Machine Learning – Review Q6

x	y
3	2
1	2
0	1
4	3

$m=4$, $\theta_0=1$, $\theta_1=0.5$ $\quad a:=b$ (dynamic assignment)

$h_\theta(x)=\theta_0+\theta_1 x$

Gradient Descent Algorithm:

$$\theta_j := \theta_j - \underbrace{\alpha}_{\text{learning rate}} \frac{\partial}{\partial \theta_j} J(\theta_0,\theta_1) \quad (\text{for } j=0 \text{ \& } j=1)$$

$\Rightarrow \alpha = 0.1$

$$J(\theta_0,\theta_1)=\frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)})-y^{(i)}\right)^2$$

$h_\theta(0)=1+0=1$

$h_\theta(1)=1+0.5=1.5$

$h_\theta(3)=1+1.5=2.5$

$h_\theta(4)=1+2=3$

$$\theta_0 := \theta_0 - \alpha \frac{\partial}{\partial \theta_0}\left[\frac{1}{2m}\sum_{i=1}^{m}\left((\theta_0+\theta_1 x^i)-y^i\right)^2\right]$$

$$\theta_0 := 1 - \frac{0.1}{4}\sum_{i=1}^{4}\left((\theta_0+\theta x^i)-y^i\right)^2$$

$$\theta_0 := 1 - \frac{1}{40}\left((1-1)^2+(1.5-2)^2+(2.5-2)^2+(3-3)^2\right)$$

$$\theta_0 := = 1-\frac{1}{40}\ (0+0.25+0.25+0)=1-\frac{1}{80}=\underline{\underline{0.988}}$$

$$\theta_1=\theta_1-\alpha\frac{\partial}{\partial\theta_1}\left[\frac{1}{2m}\sum_{i=1}^{m}\left((\theta_0+\theta_1 x^i)-y^i\right)^2\right]$$

$$\theta_1=0.5-\frac{0.1}{4}\underbrace{(0+0.25+0.25+0)}_{1/2}=\frac{1}{2}-\frac{1}{80}=\underline{\underline{0.488}}$$

$$h_\theta(x^i)=0.988+0.488\,x^i$$