Mathematical Formulation of the Hybrid Agriculture Model

1. DecayAware Module

This module handles masked, decayed input with confidence noise blending:

$$
\begin{aligned}
\gamma_x &= \sigma(W_{\gamma_2} \cdot \mathrm{ReLU}(W_{\gamma_1} \cdot \mathbf{d}_t)) \\
\hat{\mathbf{x}}_t &= \mathbf{m}_t \odot \mathbf{x}_t + (1 - \mathbf{m}_t) \odot (\gamma_x \odot \bar{\mathbf{x}}) \\
\mathbf{c}_t &= \sigma(W_c[\hat{\mathbf{x}}_t, \mathbf{d}_t]) \\
\tilde{\mathbf{x}}_t &= \hat{\mathbf{x}}_t \odot \mathbf{c}_t + \epsilon \odot (1 - \mathbf{c}_t) \quad \text{where } \epsilon \sim \mathcal{N}(0, I) \\
\mathbf{z}_t &= \sigma(W_z[\tilde{\mathbf{x}}_t, \mathbf{m}_t, \mathbf{d}_t, \mathbf{h}_{t-1}]) \\
\mathbf{r}_t &= \sigma(W_r[\tilde{\mathbf{x}}_t, \mathbf{m}_t, \mathbf{d}_t, \mathbf{h}_{t-1}]) \\
\tilde{\mathbf{h}}_t &= \tanh(W_h[\tilde{\mathbf{x}}_t, \mathbf{m}_t, \mathbf{d}_t, \mathbf{r}_t \odot \mathbf{h}_{t-1}]) \\
\mathbf{h}_t &= (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1} + \mathbf{z}_t \odot \tilde{\mathbf{h}}_t
\end{aligned}
$$

2. TimeEmbeddingGRUEncoder

Adds time embedding before applying a GRU:

$$
\begin{aligned}
\mathbf{e}_t &= W_{\text{time}} \cdot \delta_t \\
\mathbf{x}'_t &= [\mathbf{x}_t, \mathbf{e}_t] \\
\mathbf{h}_t &= \mathrm{GRU}(\mathbf{x}'_t)
\end{aligned}
$$

3. TabularEncoder

MLP over static tabular inputs:

$$
\begin{aligned}
\mathbf{h}_1 &= \mathrm{ReLU}(W_1 \cdot \mathbf{x}) \\
\mathbf{h}_2 &= \mathrm{BN}(\mathbf{h}_1) \\
\mathbf{h}_3 &= \mathrm{Dropout}(\mathbf{h}_2) \\
\mathbf{y} &= W_2 \cdot \mathbf{h}_3
\end{aligned}
$$

4. CrossModelAttentionFusion

Performs soft-attention across three modalities:

$$
\begin{aligned}
\mathbf{f}_{\text{all}} &= [\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3] \\
\mathbf{a} &= \tanh(W_a \cdot \mathbf{f}_{\text{all}}) \\
\mathbf{w} &= \mathrm{softmax}(W_w \cdot \mathbf{a}) \\
\mathbf{f}_{\text{fused}} &= \sum_i \mathbf{w}_i \cdot \mathbf{f}_i
\end{aligned}
$$

5. HybridAgriModel Summary

The full model aggregates 3 encoders:

- $\mathbf{f}_1$: from DecayAware (soil)
- $\mathbf{f}_2$: from TimeEmbeddingGRU (indoor + weather)
- $\mathbf{f}_3$: from TabularEncoder (crop)

These are fused via attention and passed through the head MLP:

$$\hat{y} = \text{Softplus}\left(W_2 \cdot \text{ReLU}(W_1 \cdot \text{Fusion}([\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3]))\right)$$