Q1 answer:

The code is copied from the .cpp text in Ubuntu:

```
#include<iostream>
#include<ctime>
#include<limits.h>
using namespace std;

void insertion_sort(int v[], int n)
{
	int value;
	int i, j;
	for (i = 1; i < n; i++)
	{
		value = v[i];
		j = i - 1;
		while (j >= 0 && v[ j ] > value)
		{
			v[j + 1 ] = v[j];
			j--;
		}
		v[j + 1] = value;
	}
}

void merge_sort(int v[], int n)
{
	int A[50];
	int B[50];
	int i, j, k;
	int value;
	for (i = 1; i <= (n / 2); i++)
	{
		A[i] = v[i - 1];
		B[i] = v[i + n / 2 - 1];
	}
	if ((n % 2) == 0)
	{

		B[i] = INT_MAX;
		A[i] = INT_MAX;
	}
	else
```

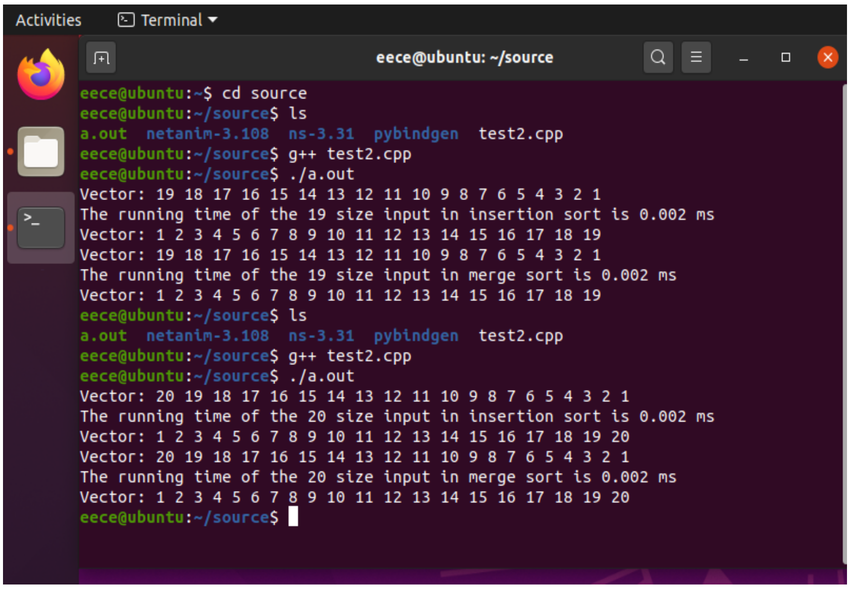
```
	{
		B[i] = v[n - 1];
		B[i + 1] = INT_MAX;
		A[i] = INT_MAX;
	}
	for (i = 2; i <= (n / 2); i++)
	{
		value = A[i];
		j = i - 1;
		while (j >= 0 && A[j] > value)
		{
			A[j + 1] = A[j];
			j--;
		}
		A[j + 1] = value;
	}
	for (i = 2; i <= ((n+1) / 2); i++)
	{
		value = B[i];
		j = i - 1;
		while (j >= 0 && B[j] > value)
		{
			B[j + 1] = B[j];
			j--;
		}
		B[j + 1] = value;
	}
	i = 1;
	j = 1;
		for (k= 0; k< n; k++)
		{
			if (A[i] <= B[j])
			{
				v[k] = A[i];
				i++;
			}
			else
			{
				v[k] = B[j];
				j++;
			}
		}
}
```

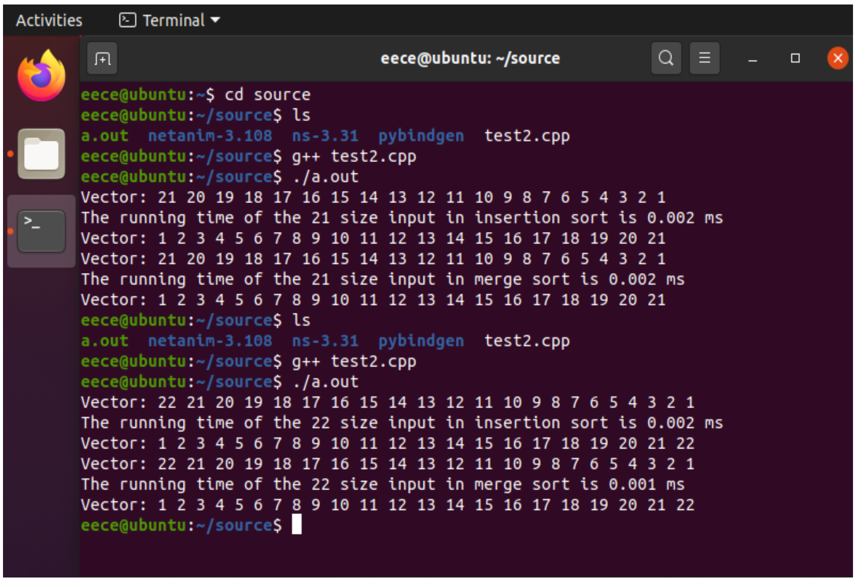
```
void print_vector(int v[], int n)
{
	int i;
	cout << "Vector:";
	for (i = 0; i < n; i++)
		cout << " " << v[i];
	cout << endl;
}

int main()
{
	int v[22];
	int u[22];
	int i;
	double n1, n2;
	clock_t start, end, start1, end1;
	for (i = 0; i < 22; i++)
	{
		v[i] = 22 - i;
		u[i] = 22 - i;
	}
	print_vector(v, 22);
	start = clock();
	insertion_sort(v, 22);
	end = clock();
	n1 = (double)(end - start) / (double)(CLOCKS_PER_SEC)*(double)(1000.000000);
	cout << "The running time of the 22 size input in insertion sort is "<<n1<<" ms"<< endl;
	print_vector(v, 22);
	print_vector(u, 22);
		start1=clock();
	merge_sort(u, 22);
		end1=clock();
		n2 = (double)(end1 - start1) / (double)(CLOCKS_PER_SEC)*(double)(1000.000000);
		cout << "The running time of the 22 size input in merge sort is "<<n2<<" ms"<<
endl;
		print_vector(u, 22);
	return 0;
}
```

Through the code and continuing to change some values, I can get when n=22, the merge sort will beat the insertion sort. As you can see In the picture below, the size of input indicates the number of n in an array. I can change the value of n in the main fuction.When n>22, the running time of insertion sort will be more than that of merge sort.

At first, I would do the experiment in Windows10. After some trials, I found that the base of CPU time is ‘ms’ which is not precise enough. So at last I use ubuntu to show it.

As you see, when n=22, the running time of insertion seat is 0.002 ms. The running time of merge sort is 0.001 ms. All inputs are set in the worst-case.





Q2 answer:

Initial array: 10,5,7,9,8,3

Iteration of insertion sort:

5,10,7,9,8,3

5,7,10,9,8,3

5,7,9,10,8,3

5,7,8,9,10,3

3,5,7,8,9,10

The last array is the result of insertion sort.

Initial array:10,5,7,9,8,3

So, the iteration of quick sort is:

10,5,7,9,8,3

5,7,9,8,3,10

3,5,7,9,8,10

3,5,7,9,8,10

3,5,7,8,9,10

Q3 answer:

$n + 3 \in \Omega(n)$ is true.

$n + 3 \in O(n^2)$ is true.

$n + 3 \in \Theta(n^2)$ is false, $n + 3 \in O(n^2)$

$2^{n+1} \in O(n + 1)$ is false, $2^{n+1} \in \Omega(n + 1)$

$2^{n+1} \in \Theta(2^n)$ is true.

Q4 answer:

$T(n) = 8T\left(\frac{n}{2}\right) + n$, for some $\varepsilon > 0$, $n = O(n^{3-\varepsilon})$, so $T(n) = \theta(n^3)$.

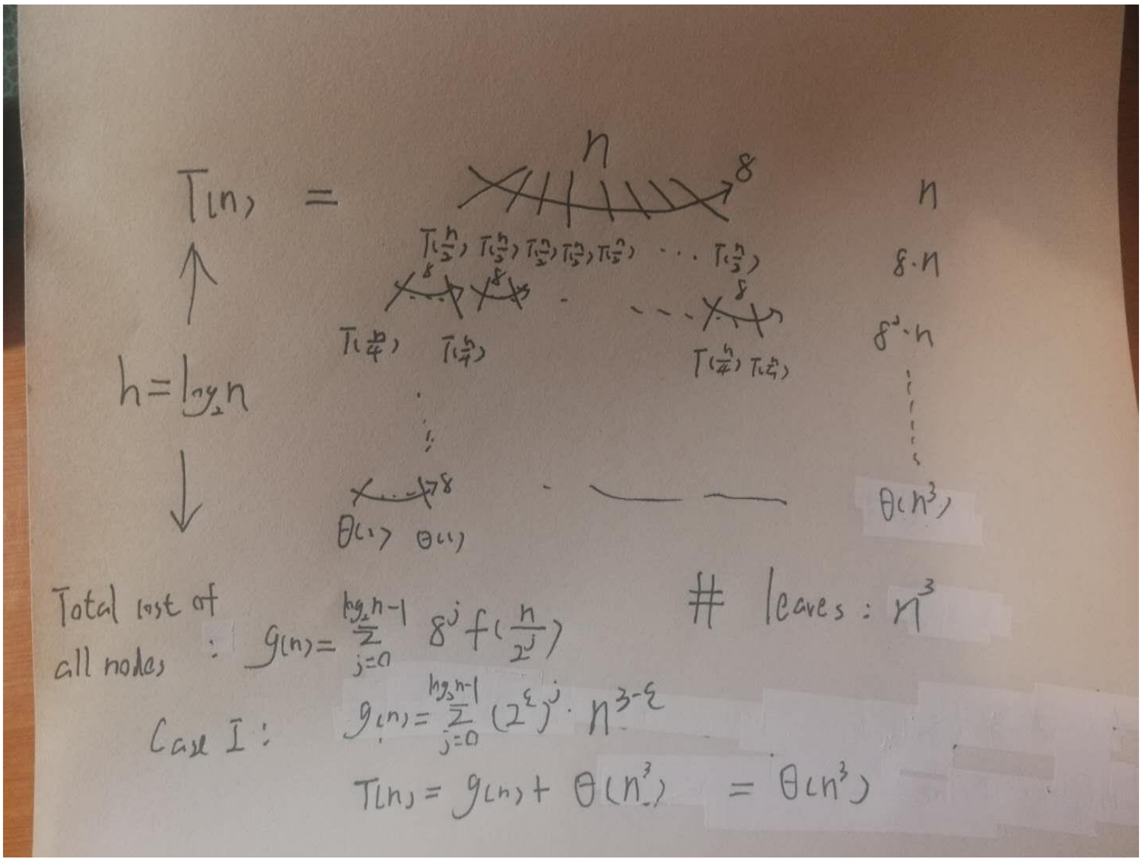
$T(n) = 8T\left(\frac{n}{2}\right) + n^2$, for some $\varepsilon > 0$, $n^2 = O(n^{3-\varepsilon})$, so $T(n) = \theta(n^3)$.

$T(n) = 8T\left(\frac{n}{2}\right) + n^3$, if $k = 0$, $n^3 = \theta(n^3)$, so $T(n) = \theta(n^3 \log n)$.

$T(n) = 8T\left(\frac{n}{2}\right) + n^4$, for some $\varepsilon > 0$, $n^4 = \Omega(n^{3+\varepsilon})$, and for some $\varepsilon^l > 0$, $\frac{n^4}{2} \leq (1 - \varepsilon^l)n^4$, so $T(n) = \theta(n^4)$

Q5 answer:

The recursion tree is as the picture below:



Prove:

Guess $T(n) = \theta(n^3)$;

Assume $T(n) \leq c_1 n^3 - c_2 n^2$ for all $n > n_0$ and $T(n) \geq c_3 n^3 + c_4 n^2$ for all $n > n_0$;

For the former $T(n) = 8T\left(\frac{n}{2}\right) + n \leq 8 * c_1 * \left(\frac{n}{2}\right)^3 - 8 * c_2 * \left(\frac{n}{2}\right)^2 + n$ $= c_1 n^3 - c_2 n^2 - n(c_2 n - 1)$ and if $c_2 = 1, n_0 = 1, n(c_2 n - 1) > 0$, so $T(n) \leq c_1 n^3 - c_2 n^2$, $T(n) = O(n^3)$

For the latter $T(n) = 8T\left(\frac{n}{2}\right) + n \geq 8 * c_3 * \left(\frac{n}{2}\right)^3 + 8 * c_4 * \left(\frac{n}{2}\right)^2 + n$ $= c_3 n^3 + c_4 n^2 + n(c_4 n + 1)$ and if $c_4 = 1, n_0 = 1, n(c_4 n + 1) > 0$, so

$T(n) \geq c_3 n^3 + c_4 n^2$, $T(n) = \Omega(n^3)$

For base case, $T(1) \leq c_1 - c_2$ and $T(1) \geq c_3 + c_4$ for that c_1 is much larger than c_2 and c_3, c_4 are very small.

In summary, $T(n) = \theta(n^3)$