Real-Time Eulerian Water Simulation Using a Restricted Tall Cell Grid

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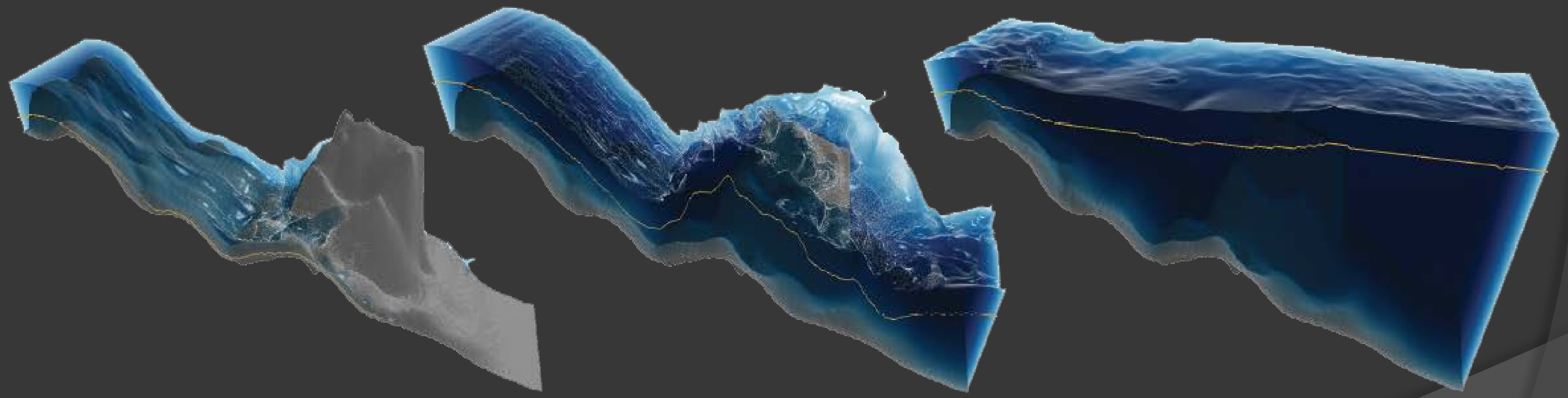
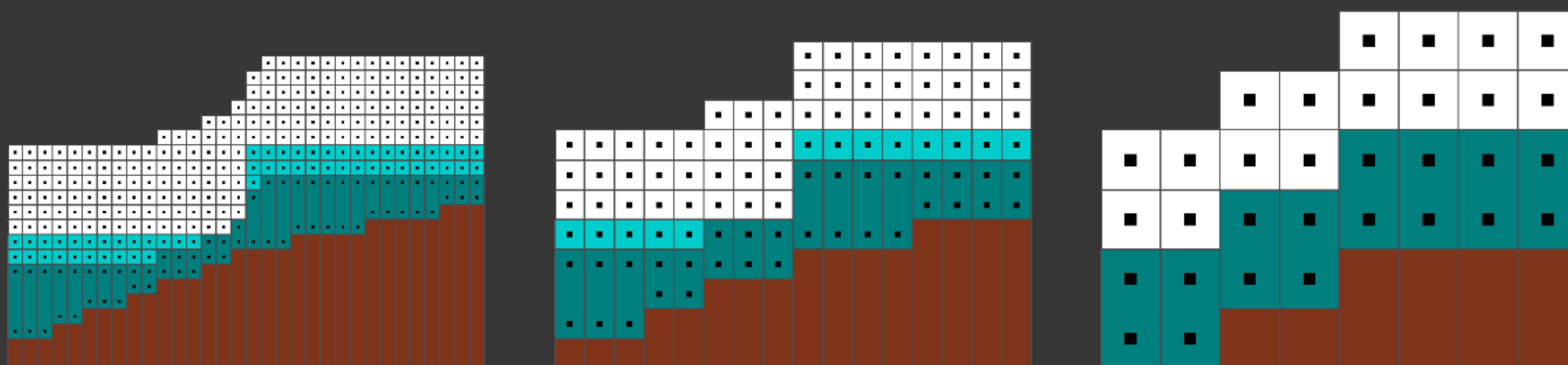
- Tall cells
- Cubic cells





Summarize

- Tall cell grid data structure
- Modified level set method
- Multigrid Poisson solver

Methods

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \frac{\mathbf{f}}{\rho} - \frac{\nabla p}{\rho}, \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

$$\frac{\partial \phi}{\partial t} = -\mathbf{u} \cdot \nabla \phi. \tag{3}$$

Level Set

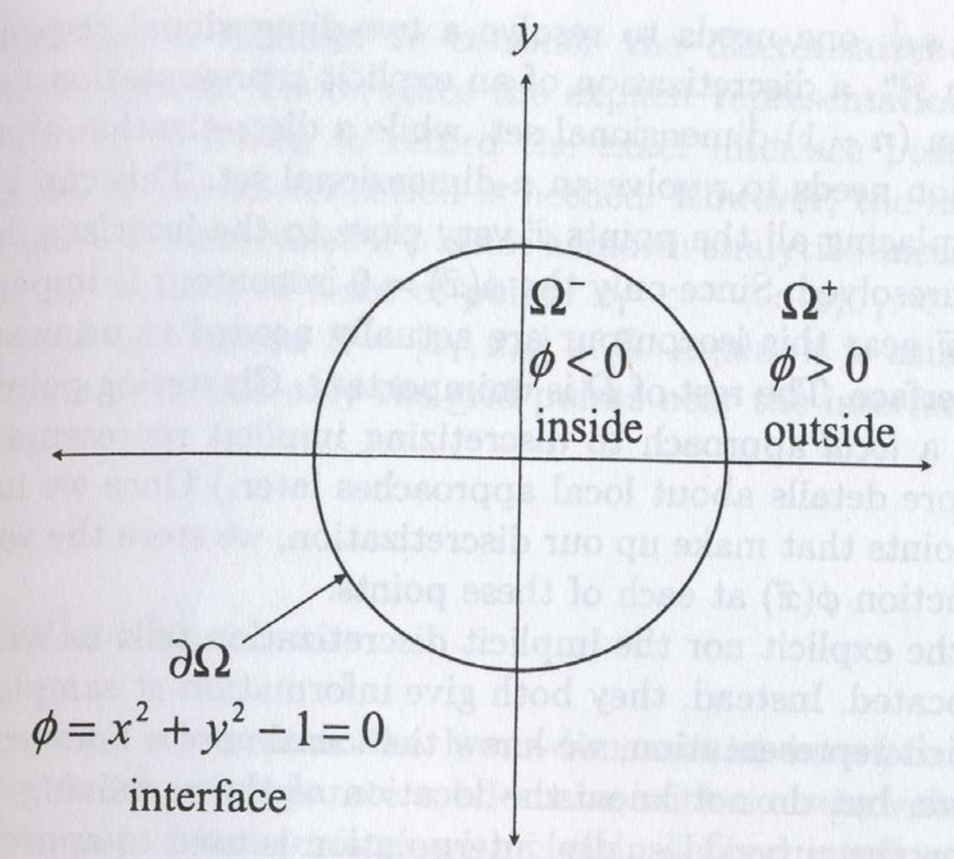


Figure 1.2. Implicit representation of the curve $x^2 + y^2 = 1$.

Discretization

- Collocated grid
- y-coordinate of uncompressed position of element is

$$y_{i,j,k} = \begin{cases} H_{i,k} + 1 & \text{if } j = 1 \text{ (tall cell bottom)} \\ H_{i,k} + h_{i,k} & \text{if } j = 2 \text{ (tall cell top)} \\ H_{i,j} + h_{i,k} + j - 2 & \text{if } j \geq 3 \text{ (regular cell).} \end{cases}$$

Algorithm

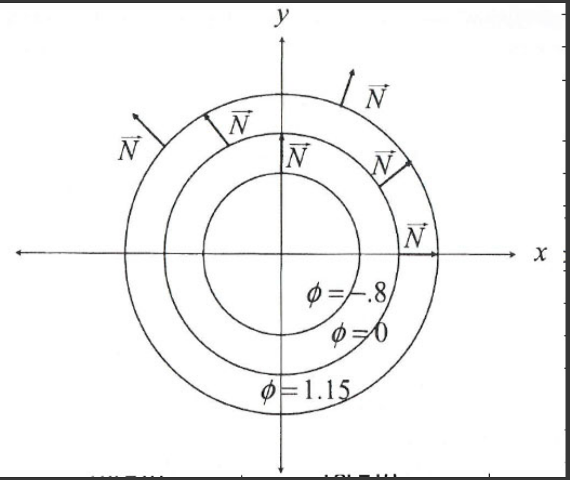
- For each time step
  - Velocity extrapolation
  - Level set reinitialization
  - Advection and external force integration
  - Remeshing
  - Incompressibility enforcement

1.Velocity Extrapolation

- Eikonal solver on GPU [Jeong 2007]

$$\frac{\partial u}{\partial \tau} = -\frac{\nabla \phi}{|\nabla \phi|} \cdot \nabla u,$$

- Narrow band [Enright 2002]



2.Level Set Reinitialization

- Recompute signed distance field
  - Run reinitialization step only every ten frames.
  - Don’t modify the ϕ values next to the surface in order to avoid moving it.
  - Clamp the ϕ value next to surface to not exceed the grid space h

3. Remeshing

- Define new $h_{i,k}$, where $\phi \leq 0$.
- Constraints:
  - At least Gl regular cells below liquid
  - At least Ga regular cells above
  - The heights of adjacent tall cells must not differ than D

4. Enforcing Incompressibility

$$\nabla \cdot \left(\mathbf{u}^* - \frac{\Delta t}{\rho} \nabla p\right) = 0.$$

- Ghost-fluid method [Enright 2002]
- Solid fraction [Batty 2007]

Multigrid vs PCG(Preconditioned conjugate gradient)

64^3 Cases	IC(0) PCG				Multi-grid			
	Tol $=10^{-4}$		Tol $=10^{-8}$		Tol $=10^{-4}$		Tol $=10^{-8}$	
	Iteration	Time	Iteration	Time	Iteration Full-cycle	Time	Iteration Full-cycle	Time
Low	57	0.75	92	1.21	9	0.75	16	1.31
Mid	97	1.35	156	2.18	8	0.68	14	1.18
High	124	1.92	198	3.06	7	0.61	12	1.02

128^3 Case	IC(0) PCG				Multi-grid			
	Tol $=10^{-4}$		Tol $=10^{-8}$		Tol $=10^{-4}$		Tol $=10^{-8}$	
	Iteration	Time	Iteration	Time	Iteration Full-cycle	Time	Iteration Full-cycle	Time
Low	102	10.95	162	17.32	9	6.32	16	11.10
Mid	211	25.31	327	39.14	8	5.64	13	9.03
High	251	32.39	435	55.96	8	5.74	13	9.16

256^3 Case	IC(0) PCG				Multi-grid			
	Tol $=10^{-4}$		Tol $=10^{-8}$		Tol $=10^{-4}$		Tol $=10^{-8}$	
	Iteration	Time	Iteration	Time	Iteration Full-cycle	Time	Iteration Full-cycle	Time
Low	217	183.31	334	281.83	7	39.49	11	55.89
Mid	450	424.27	675	635.03	7	39.77	12	67.06
High	523	542.32	918	951.08	8	46.08	12	68.04

Multigrid

$$-u_{j-1} + 2u_j - u_{j+1} = h^2 f_j = F_j$$

$$Au = F$$

$$\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-2} \\ u_{N-1} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_{N-2} \\ F_{N-1} \end{bmatrix}$$

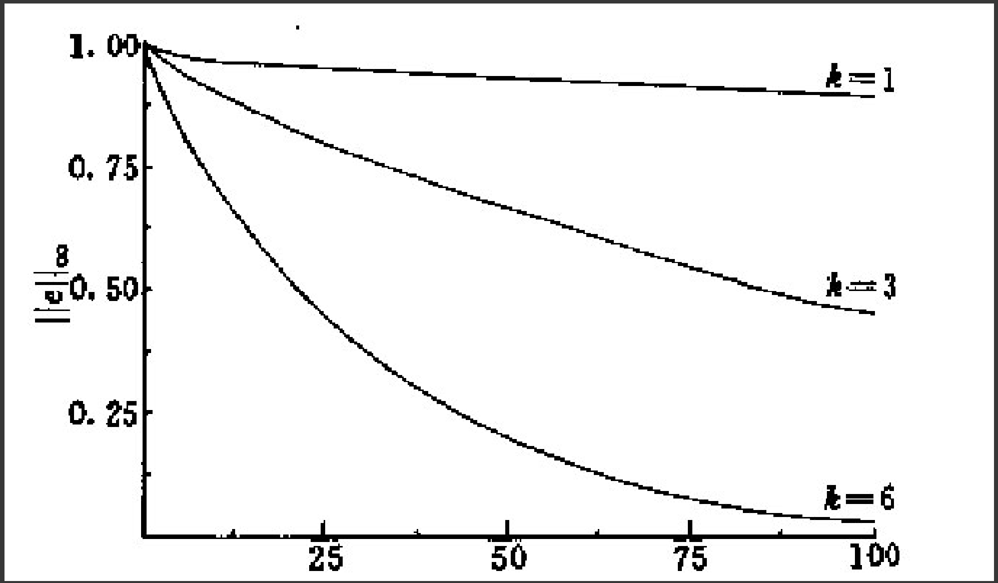
$$A = D - L - U$$

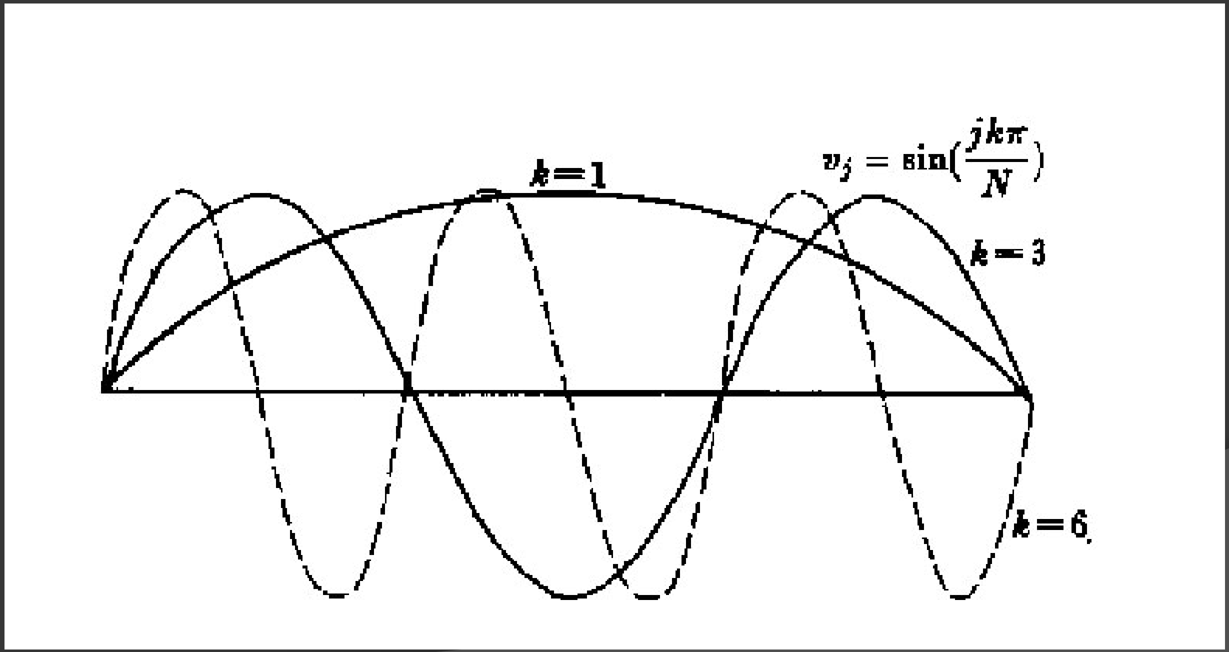
$$M = [(1 - \omega)I + \omega D^{-1}(L + U)]$$

$$E^{(n)} = M^n E^{(0)}$$

$$\lambda_k = 1 - 2\omega \sin^2 \frac{k\pi}{2N}$$

$$k = 1, 2, \cdots, N - 1$$


1.00
0.75
0.50
0.25
$\|e\|_\infty$
25
50
75
100
k=1
k=3
k=6


k=1
$v_j = \sin(\frac{jk\pi}{N})$
k=3
k=6

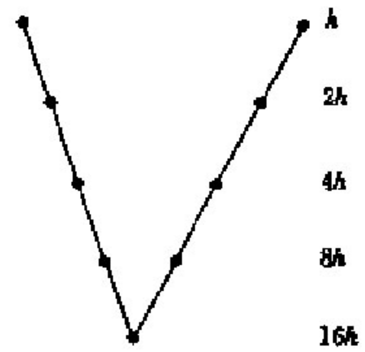
$$E^{(n+1)} \approx \lambda_1^{n+1} a_1 v_1$$

$$E^{(n)} \approx \lambda_1^{n} a_1 v_1$$

$$E^{(n+1)} \approx \lambda_1 E^{(n)}$$

$$\lambda_1 = 1 - 2\omega \sin^2\left(\frac{\pi}{2N}\right)$$

$$\approx 1 - \frac{\omega \pi^2 h^2}{2}$$



(a)