STAT230 Homework 5

Thibault Doutre

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1

I note E the elite tolerance, R the repression and M the mass tolerance.

$$\begin{align} R &= \beta_1 M + \beta_2 E + \delta \tag{1} \\ &= X\beta + \delta \tag{2} \end{align}$$

Where $X = \begin{bmatrix} M & E \end{bmatrix}$ and $\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$.

Using the OLS estimate:

$$\hat{\beta} = (X^T X)^{-1} X^T R \tag{3}$$

Here, we have:

$$X^T X = n \begin{bmatrix} 1 & 0.52 \\ 0.52 & 1 \end{bmatrix} \tag{4}$$

$$X^T R = n \begin{bmatrix} -0.26 \\ -0.42 \end{bmatrix} \tag{5}$$

Therefore:

$$\hat{\beta} = \begin{bmatrix} 1 & 0.52 \\ 0.52 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -0.26 \\ -0.42 \end{bmatrix} = \begin{bmatrix} -0.057 \\ -0.390 \end{bmatrix} \tag{6}$$

2

Using the fact that:

$$1 = \hat{\beta_1}^2 + \hat{\beta_2}^2 + 2\hat{\beta}_1\hat{\beta}_2 * 0.52 + \hat{\sigma}^2 \tag{7}$$

We come up with:

$$\hat{\sigma}^2 = 0.821 \tag{8}$$

3

Since $n = 36$ understate SE, I choose $p = 3$ to correct the bias.

$$\begin{aligned} SE &= \left(\frac{n}{n-p}\hat{\sigma}^2\right)^{1/2} && (9) \\ &= 0.946 && (10) \end{aligned}$$

4

$$\begin{aligned} Var(\hat{\beta}_1 - \hat{\beta}_2|X) &= Var(\hat{\beta}_1) + Var(\hat{\beta}_2|X) - 2Cov(\hat{\beta}_1, \hat{\beta}_2|X) && (11) \\ &= V_{11} + V_{22} - 2V_{12} && (12) \end{aligned}$$

Where:

$$\begin{aligned} V &= \hat{\sigma}^2(X^TX)^{-1} && (13) \\ &= \frac{\hat{\sigma}^2}{n}\begin{bmatrix} 1 & 0.52 \\ 0.52 & 1 \end{bmatrix}^{-1} && (14) \\ &= \begin{bmatrix} 0.031 & -0.016 \\ -0.016 & 0.031 \end{bmatrix} && (15) \end{aligned}$$

Therefore,

$$\begin{aligned} Var(\hat{\beta}_1 - \hat{\beta}_2|X) &= 0.095 && (16) \\ SE(\hat{\beta}_1 - \hat{\beta}_2) &= 0.308. && (17) \end{aligned}$$

Since $\hat{\beta}_1 - \hat{\beta}_2 = 0.33$, the difference is not significant, the standard error is too high to state that $\hat{\beta}_2$ is significant and not $\hat{\beta}_1$.