

ASIC & AI

Stage 1: AI introduction & algorithms

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Content

- Gradient descent
- Perceptron learning algorithm
- Logistic regression
- Softmax regression
- Multilayer neural network and Backpropagation
- Multilayer perceptron
- Support Vector machine

Gradient descent

- Is an optimization algorithm used to minimize some function by iteratively moving in the direction of steepest as defined by the negative of the gradient.
- ⇒ In machine learning we use this algorithm to update the parameters of our model.
- Math, give the cost function:

$$f(m, b) = \frac{1}{N} \sum_{i=1}^n (y_i - (mx_i + b))^2$$

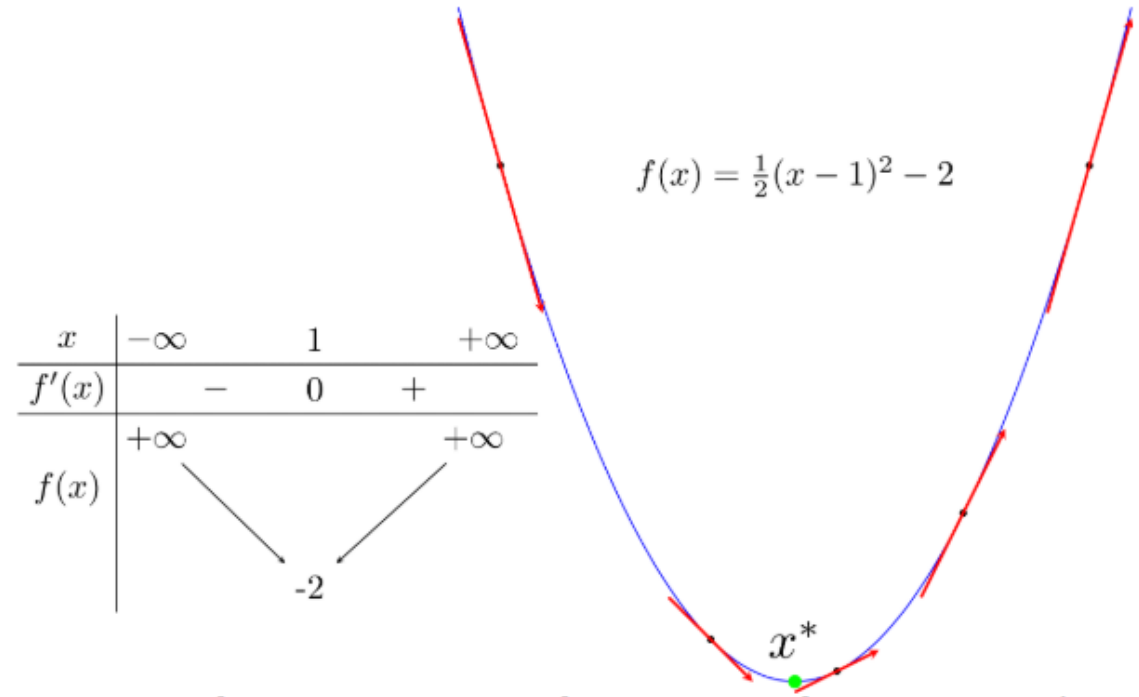
The gradient will be calculated by:

$$f'(m, b) = \begin{bmatrix} \frac{df}{dm} \\ \frac{df}{db} \end{bmatrix} = \begin{bmatrix} \frac{1}{N} \sum -2x_i(y_i - (mx_i + b)) \\ \frac{1}{N} \sum -2(y_i - (mx_i + b)) \end{bmatrix}$$

⇒ Solve $f'(m, b) = 0$ to find out the value of parameters m, b

Gradient Descent

- Example
<source code>



Graph gradient descent of function $f(x)$

Perceptron learning algorithm

- The perceptron is an algorithm for supervised learning of binary classifier. A binary classifier is a function which can decide whether or not an input, represented by a vector of numbers, belongs to some specific class.
- Definition of a threshold function:

$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

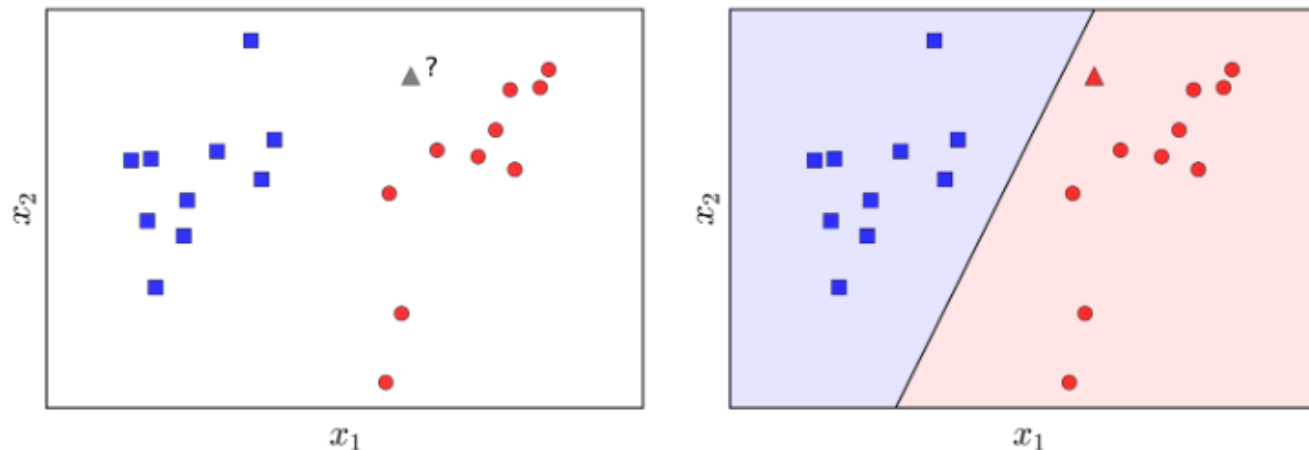
w is a vector of real-valued weights; $w \cdot x$ is the dot product $\sum_{i=1}^m w_i x_i$

m is the number of input to the perceptron

b is the bias

$\Rightarrow f(x)$ is used to classify x as either a positive or a negative instance.

Figure: Binary classification for 2 classes in 2 dimension



Perceptron learning algorithm

- $X = [x_1, x_2, \dots, x_N] \in \mathbb{R}^{d \times N}$ is matrix combine training data, each column x_i is a data point in d dimension space.
- $y = [y_1, y_2, \dots, y_N] \in \mathbb{R}^{1 \times N}$ is label of each data point, $y_i = 1$ if x_i belong to 1st class, $y_i = -1$ if x_i belong to the 2nd class.
- Equation function:

$$f_w(x) = w_1 x_1 + \dots + w_d x_d + w_0 = 0$$

$$\Rightarrow f_w(x) = w^T x = 0$$

\Rightarrow We can define label of input data by:

$$label(x) = \begin{cases} 1 & \text{if } w^T x \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

In other word: $label(x) = \text{sgn}(w^T x)$

Perceptron learning algorithm

- ***Loss function***

Calculate the number of data point was set in wrong class

$$J_1(w) = \sum_{x_i \in \mathcal{M}} (-y_i \operatorname{sgn}(w^T x_i))$$

\mathcal{M} is group of wrong classify data.

\Rightarrow Find out w satisfy $J_1(w) = 0$

\Rightarrow Apply Gradient Descent algorithm to update w :

$$J(w; x_i; y_i) = -y_i w^T x_i; \quad \nabla_w J(w; x_i; y_i) = -y_i x_i$$

Update w by:

$$w \leftarrow w - \eta(-y_i x_i) = w + \eta y_i x_i$$

$\Rightarrow \quad \eta = 1$ we have:

$$\Rightarrow \quad w_{t+1} = w_t + y_i x_i$$

Perceptron learning algorithm

- 1. at $t = 0$, chose random vector w_0
- 2. at t , if don't have any miss classify data, stop algorithm
- 3. if x_i is a miss classify, update:
$$w_{t+1} = w_t + y_i x_i$$
- 4. change $t = t+1$ and back to step 2.

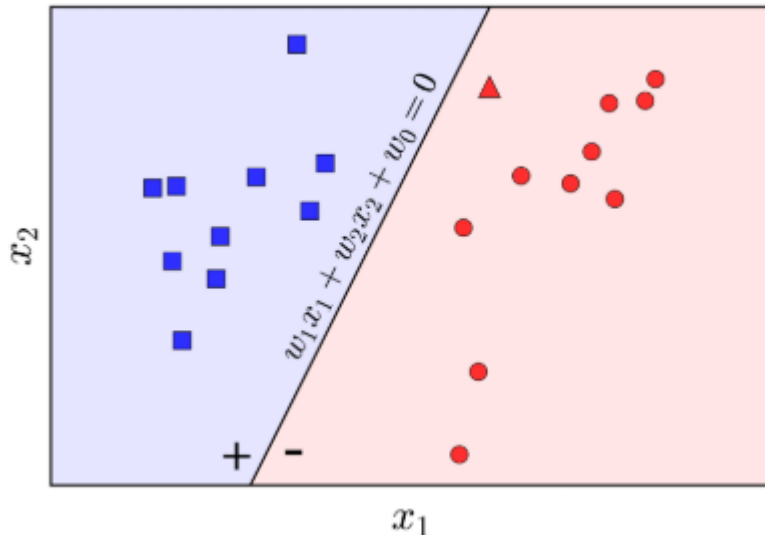


Figure: Boundary equation

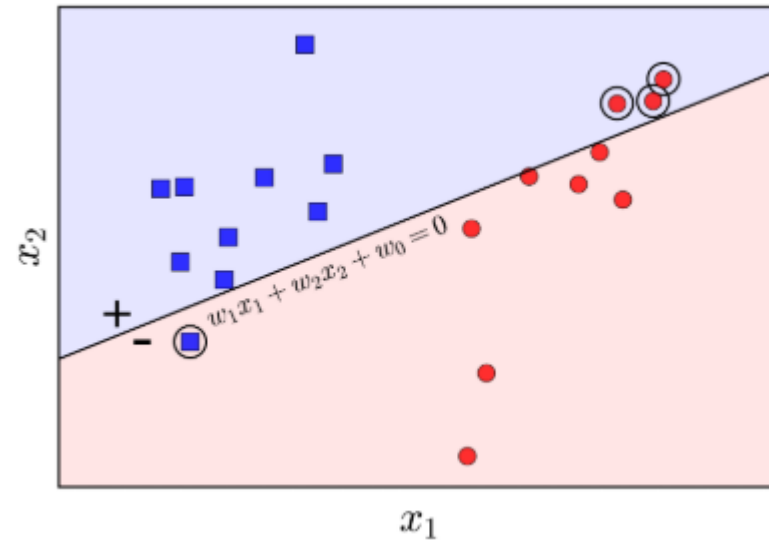


Figure: Random line with misclassified data

Logistic regression

- **Recap:**

- Linear regression to predict output y without limited upper or lower.
 - PLA (perceptron learning) to predict output y only in 1 or -1
- ⇒ Logistic regression will predict output y in between $[0, 1]$

- **Formulation of logistic regression**

$$f(x) = \theta(w^T x)$$

θ is logistic function

We use sigmoid function to describe logistic regression

$$f(s) = \frac{1}{1 + e^{-s}} \triangleq \sigma(s)$$

Loss function:

$$J(w) = - \sum_{i=1}^N (y_i \log z_i + (1 - y_i) \log(1 - z_i))$$

With:

$$z_i = f(w^T x_i)$$
$$P(y_i | x_i; w) = z_i^{y_i} (1 - z_i)^{1 - y_i}$$

Logistic regression

- Example
(source code)

Softmax regression

- We need a probability formulation satisfy each input x , a_i show that input belong to class i

⇒ Each a_i should be positive and $\sum a_i = 1$

⇒ If $z_i = w_i^T x$ larger, the probability of data fall into class i will be more high

⇒ Function :

$$a_i = \frac{\exp(z_i)}{\sum_{j=1}^C \exp(z_j)}, \forall i = 1, 2, \dots, C$$

⇒ This function was called by **Softmax** function.

- **Loss function**

$$J(W) = \sum_{i=1}^N \|a_i - y_i\|_2^2$$

Solve equation by minimize **$J(W)$** with **SDG** algorithms:

$$W = W + \eta x_i (y_i - a_i)^T$$

Softmax regression

- Example (source code)

Multilayer neural network and Backpropagation

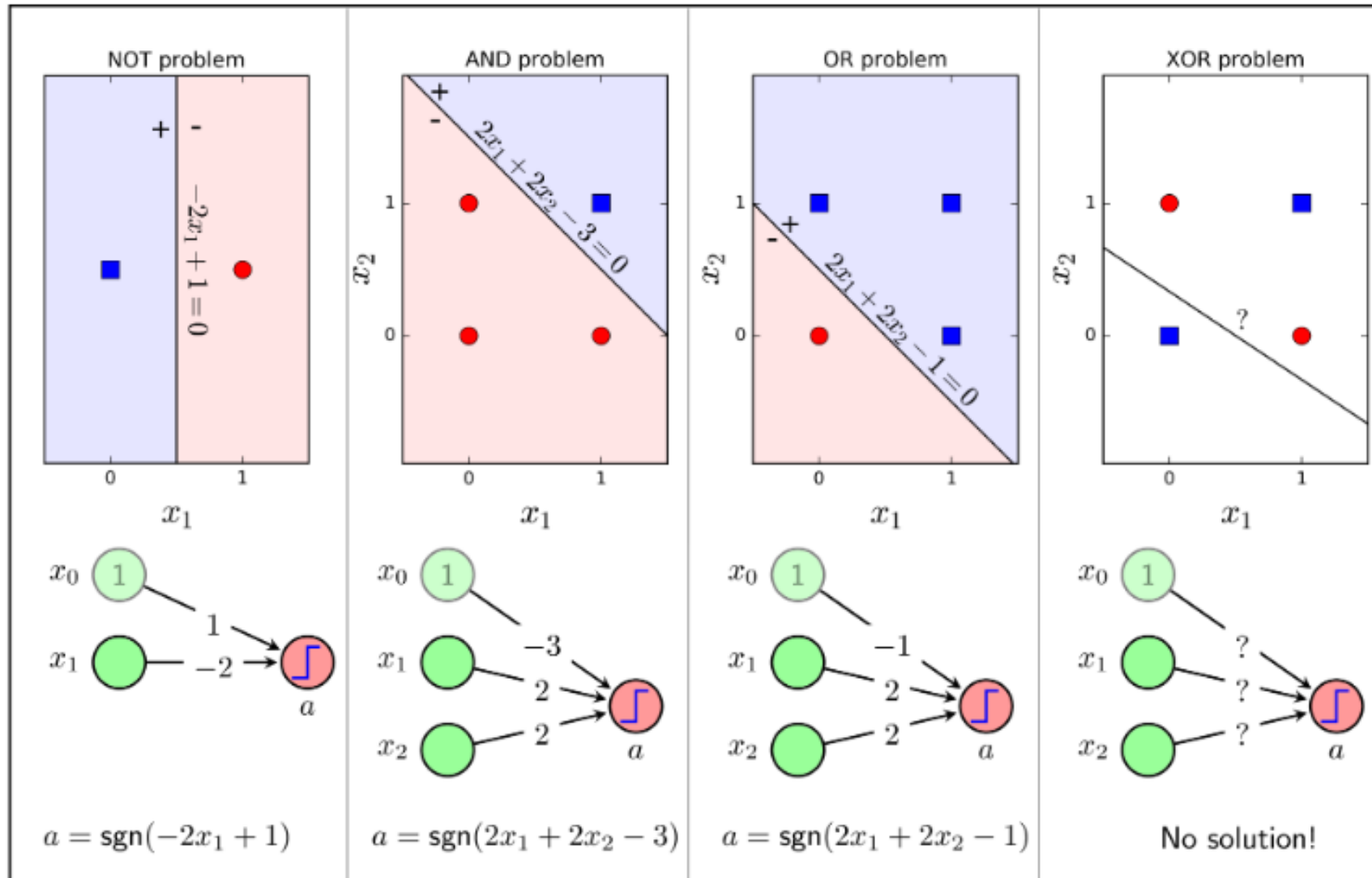


Figure: PLA with logic function basic.

Multilayer neural network and backpropagation

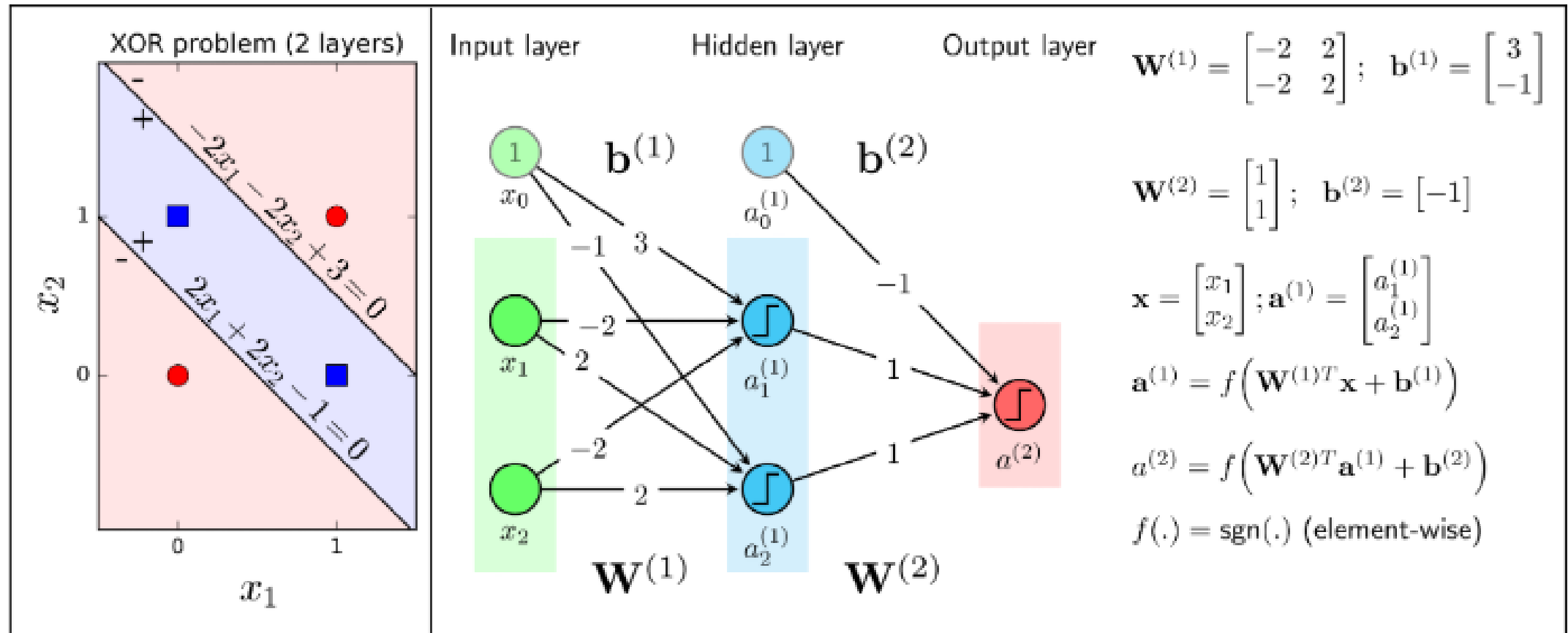


Figure: Multilayer Perceptron expression of XOR function

Multilayer neural network and backpropagation

- **Layers**

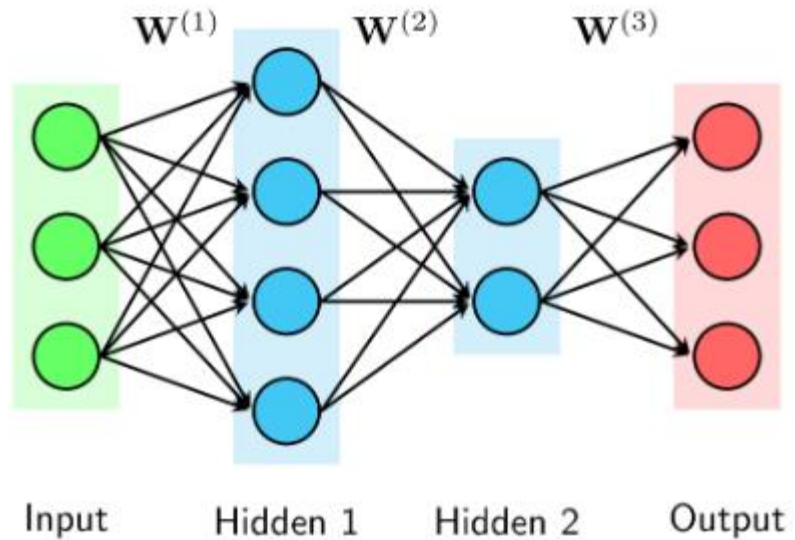


Figure: MLP with 2 hidden layers (biases were hided)

- **Weights and biases: W, b**
- **Activation functions**

Function output of each unit:

$$a_i^l = f(w_i^{(l)T} a^{l-1} + b_i^l)$$

Vector:

$$a^{(l)} = f(W^{(l)T} a^{(l-1)} + b^{(l)})$$

- **Units**

Each node in circle in layer we call is a unit.

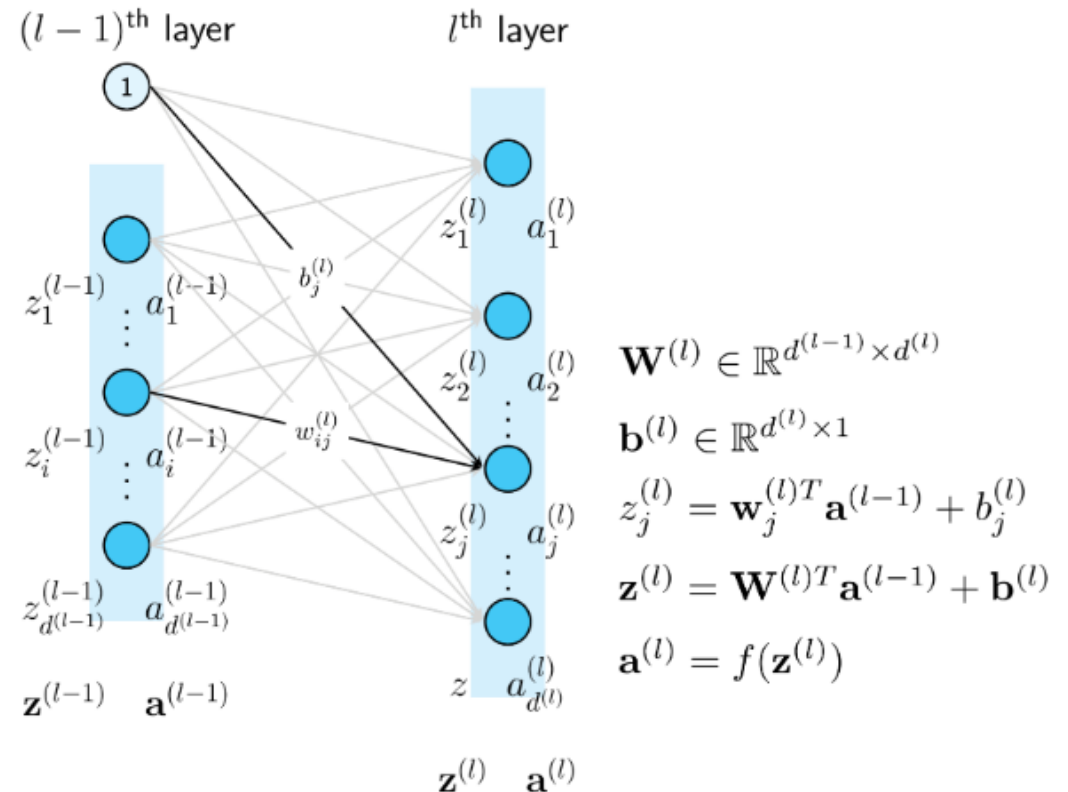


Figure: Symbols uses in MLP

Multilayer neural network and backpropagation

- **Backpropagation**

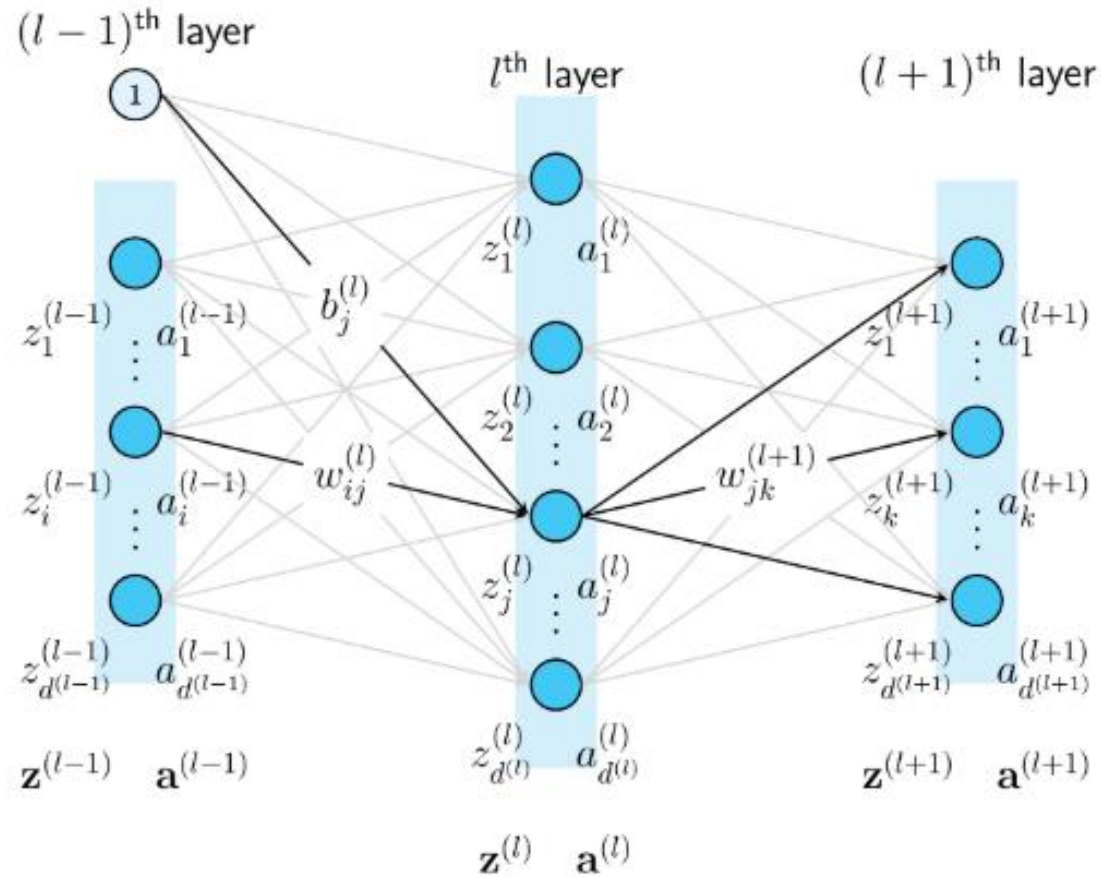
For optimization of loss function with using Gradient Descent algorithm for Weights and bias

$$\frac{\partial J}{\partial W^{(l)}} ; \frac{\partial J}{\partial b^{(l)}} , l = 1, 2, \dots, L$$

$$\begin{aligned} \frac{\partial J}{\partial w_{ij}^{(l)}} &= \frac{\partial J}{\partial z_j^{(l)}} \frac{\partial z_j^{(l)}}{\partial w_{ij}^{(l)}} \\ &= e_j^{(l)} a_i^{(l-1)} \end{aligned}$$

$$e_j^{(l)} = \left(w_j^{(l+1)} e^{(l+1)} \right) f' \left(z_j^{(l)} \right)$$

$$\frac{\partial J}{\partial b_j^{(l)}} = e_j^{(l)}$$



$$\mathbf{W}^{(l)} \in \mathbb{R}^{d^{(l-1)} \times d^{(l)}}$$

$$\mathbf{b}^{(l)} \in \mathbb{R}^{d^{(l)} \times 1}$$

$$z_j^{(l)} = \mathbf{w}_j^{(l)T} \mathbf{a}^{(l-1)} + b_j^{(l)}$$

$$\mathbf{z}^{(l)} = \mathbf{W}^{(l)T} \mathbf{a}^{(l-1)} + \mathbf{b}^{(l)}$$

$$\mathbf{a}^{(l)} = f(\mathbf{z}^{(l)})$$

Figure: Illustrates method of calculating backpropagation from the last layer

Multilayer neural network and backpropagation

- Example (source code)

Schedule and plan

- *Time*: 30/Sept – 30/Nov
- *Machine learning*: 1/Oct – 18/Oct
- *Deep learning*: 21/Oct – 29/Nov
- 2 times per week, 2 hours per time.

Time	Contents
1 – 4/Oct	AI Introduction & requirements & applications Linear regression Overfitting K-nearest neighbors K-mean clustering Naïve Bayes classifier Gradient descent
7 – 11/Oct	Perceptron learning algorithm Logistic regression Softmax regression Multilayer neural network and Backpropagation Multilayer perceptron Support Vector machine

Time	Contents
14 – 18/Oct	Deep neural network Convolutional neural network: faster R-CNN, SSD (Single Shot MultiBox Detector, YOLO (You Look Only Once)
21 – 25/Oct	Deep neural network Convolutional neural network: faster R-CNN, SSD (Single Shot MultiBox Detector, YOLO (You Look Only Once)
28 – 1/Nov	Deep neural network Convolutional neural network: faster R-CNN, SSD (Single Shot MultiBox Detector, YOLO (You Look Only Once)
4 – 8/Nov	Convolutional neural network: faster R-CNN, SSD (Single Shot MultiBox Detector, YOLO (You Look Only Once)
11 – 15/Nov	Time Series: seq-to-seq modeling, RNN, LSTM, GRU Time Series: seq-to-seq modeling, RNN, LSTM, GRU Introduction to Reinforcement learning

THANK YOU SO MUCH!