STAT230 Homework 1

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1 Problem B.15

Using the bi-linearity of $(x, y) \rightarrow cov(x, y)$, and using the fact that $cov(x, a) = 0$ for a constant, we have:

$$\begin{aligned} r(x^*, y^*) &= cov(x^*, y^*) && (1) \\ &= cov(\frac{x-\mu_x}{\sigma_x}, \frac{y-\mu_y}{\sigma_y}) && (2) \\ &= \frac{cov(x, y)}{\sigma_x \sigma_y} && (3) \\ &= r(x, y) && (4) \end{aligned}$$

2 Problem B.16

2.1

We have:

$$\begin{aligned} \frac{1}{n}\sum_{i=1}^{n}(x_i + y_i)^2 &= \frac{1}{n}(x+y)(x+y)^T && (5) \\ &= \frac{1}{n}(xx^T + yy^T + 2xy^T) && (6) \end{aligned}$$

Since x and y are $\in R^n$, we have:

$$\begin{aligned} \frac{1}{n}\sum_{i=1}^{n}(x_i + y_i)^2 &= E[xx^T + yy^T + 2xy^T] && (7) \\ &= E[xx^T] + E[yy^T] + 2E[xy^T] && (8) \end{aligned}$$

Since $s_x = s_y = 1$ and $E(x) = E(y) = 0$, we have:

$$\frac{1}{n}\sum_{i=1}^{n}(x_i + y_i)^2 = 1 + 1 + 2Cov(x, y) \tag{9}$$

$$= 2(1 + r) \tag{10}$$

Similarly,

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - y_i)^2 = \frac{1}{n}(x - y)(x - y)^T \tag{11}$$

$$= E[xx^T + yy^T - 2xy^T] \tag{12}$$

$$= E[xx^T] + E[yy^T] - 2E[xy^T] \tag{13}$$

$$= 2(1 - r) \tag{14}$$

2.2

$\frac{1}{n}\sum_{i=1}^{n}(x_i - y_i)^2$ and $\frac{1}{n}\sum_{i=1}^{n}(x_i + y_i)^2$ are non negative as sums of squared numbers. Therefore, $2(1 - r)$ and $2(1 + r)$ are also non negative, i.e. respectively $r \leq 1$ and $-1 \leq r$.