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1. Question

a. $h_\theta = [\theta_0\ \theta_1 \dots\ \theta_n]\begin{bmatrix} x_0^i \\ x_1^i \\ \vdots \\ x_n^i \end{bmatrix}$

b. $J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(\left([\theta_0\ \theta_1 \dots\ \theta_n]\begin{bmatrix} x_0^i \\ x_1^i \\ \vdots \\ x_n^i \end{bmatrix}\right) - y^i\right)^2$

c. $\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{m}\sum_{i=1}^{m}\left(\left([\theta_0\ \theta_1 \dots\ \theta_n]\begin{bmatrix} x_0^i \\ x_1^i \\ \vdots \\ x_n^i \end{bmatrix}\right) - y^i\right)\begin{bmatrix} x_0^i \\ x_1^i \\ \vdots \\ x_n^i \end{bmatrix}$

d. $\begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(\left([\theta_0\ \theta_1 \dots\ \theta_n]\begin{bmatrix} x_0^i \\ x_1^i \\ \vdots \\ x_n^i \end{bmatrix}\right) - y^i\right)\begin{bmatrix} x_0^i \\ x_1^i \\ \vdots \\ x_n^i \end{bmatrix}$

3. Question

a. $\mu = \frac{2+5+7+7+9+25}{6} = 9.17$

$\sigma^2 = \frac{(2-9.17)^2+(5-9.17)^2+(7-9.17)^2+(7-9.17)^2+(9-9.17)^2+(25-9.17)^2}{5} = 65.77$

b. $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{8.12\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{x-9.17}{8.12}\right)^2}$,

$f_x(20) = \frac{1}{8.12\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{20-9.17}{8.12}\right)^2} = 0.049 * 0.410 = 0.020$

c. $f_{x_1 \dots x_6}(2,5,7,7,9,25) =$

$\frac{1}{8.12\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{2-9.17}{8.12}\right)^2} * \frac{1}{8.12\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{5-9.17}{8.12}\right)^2} * \frac{1}{8.12\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{7-9.17}{8.12}\right)^2} * \frac{1}{8.12\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{7-9.17}{8.12}\right)^2} * \frac{1}{8.12\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{9-9.17}{8.12}\right)^2} * \frac{1}{8.12\sqrt{2\pi}}e^{\frac{-1}{2}\left(\frac{25-9.17}{8.12}\right)^2} =$

$0.295 * e^{0.781-0.390-0.132-0.036-0.036-0.000-1.906} = 0.295 * 0.180 = 0.053$

d. $f_{x_1 \dots x_6}(2,5,7,7,9,25)$ is greater than $f_x(20)$.

e. $cov(X,Y) = \frac{1}{N-1}\left(\sum_{i=1}^{N} x_i y_i - \frac{\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N}\right) = \frac{1}{5}\left(427 - \frac{55*37}{6}\right) = 17.57$

f. $cov(x,y) = \frac{\sum_{i=1}^{N}(x_i-\tilde{x})(y_i-\tilde{y})}{N-1}, MSE = \frac{1}{N}\sum_{i=1}^{N}(x_i - \tilde{x}_i)^2$

Both functions use the difference between the mean or actual value and the data. They are related in the method they use to compare. However, while covariance is divided by one minus sample size, MSE is divided by the sample size.