

# INFO0948

## Feature Extraction

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These slides are based on Chapter 13 of the book *Robotics, Vision and Control: Fundamental Algorithms in MATLAB* by Peter Corke, published by Springer in 2011.

The Bag-of-feature section is based on a presentation by Cordelia Schmid  
[http://www.di.ens.fr/willow/events/cvml2011/materials/  
CVML2011\\_Cordelia\\_bof.pdf](http://www.di.ens.fr/willow/events/cvml2011/materials/CVML2011_Cordelia_bof.pdf)

# Motivation

Raw images contain too much data to be of direct practical use for high(er)-level robot vision (object recognition, pose estimation, tracking, ...).

We need to reduce the dimensionality of raw image data, ideally focusing on

- ▶ discarding redundant information.
- ▶ extracting entities that are invariant to the conditions that typically change while a robot is working (viewpoint, illumination, ...).

Feature extraction is an information concentration step that reduces the data rate from  $10^6$ – $10^8$  bytes  $s^{-1}$  at the output of a camera to something of the order of tens of features (vectors of a few dozen scalars) per frame that can be used as input to a robot's control system.

# Plan

## Region Features

### Point Features

#### Scale-space Features

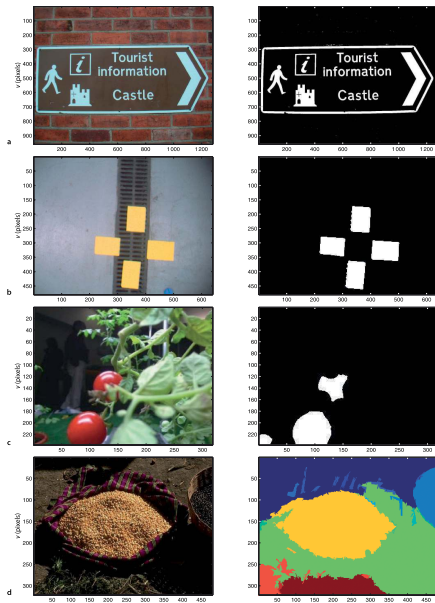
### Bag-of-features for Image Classification

Step 1: Extract Features

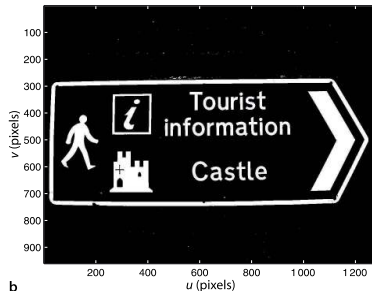
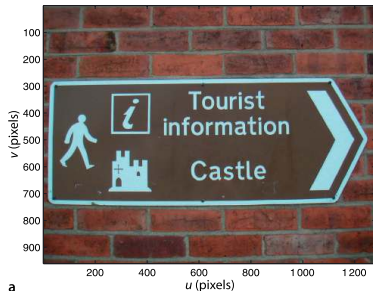
Step 2: Cluster Features, Compute Feature Frequencies

Step 3: Classification

# Region Features



# Thresholding



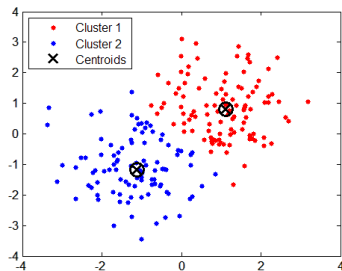
$$c[u, v] = \begin{cases} 0 & I[u, v] < t \\ 1 & I[u, v] \geq t \end{cases} \quad \forall (u, v) \in I$$

# The $k$ -means Algorithm

Given a set of observations  $(x_1, x_2, \dots, x_n)$ , where each observation is a  $d$ -dimensional real vector,  $k$ -means clustering aims to partition the  $n$  observations into  $k$  sets ( $k \leq n$ )  $S = \{S_1, S_2, \dots, S_k\}$  so as to minimize the within-cluster sum of squares (WCSS)

$$\arg \min_{\mathbf{S}} \sum_{i=1}^k \sum_{\mathbf{x}_j \in S_i} \|\mathbf{x}_j - \boldsymbol{\mu}_i\|^2$$

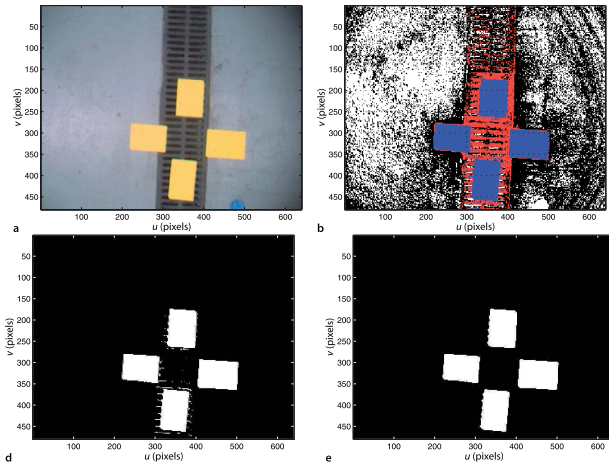
where  $\mu_i$  is the mean of points in  $S_i$ .



```
>> [cls,centre,r] = kmeans(a, 3);
```

# Color Clustering and Classification

```
>> [cls, cxy, resid] = colorkmeans(im_targets, 4);
```





# Plan

Region Features

Point Features

Scale-space Features

Bag-of-features for Image Classification

Step 1: Extract Features

Step 2: Cluster Features, Compute Feature Frequencies

Step 3: Classification

# Point Features

*Point features* relate to two problems:

- ▶ Identifying “interesting” points (also called salient points or keypoints).
- ▶ Characterizing patches of pixels (centered on an interest point or not).

# Corner Detectors

Corners are characterized by a strong gradient in two orthogonal directions. Let us define

$$s(u, v, \delta_u, \delta_v) = \sum_{(i,j) \in \mathcal{W}} w[i, j] \left( \underbrace{I[u + \delta_u + i, v + \delta_v + j]} - I[u + i, v + j] \right)^2$$

An interest point is one for which  $s$  is high for all directions of the vector  $(\delta_u, \delta_v)$ .

The underlined term can be approximated (Taylor) with

$$I[u + \delta_u + j, v + \delta_v + i] \approx I[u + i, v + j] + I_u[u + i, v + j]\delta_u + I_v[u + i, v + j]\delta_v$$

# Corner Detectors

$$s(u, v, \delta_u, \delta_v) = \sum_{(i,j) \in \mathcal{W}} \mathbf{W}[i, j] \left( \underbrace{I[u + \delta_u + i, v + \delta_v + j]} - I[u + i, v + j] \right)^2$$

$$I[u + \delta_u + j, v + \delta_v + i] \approx I[u + i, v + j] + I_u[u + i, v + j]\delta_u + I_v[u + i, v + j]\delta_v$$

$$\begin{aligned} s(u, v, \delta_u, \delta_v) &= \sum_{(i,j) \in \mathcal{W}} \mathbf{W}[i, j] (I_u[u + i, v + j]\delta_u + I_v[u + i, v + j]\delta_v)^2 \\ &= \delta_u^2 \sum_{(i,j) \in \mathcal{W}} \mathbf{W}[i, j] I_u^2[u + i, v + j] + \delta_v^2 \sum_{(i,j) \in \mathcal{W}} \mathbf{W}[i, j] I_v^2[u + i, v + j] \\ &\quad + 2\delta_u\delta_v \sum_{(i,j) \in \mathcal{W}} \mathbf{W}[i, j] I_u[u + i, v + j] I_v[u + i, v + j] \end{aligned}$$

$$\mathbf{A} = \begin{pmatrix} \sum \mathbf{W}[i, j] I_u^2[u + i, v + j] & \sum \mathbf{W}[i, j] I_u[u + i, v + j] I_v[u + i, v + j] \\ \sum \mathbf{W}[i, j] I_u[u + i, v + j] I_v[u + i, v + j] & \sum \mathbf{W}[i, j] I_v^2[u + i, v + j] \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{G}(\sigma_I) \otimes I_u^2 & \mathbf{G}(\sigma_I) \otimes I_u I_v \\ \mathbf{G}(\sigma_I) \otimes I_u I_v & \mathbf{G}(\sigma_I) \otimes I_v^2 \end{pmatrix}$$

$$s(u, v, \delta_u, \delta_v) = (\delta_u \quad \delta_v) \mathbf{A} \begin{pmatrix} \delta_u \\ \delta_v \end{pmatrix}$$

# The Harris Corner Detector

$$s(u, v, \delta_u, \delta_v) = (\delta_u \quad \delta_v) \mathbf{A} \begin{pmatrix} \delta_u \\ \delta_v \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} \mathbf{G}(\sigma_I) \otimes \mathbf{I}_u^2 & \mathbf{G}(\sigma_I) \otimes \mathbf{I}_u \mathbf{I}_v \\ \mathbf{G}(\sigma_I) \otimes \mathbf{I}_u \mathbf{I}_v & \mathbf{G}(\sigma_I) \otimes \mathbf{I}_v^2 \end{pmatrix}$$

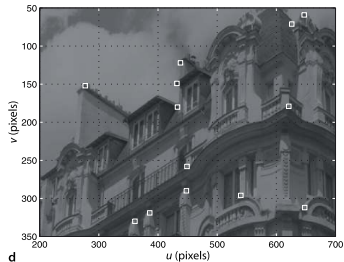
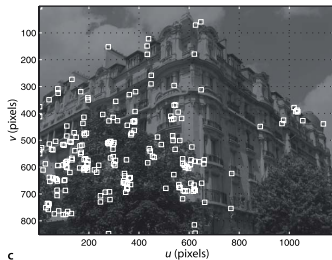
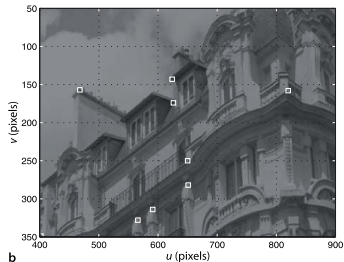
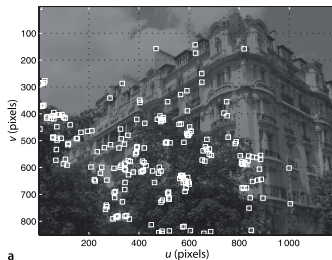
$s$  is high for all directions of the vector  $(\delta_u, \delta_v)$  when  $\mathbf{A}$  has two large eigenvalues.

The Harris detector is defined as

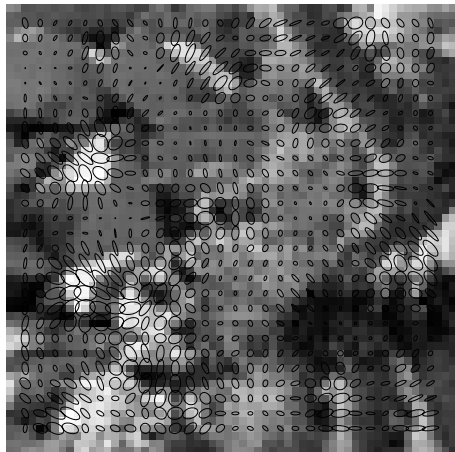
$$C_H(u, v) = \det(\mathbf{A}) - k \operatorname{tr}^2(\mathbf{A})$$

It has a large value around sharp corners.

# The Harris Corner Detector



# The Harris Corner Detector



Ellipses  $(u, v)^T \mathbf{A}^{-1} (u, v)$ .

Their major and minor axes are along the eigenvectors of  $\mathbf{A}$ , and the extent of the ellipses along their major or minor axes corresponds to the size of the eigenvalues.

A large circle corresponds to a corner and a narrow extended ellipse indicates an edge.

D. Forsyth and J. Ponce. Computer vision: a modern approach. PHPTR, 2002.

# The Harris Corner Detector

The Harris detector is computed from image gradients and is therefore robust to offsets in illumination, and the eigenvalues of the structure tensor  $A$  are invariant to rotation. However the detector is not invariant to changes in scale.

The Harris detector responds strongly to fine texture (see leaves above), but does not detect larger structures.



# Plan

Region Features

Point Features

Scale-space Features

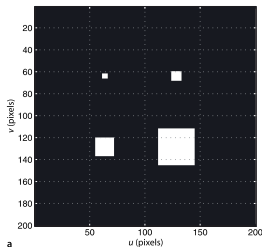
Bag-of-features for Image Classification

Step 1: Extract Features

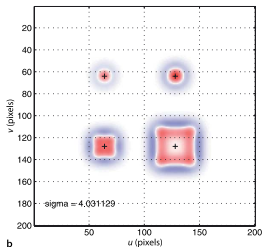
Step 2: Cluster Features, Compute Feature Frequencies

Step 3: Classification

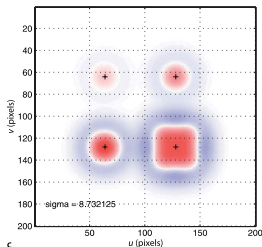
# Scale-space Detector



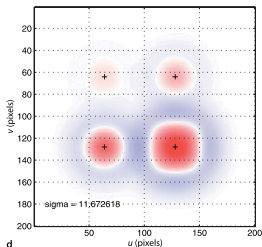
a



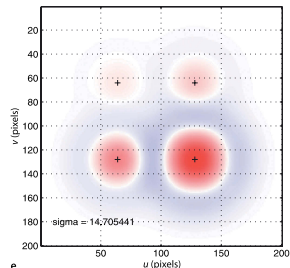
b



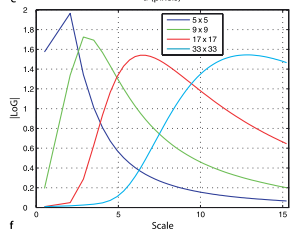
c



d



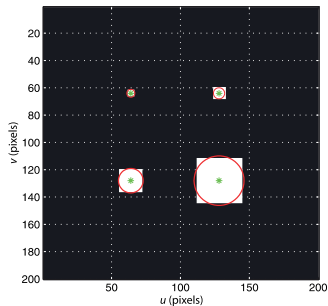
e



f

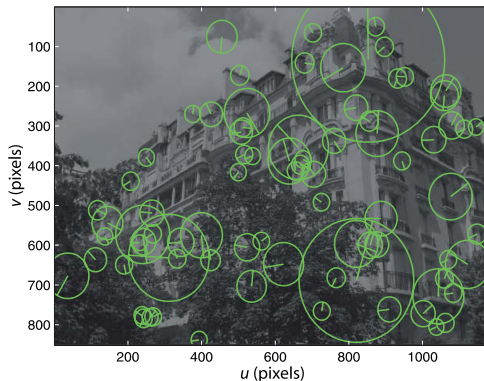
$$\nabla^2 \mathbf{I} = \frac{\partial^2 \mathbf{I}}{\partial u^2} + \frac{\partial^2 \mathbf{I}}{\partial v^2} = \mathbf{I}_{uu} + \mathbf{I}_{vv} = \mathbf{L} \otimes \mathbf{I}$$

# Scale-space Detector



# Scale-Space Point Feature

Scale-space maximum detection is at the root of SIFT and SURF features.



# Plan

Region Features

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Scale-space Features

Bag-of-features for Image Classification

Step 1: Extract Features

Step 2: Cluster Features, Compute Feature Frequencies

Step 3: Classification

# Object/Category Recognition

Image classification: assigning a class label to the image



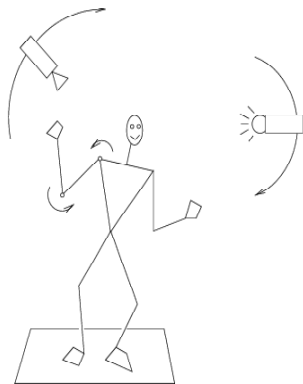
Car: present  
Cow: present  
Bike: not present  
Horse: not present  
...

Object localization: define the location and the category



Location  
Category

# Challenge 1: Intra-instance Variations



Viewpoint, illumination, kinematic configuration, ...

## Challenge 2: Intra-class Variations





# Image Classification, Case 1: Detection

## Given

- Positive examples (containing bike)



- Negative examples (containing **no** bike)



**Decide** whether a test image contains a bike or not



# Image Classification, Case 2: Recognition

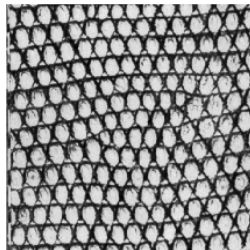
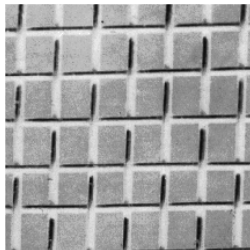
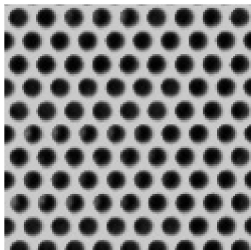
Given



Decide what object is the most likely object in an image

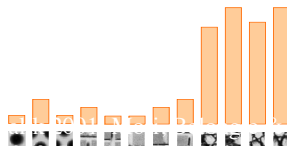
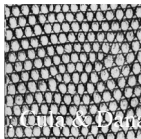
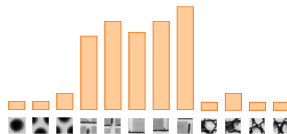
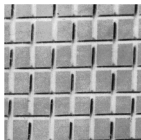
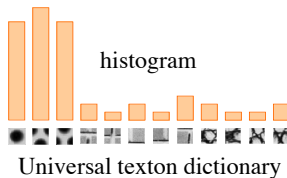
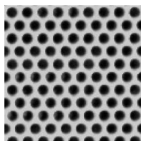


# BoF Origin: Texture Classification



Julesz, 1981; Cula & Dana, 2001; Leung & Malik 2001; Mori, Belongie & Malik, 2001; Schmid 2001; Varma & Zisserman, 2002, 2003; Lazebnik, Schmid & Ponce, 2003

# Texture Classification: Histograms over Textons



# Bag-of-features for Image Classification

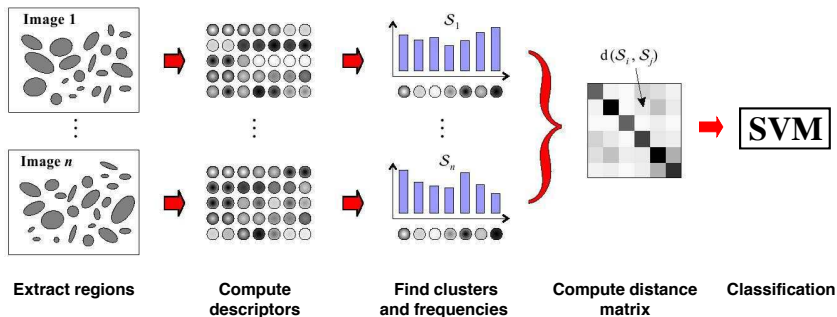


Image 1 contains a bike, image 2 contains a horse, image 3 contains a car, etc...

[Csurka et al., ECCV Workshop'04], [Nowak, Jurie, Triggs, ECCV'06], [Zhang, Marszalek, Lazebnik, Schmid, IJCV'07]

# Plan

Region Features

Point Features

Scale-space Features

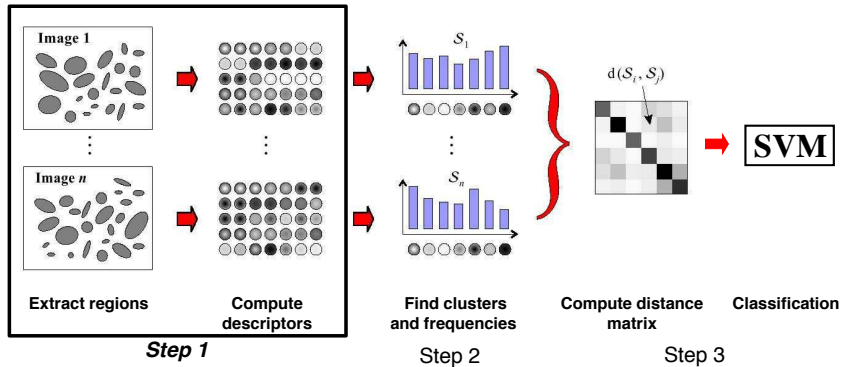
Bag-of-features for Image Classification

Step 1: Extract Features

Step 2: Cluster Features, Compute Feature Frequencies

Step 3: Classification

# Step 1: Extract Features



SIFT features are a popular choice.

# Plan

Region Features

Point Features

Scale-space Features

## Bag-of-features for Image Classification

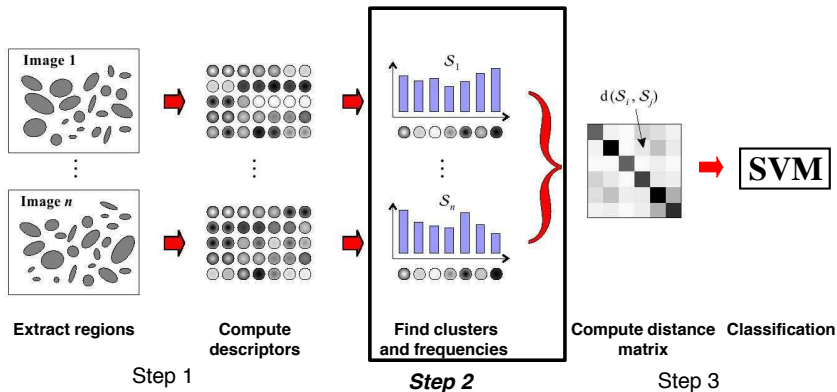
Step 1: Extract Features

Step 2: Cluster Features, Compute Feature Frequencies

Step 3: Classification



# Step 2: Cluster Features, Compute Feature Frequencies

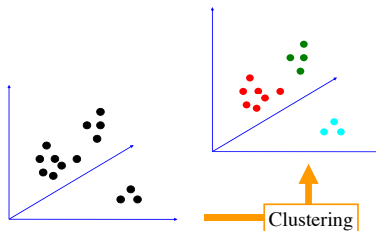


## Clustering Features ( $k$ -means)

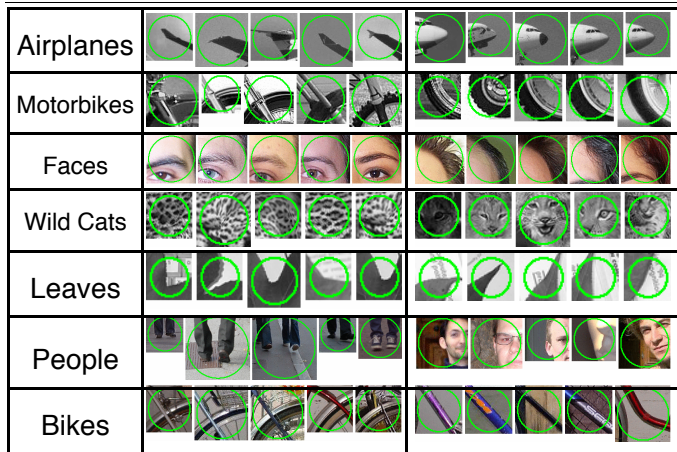
Because of viewpoint and lighting changes, it is unlikely that two images of even the same object will produce exactly the same SIFT features. This gets even worse when working with different instances of a class (e.g., different cars).

As a result, it is not a good idea to model an object (or object class) with a histogram over all the features that the object produces.

Instead, we **cluster** all the features that come from the training images (of all classes), and keep only the cluster centers. The set of cluster centers is called a **codebook**, or **dictionary**. The elements of the codebook are called **codewords**.



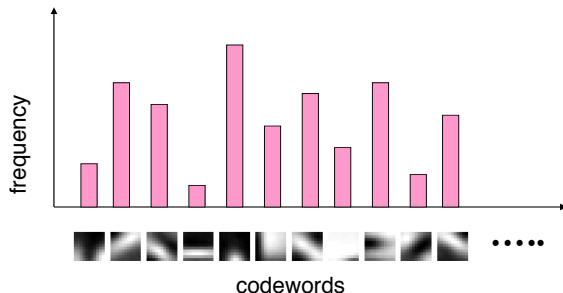
# Examples of Clusters of Features



↓  
Average of these  
becomes one  
codeword

↓  
Average of these  
becomes one  
codeword

# Object/Class Instance Representation: Codeword Frequencies



Typically: 1000–4000 codewords:

- ▶ More codewords: towards **object** representation
- ▶ Less codewords: towards **object class** representation

One image of an instance of an object/class is represented with a vector  $V$  of frequencies of the codewords. (L1/L2 normalization)

# Plan

Region Features

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Scale-space Features

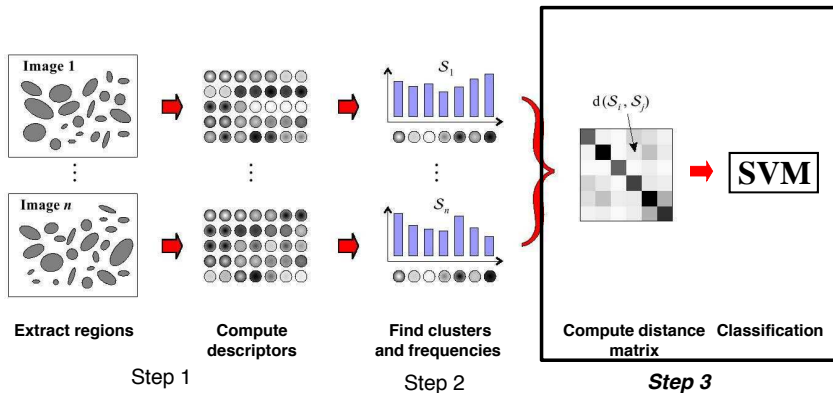
**Bag-of-features for Image Classification**

Step 1: Extract Features

Step 2: Cluster Features, Compute Feature Frequencies

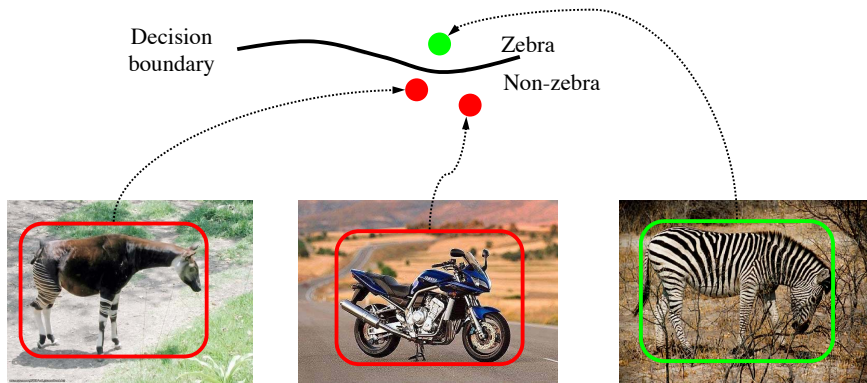
Step 3: Classification

# Step 3: Image Classification

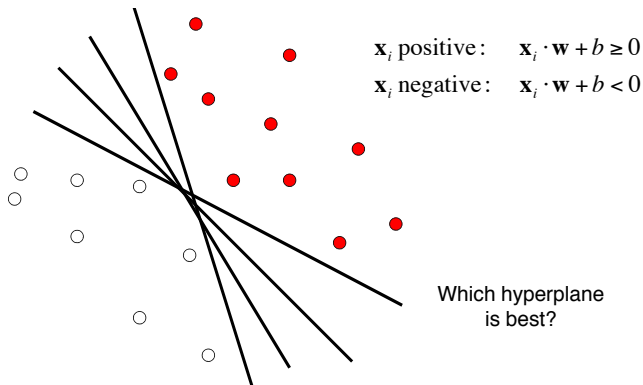


# Image Classification

Goal: Learn a decision rule (**classifier**) to assign  $V$  to an object/class.



# Linear Classification

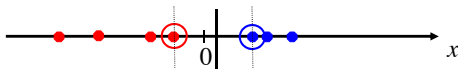


For instance: support vector machines (SVM), logistic regression, linear discriminant analysis (LDA), naive Bayes classification, ...

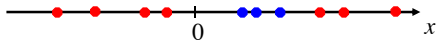


# Nonlinear Classification

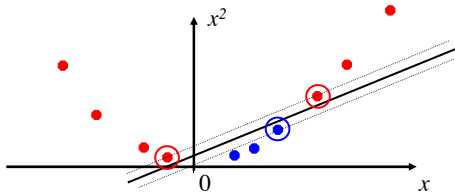
Datasets that are linearly separable work out great:



But what if the data set is just too hard?



Map the data to a higher dimensional space where it is linearly separable:



# Nonlinear Classification

For instance: Kernel-SVM, kernel logistic regression. A radial basis function usually works well.

See

*ELEN0062-1*

*Apprentissage inductif appliqué – Pierre Geurts, Louis Wehenkel*

# Bag-of-features: Summary

## Advantages:

- ▶ Robust to position and orientation
- ▶ Very successful at modeling inter/intra-class variability.

## Shortcomings:

- ▶ No explicit encoding of an object's **structure**
  - ▶ Two objects of similar local appearance but different global shapes are not discernible.
  - ▶ Poor at computing an object's position or orientation in an image.