GDP	11.9	8.2	10.9	9.6	9.8	14.8	4.6	4.9	8.5	9.4	5.0	12.2	7.2	7.2	9.1	11.5	7.6
	24.5	13.5	22.5	21.9	21.0	27.3	14.3	9.4	18.8	19.1	12.2	23.1	21.3	13.5	19.7	27.5	15.1
	46.7	41.2	46.2	46.6	45.5	47.3	46.6	40.0	45.1	44.5	43.4	46.0	46.6	43.8	45.4	47.3	43.5
Real I-GDP	4.0	11.0	12.8	13.1	20.3	19.6	7.6	16.3	9.4	9.6	8.8	5.6	13.8	4.7	3.5	8.2	4.3
	20.7	28.2	30.4	30.6	37.0	37.0	24.0	33.1	28.4	27.8	23.5	23.8	31.6	16.2	7.9	33.2	14.6
	46.5	47.5	47.8	48.1	48.7	48.8	47.0	48.1	47.6	47.5	46.9	47.9	47.9	45.9	41.7	48.4	45.9
Loan-GDP	6.9	16.2	13.1	13.4	10.3	16.6	11.8	10.4	8.5	15.0	3.8	10.0	18.4	15.0	15.0	6.9	17.6
	15.7	32.6	30.3	28.7	23.4	34.1	28.2	21.6	29.1	31.9	9.6	27.2	35.0	32.2	30.6	18.4	32.2
	45.1	47.8	47.9	47.9	46.3	48.5	47.3	46.2	47.6	48.4	43.2	47.3	48.5	48.2	47.6	46.2	47.8
Real C-I	18.1	4.1	4.4	22.5	19.3	5.9	21.5	8.8	8.4	13.1	3.1	15.1	6.5	21.0	3.8	8.2	9.1
	34.5	21.6	21.5	37.8	37.2	12.9	33.8	26.5	18.1	32.2	7.9	33.7	14.6	38.4	7.1	20.6	25.7
	48.5	46.9	46.7	49.1	49.0	43.4	48.0	47.3	44.8	48.2	46.3	48.5	43.5	48.8	41.5	46.0	46.9

Note: The first row is 3-letter ISO country code.

Table 10: The 5th, 50th and 95th percentiles of α_T for $p = 4$.

Series	**USA**	**GBR**	**CHE**	**SWE**	**ESP**	**POR**	**NOR**	**NLD**	**JPN**	**ITA**	**DEU**	**FRA**	**FIN**	**DNK**	**CAN**	**BEL**	**AUS**
Real GDP	0.00	0.04	0.00	0.00	0.71	0.63	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
	0.54	0.96	0.51	0.95	0.98	0.98	0.98	0.95	0.95	0.97	0.90	0.96	0.94	0.87	0.77	0.97	0.95
	0.97	0.99	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.99	0.98	0.99	0.97	0.99	0.98
Int. rate spr.	0.88	0.08	0.64	0.81	0.87	0.74	0.74	0.51	0.71	0.79	0.75	0.89	0.00	0.83	0.84	0.85	0.74
	0.94	0.94	0.94	0.94	0.96	0.95	0.93	0.94	0.95	0.93	0.94	0.94	0.95	0.94	0.93	0.94	0.93
	0.97	0.97	0.97	0.97	0.98	0.98	0.97	0.97	0.98	0.97	0.97	0.97	0.98	0.97	0.97	0.97	0.97
Inflation	0.92	0.92	0.93	0.92	0.93	0.96	0.90	0.92	0.92	0.94	0.91	0.91	0.93	0.92	0.91	0.92	0.87
	0.96	0.95	0.97	0.96	0.97	0.98	0.96	0.95	0.95	0.98	0.96	0.96	0.96	0.95	0.95	0.95	0.96
	0.98	0.97	1.00	0.98	0.98	0.99	0.99	0.97	0.98	0.99	0.98	0.98	0.98	0.97	0.98	0.98	0.98
Real ex. rate	N.A.	0.00	0.69	0.00	0.00	0.43	0.00	0.00	0.95	0.44	0.00	0.61	0.66	0.13	0.48	0.76	0.36
	N.A.	0.90	0.96	0.92	0.93	0.94	0.93	0.94	0.98	0.93	0.93	0.93	0.93	0.95	0.94	0.95	0.93
	N.A.	0.98	0.98	0.97	0.97	0.97	0.98	0.98	0.99	0.97	0.98	0.97	0.97	0.98	0.97	0.97	0.98
Real housing	0.55	0.51	0.63	0.23	0.00	0.00	0.00	0.00	0.56	0.00	0.00	0.00	0.75	0.68	0.50	0.80	0.81
	0.97	0.97	0.97	0.97	0.90	0.00	0.97	0.96	0.97	0.92	0.94	0.97	0.97	0.98	0.97	0.96	0.97
	0.99	0.99	0.99	0.99	0.98	0.61	0.99	0.99	0.99	0.97	0.98	0.99	0.99	0.99	0.99	0.98	0.99
Debt-GDP	0.67	0.00	0.71	0.65	0.13	0.50	0.76	0.00	0.27	0.19	0.00	0.73	0.68	0.41	0.17	0.65	0.00
	0.98	0.95	0.97	0.97	0.97	0.98	0.96	0.92	0.96	0.97	0.94	0.98	0.96	0.96	0.98	0.97	0.96
	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.99	0.99	0.98	0.99	0.98	0.99	0.99	0.99	0.99
Real I-GDP	0.77	0.73	0.91	0.88	0.27	0.94	0.44	0.93	0.87	0.64	0.66	0.35	0.89	0.42	0.00	0.90	0.36
	0.93	0.97	0.97	0.95	0.97	0.98	0.96	0.97	0.96	0.97	0.96	0.96	0.95	0.95	0.91	0.96	0.95
	0.97	0.98	0.98	0.98	0.98	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.98	0.98	0.98	0.98
Loan-GDP	0.21	0.78	0.74	0.90	0.38	0.71	0.61	0.26	0.85	0.88	0.00	0.83	0.88	0.78	0.82	0.63	0.95
	0.96	0.98	0.98	0.99	0.97	0.98	0.97	0.97	0.94	0.98	0.92	0.96	0.99	0.99	0.98	0.96	0.98
	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.97	0.99	0.98	0.98	0.99	1.00	0.99	0.98	0.99
Real C-I	0.93	0.01	0.61	0.90	0.91	0.00	0.73	0.86	0.23	0.92	0.03	0.89	0.00	0.93	0.00	0.23	0.48
	0.96	0.95	0.94	0.98	0.95	0.95	0.98	0.94	0.97	0.98	0.94	0.94	0.95	0.96	0.91	0.96	0.97
	0.98	0.98	0.98	0.99	0.97	0.99	0.99	0.96	0.99	0.99	0.97	0.96	0.99	0.98	0.98	0.98	0.99

Notes: The first row is 3-letter ISO country code. α_T is downcoded by 0.

Table 11: The 5th, 50th and 95th percentiles of τ_T for $p = 4$.

Series	USA	GBR	CHE	SWE	ESP	POR	NOR	NLD	JPN	ITA	DEU	FRA	FIN	DNK	CAN	BEL	AUS
Real GDP	3.1	8.5	5.1	7.5	14.4	14.8	15.0	6.9	7.5	10.0	3.8	8.1	5.9	6.2	4.0	9.3	6.5
	4.0	25.9	7.8	13.5	27.3	27.0	30.6	12.3	14.1	17.8	7.3	17.6	14.3	9.1	6.6	20.4	14.6
	35.9	47.0	27.8	39.7	47.0	47.2	47.9	40.4	41.2	42.9	41.3	44.4	42.9	37.2	28.2	45.4	42.9
Int. rate spr.	4.9	3.2	3.5	4.9	7.6	3.4	4.0	3.2	5.4	4.3	3.7	8.4	4.0	4.3	4.6	5.0	4.1
	32.3	13.2	21.0	24.3	32.9	29.1	22.0	24.5	22.1	19.0	25.3	28.8	16.3	28.7	23.5	26.8	21.6
	48.7	46.3	47.0	47.5	48.4	48.4	46.6	47.5	46.7	46.0	47.5	47.6	46.2	47.9	46.7	47.8	45.4
Inflation	6.3	8.5	4.6	17.5	12.8	22.8	14.4	17.1	6.0	23.1	5.1	8.5	9.4	7.9	8.2	19.8	6.9
	34.7	37.0	31.0	38.7	37.9	41.0	39.1	38.8	33.7	42.3	32.9	36.2	35.3	36.2	34.8	37.3	32.8
	48.8	49.1	48.4	49.0	49.1	49.4	49.2	49.1	48.7	49.5	48.7	48.8	48.8	48.8	49.0	49.0	48.4
Real ex. rate	N.A.	3.1	11.3	3.2	3.5	5.1	4.0	4.3	23.5	3.4	3.1	3.5	3.5	5.6	3.7	3.5	4.7
	N.A.	7.1	29.4	10.6	14.7	21.3	12.5	17.1	39.1	21.9	7.1	23.4	20.3	18.8	20.9	28.1	17.1
	N.A.	44.2	48.1	44.4	44.5	46.5	42.8	45.7	49.2	47.0	46.2	46.3	45.3	45.9	46.6	47.9	45.0
Real housing	10.6	14.0	7.6	7.9	5.3	4.0	8.5	7.2	7.8	3.4	4.3	8.8	6.5	16.0	10.0	9.4	13.4
	27.0	29.5	25.4	21.3	8.8	30.3	21.2	18.5	18.7	8.7	15.0	15.7	18.8	31.8	23.8	28.5	28.1
	47.2	47.3	46.7	46.6	35.7	39.4	46.3	45.7	45.4	43.2	45.3	42.6	45.7	48.2	47.0	47.8	47.5
Debt-GDP	10.9	7.5	10.4	9.1	9.3	14.0	4.7	5.0	8.4	9.0	5.1	11.5	7.2	6.9	8.4	10.6	7.3
	24.5	12.9	23.4	21.0	20.7	28.5	14.8	9.1	19.4	19.6	12.8	23.2	22.6	12.2	18.4	27.5	14.8
	46.2	41.0	46.3	46.2	45.6	47.3	45.9	39.7	44.7	44.4	43.5	45.6	47.2	44.4	45.4	47.5	44.1
Real I-GDP	4.1	10.3	12.8	12.9	20.4	19.0	7.4	16.2	9.4	9.3	9.0	6.0	13.1	5.1	3.8	7.1	4.7
	20.4	29.7	30.3	30.6	37.3	36.6	24.3	31.6	27.6	28.5	25.0	25.3	31.0	17.8	8.4	32.0	16.5
	46.0	47.2	47.9	48.1	48.8	48.8	46.9	48.1	47.5	47.3	47.2	47.3	47.8	45.6	42.3	48.4	46.3
Loan-GDP	6.8	15.1	11.6	12.8	9.8	15.7	10.9	9.7	8.7	14.4	4.1	10.1	17.5	14.7	14.8	6.9	17.9
	14.8	31.5	29.1	29.7	23.2	33.2	28.4	22.0	28.8	33.2	10.1	27.6	34.5	30.3	30.4	20.3	32.2
	44.1	48.2	47.5	47.9	46.3	48.4	47.6	46.2	47.8	48.1	42.6	47.5	48.5	47.6	47.6	46.7	48.2
Real C-I	17.9	4.4	4.7	22.2	19.8	6.0	21.0	9.1	7.9	14.7	3.2	15.6	6.8	19.3	4.1	8.1	9.6
	35.6	25.4	22.6	37.9	38.1	12.8	34.4	27.8	18.4	33.8	19.8	33.5	15.1	37.9	7.6	21.6	32.0
	48.7	47.0	47.0	48.7	49.1	43.8	48.4	47.3	45.6	48.4	46.6	48.5	43.7	49.1	42.8	45.7	48.1

Note: The first row is 3-letter ISO country code.

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