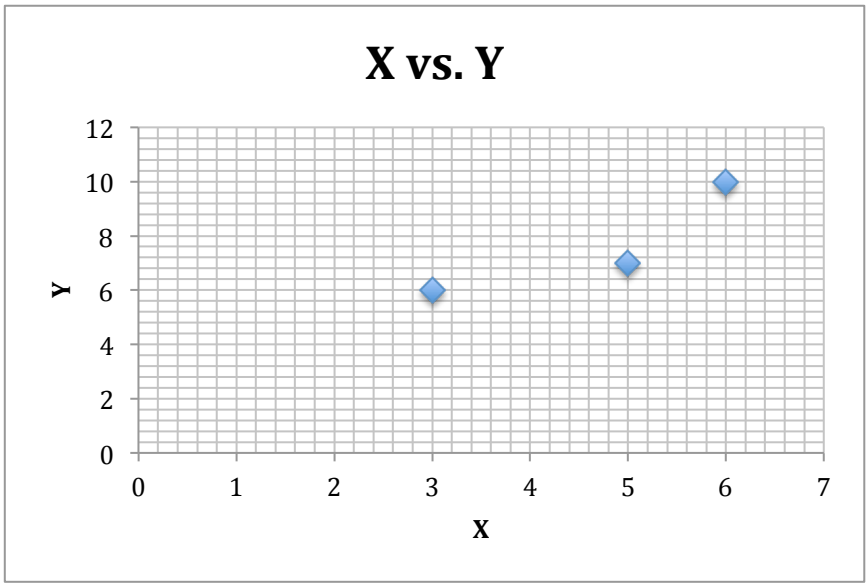
2)

X	Y
3	6
5	7
6	10



a)

$$\Theta_j := \Theta_j - \alpha \frac{\partial}{\partial \Theta_j} J(\Theta_0, \Theta_1) \quad for\ j = 0\ and\ j = 1$$

$$J(\Theta_0, \Theta_1) = \frac{1}{2m} \sum_{i=1}^{m} ((\Theta_0 + \Theta_1 x^i) - y^i)^2$$

$$h_\theta(x) = \Theta_0 + \Theta_1 x$$

$$\Theta_0 = 0.0, \Theta_1 = 1.0, m = 3, \alpha = 0.1$$

$$***$$

$$h_\theta(3) = 0 + 3 = 3$$

$$h_\theta(5) = 0 + 5 = 5$$

$$h_\theta(6) = 0 + 6 = 6$$

$$\Theta_0 := \Theta_0 - \alpha \left(\frac{1}{m} \sum_{i=1}^{m} (\Theta_0 + \Theta_1 x^i) - y^i \right)$$

$$\Theta_1 := \Theta_1 - \alpha \left(\frac{1}{m} (\sum_{i=1}^{m} \left((\Theta_0 + \Theta_1 x^i) - y^i \right) x^i) \right)$$

By substituting the variables, 1st Iteration:

$$\Theta_0 := -0.1 \left(\frac{1}{3} [(3-6) + (5-7) + (6-10)] \right) = -\frac{-9}{30} = 0.3$$

$$\Theta_1 := 1 - 0.1 \left(\frac{1}{3} ([((3-6) * 3) + ((5-7) * 5) + ((6-10) * 6)]) \right)$$

$$= 1 - \frac{-43}{30} = 2.4$$

$$h_\theta(x) = 0.3 + 2.4x$$

$$h_\theta(3) = 0.3 + 2.4 * (3) = 7.5$$

$$h_\theta(5) = 0.3 + 2.4 * (5) = 12.3$$
$$h_\theta(6) = 0.3 + 2.4 * (6) = 14.7$$

2nd Iteration:

$$\Theta_0 := 0.3 - 0.1\left(\frac{1}{3}[(3-7.5)+(5-12.3)+(6-14.7)]\right) = 0.3 - \frac{-20.5}{30} = 0.3 + 0.7 = 1$$

$$\Theta_1 := 2.4 - 0.1\left(\frac{1}{3}([((3-7.5)*3)+((5-12.3)*5)+((6-14.7)*6)])\right)$$
$$= 2.4 - \frac{-13.5-36.5-52.2}{30} = 2.4 + 3.4 = 5.8$$
$$h_\theta(x) = 1 + 5.8x$$

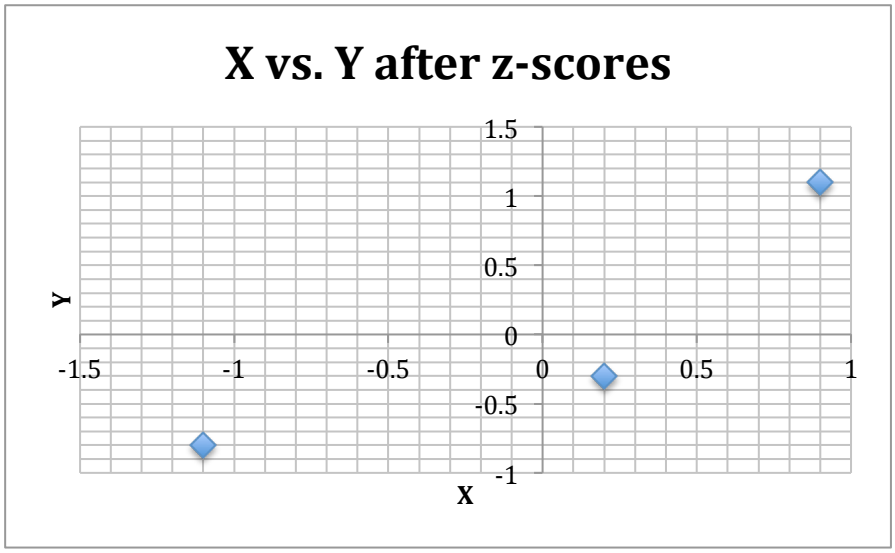
***

$$MSE_1 = \frac{1}{m}\sum_{i=1}^{m}((\Theta_0 + \Theta_1 x^i) - y^i)^2 = \frac{1}{3}((3-6)^2 + (5-7)^2 + (6-10)^2) = 9.7$$

$$MSE_2 = \frac{1}{3}((3-7.5)^2 + (5-12.3)^2 + (6-14.7)^2) = \frac{1}{3}(20.25 + 53.29 + 75.69) = 49.75$$

b)

	X	Y
	-1.1	-0.8
	0.2	-0.3
	0.9	1.1
Values used in z-score calculation	μ=4.7	μ=7.7
	σ=1.5	σ=2.1



$$h_\theta(-1.1) = 0 + -1.1 = -1.1$$
$$h_\theta(0.2) = 0 + 0.2 = 0.2$$
$$h_\theta(0.9) = 0 + 0.9 = 0.9$$

By substituting the variables, 1st Iteration:

$$\Theta_0 := -0.1\left(\frac{1}{3}[(-1.1-(-0.8))+(0.2-(-0.3))+(0.9-1.1)]\right) = -\frac{0}{30} = 0$$

$$\Theta_1 := 1 - 0.1\left(\frac{1}{3}[((-1.1-(-0.8))*-1.1)+((0.2-(-0.3))*0.2)\right.$$
$$\left.+((0.9-1.1)*0.9)]\right) = 1-\frac{0.25}{30} = 0.99$$
$$h_\theta(x) = 0.99x$$
$$h_\theta(-1.1) = 0.99*(-1.1) = -1.09$$
$$h_\theta(0.2) = 0.99*(0.2) = 0.19$$
$$h_\theta(0.9) = 0.99*(0.9) = 0.89$$

2nd Iteration:

$$\Theta_0 := -0.1\left(\frac{1}{3}[(-1.09-(-0.8))+(0.19-(-0.3))+(0.89-1.1)]\right) = \frac{-0.01}{30} \approx 0$$
$$\Theta_1 := 0.99 - 0.1\left(\frac{1}{3}[((-1.09—0.8))*-1.09)+((0.19—0.3))*0.19\right)$$
$$+\left((0.89-1.1)*0.89\right) = 0.99-\frac{0.22}{30} = 0.98$$
$$h_\theta(x) = 0.98x$$

***

$$MSE_1 = \frac{1}{m}\sum_{i=1}^{m}((\Theta_0+\Theta_1x^i)-y^i)^2$$
$$= \frac{1}{3}((-1.1-(-0.8))^2+(0.2-(-0.3))^2+(0.9-1.1)^2) = 0.13$$
$$MSE_2 = \frac{1}{3}((-1.09-(-0.8))^2+(0.19-(-0.3))^2+(0.89-1.1)^2) = 0.12$$

Comparison: When the z-scores are used the MSE's are very small.

4)

$$\frac{\partial}{\partial\Theta_1}J(\Theta_0,\Theta_1) = \frac{\partial}{\partial\Theta_1}[\frac{1}{2m}\sum_{i=1}^{m}\left((\Theta_0+\Theta_1x^i)-y^i\right)^2] = 0$$
$$=\frac{1}{2m}[\left((\Theta_0+\Theta_1x^1)-y^1\right)^2+\left((\Theta_0+\Theta_1x^2)-y^2\right)^2+\cdots]$$
$$=\frac{1}{2m}2\left[\left((\Theta_0+\Theta_1x^1)-y^1\right)x^1+\left((\Theta_0+\Theta_1x^2)-y^2\right)x^2+\cdots\right]$$
$$\frac{\partial}{\partial\Theta_1}J(\Theta_0,\Theta_1) = \frac{1}{m}\sum_{i=1}^{m}((\Theta_0+\Theta_1x^i)-y^i)] = 0$$