Thibault Doutre

What would you like to be called in class?
Thibault

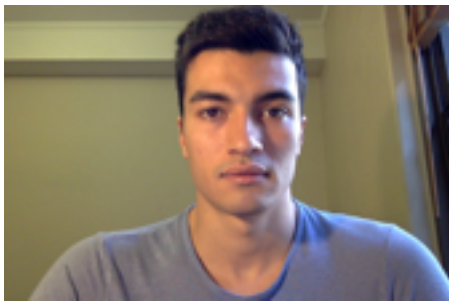
Have you ever taken a statistics class from me? If so, what?
No

Which operating system do you use most often (Unix/Windows/Mac)?
Mac OSX

Name one thing you hope to learn about in this class.
Fit performant models on real dataset

Anything else you want me to know about yourself?
No

Paste your photo here. Resize so that everything fits on one page, delete the questions above (leaving your answers, of course) if necessary.



STAT230 Homework 1

Thibault Doutre

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1 Problem B.15

Using the bi-linearity of $(x, y) \to cov(x, y)$, and using the fact that $cov(x, a) = 0$ for a constant, we have:

$$\begin{aligned} r(x^*, y^*) &= cov(x^*, y^*) && (1)\\ &= cov(\frac{x - \mu_x}{\sigma_x}, \frac{y - \mu_y}{\sigma_y}) && (2)\\ &= \frac{cov(x, y)}{\sigma_x \sigma_y} && (3)\\ &= r(x, y) && (4) \end{aligned}$$

2 Problem B.16

2.1

We have:

$$\begin{aligned} \frac{1}{n}\sum_{i=1}^{n}(x_i + y_i)^2 &= \frac{1}{n}(x + y)(x + y)^T && (5)\\ &= \frac{1}{n}(xx^T + yy^T + 2xy^T) && (6) \end{aligned}$$

Since x and y are $\in R^n$, we have:

$$\begin{aligned} \frac{1}{n}\sum_{i=1}^{n}(x_i + y_i)^2 &= E[xx^T + yy^T + 2xy^T] && (7)\\ &= E[xx^T] + E[yy^T] + 2E[xy^T] && (8) \end{aligned}$$

Since $s_x = s_y = 1$ and $E(x) = E(y) = 0$, we have:

$$\frac{1}{n}\sum_{i=1}^{n}(x_i + y_i)^2 = 1 + 1 + 2Cov(x, y) \tag{9}$$

$$= 2(1 + r) \tag{10}$$

Similarly,

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - y_i)^2 = \frac{1}{n}(x - y)(x - y)^T \tag{11}$$

$$= E[xx^T + yy^T - 2xy^T] \tag{12}$$

$$= E[xx^T] + E[yy^T] - 2E[xy^T] \tag{13}$$

$$= 2(1 - r) \tag{14}$$

2.2

$\frac{1}{n}\sum_{i=1}^{n}(x_i - y_i)^2$ and $\frac{1}{n}\sum_{i=1}^{n}(x_i + y_i)^2$ are non negative as sums of squared numbers. Therefore, $2(1-r)$ and $2(1+r)$ are also non negative, i.e. respectively $r \leq 1$ and $-1 \leq r$.