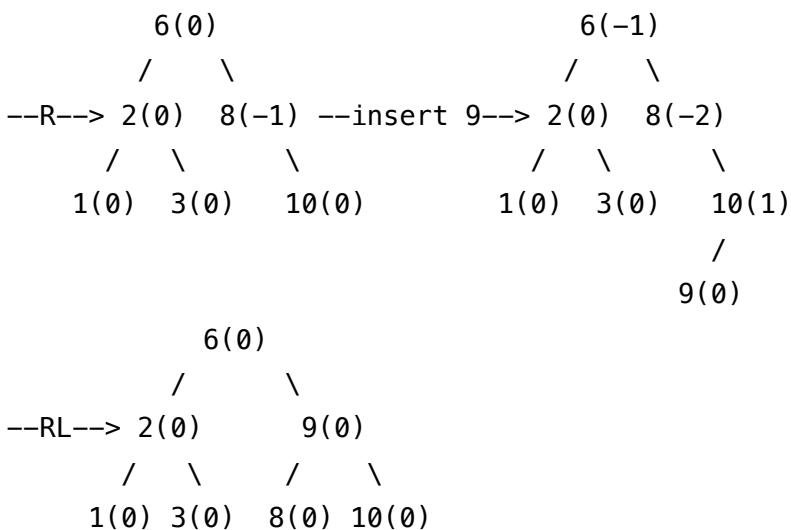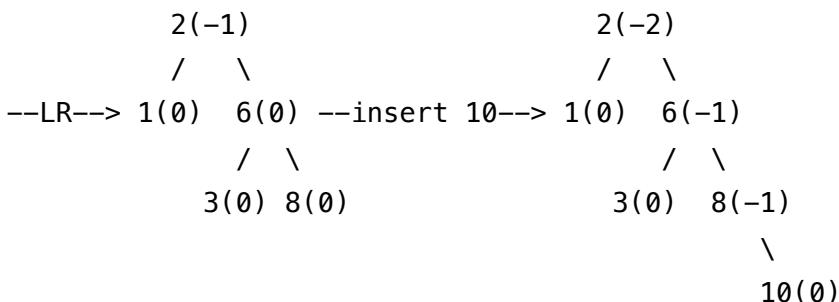# Exercise Sheet 10
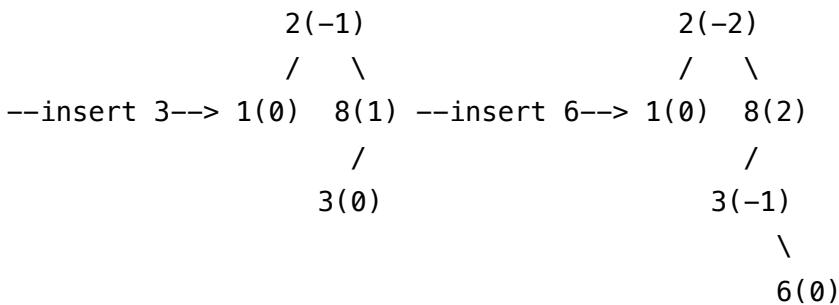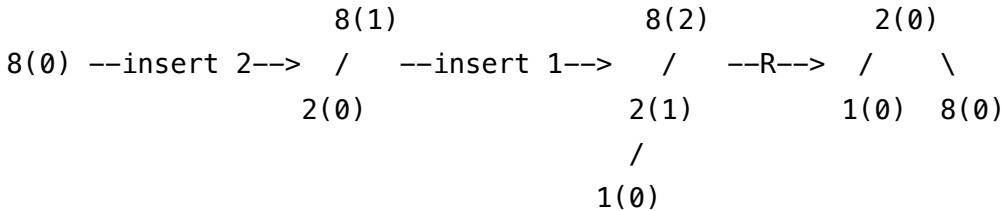
Handout: November 11th — **Deadline:** November 25th before 4pm

Question 10.1 (0.5 Marks)

Insert the keys 8, 2, 1, 3, 6, 10, 9 in this order into an empty AVL tree. Draw the tree constructed after each insertion and after each (double-)rotation (cf. the example in the lecture notes). Write down the balance degree for each node next to the node as shown in the lecture notes.

```
                    8(1)                  8(2)            2(0)
8(0) --insert 2-->  /    --insert 1-->   /    --R-->  /     \
                  2(0)                  2(1)         1(0)  8(0)
                                         /
                                       1(0)


                 2(-1)                      2(-2)
                 /   \                      /   \
--insert 3--> 1(0)  8(1) --insert 6--> 1(0)  8(2)
                      /                        /
                    3(0)                    3(-1)
                                                \
                                               6(0)


         2(-1)                      2(-2)
         /   \                      /   \
--LR--> 1(0)  6(0) --insert 10--> 1(0)   6(-1)
              / \                        / \
            3(0) 8(0)                  3(0)  8(-1)
                                                \
                                              10(0)


        6(0)                          6(-1)
        /    \                        /    \
--R--> 2(0)   8(-1) --insert 9--> 2(0)   8(-2)
       / \       \                / \       \
     1(0)  3(0)  10(0)          1(0)  3(0)   10(1)
                                              /
                                            9(0)


            6(0)
            /     \
--RL--> 2(0)       9(0)
        / \       /   \
     1(0) 3(0)  8(0) 10(0)
```

Question 10.2 (0.5 marks)

Say the minimum number of nodes that an AVL tree of height h = 10 must contain.

Answer:

Due to the theorem discussed in the lecture, we know that the minimum number of nodes `A(h)` in an AVL tree of height `h` satisfies the recurrence relation:

$$A(h) = A(h-1) + A(h-2) + 1$$

with base cases:

- `A(0)` = 1 (an AVL tree of height 0 has 1 node)
- `A(1)` = 2 (an AVL tree of height 1 has 2 nodes)
  Calculating the values up to `h` = `10` :
- `A(2)` = `A(1)` + `A(0)` + 1 = 2 + 1 + 1 = 4
- `A(3)` = `A(2)` + `A(1)` + 1 = 4 + 2 + 1 = 7
- `A(4)` = `A(3)` + `A(2)` + 1 = 7 + 4 + 1 = 12
- `A(5)` = `A(4)` + `A(3)` + 1 = 12 + 7 + 1 = 20
- `A(6)` = `A(5)` + `A(4)` + 1 = 20 + 12 + 1 = 33
- `A(7)` = `A(6)` + `A(5)` + 1 = 33 + 20 + 1 = 54
- `A(8)` = `A(7)` + `A(6)` + 1 = 54 + 33 + 1 = 88
- `A(9)` = `A(8)` + `A(7)` + 1 = 88 + 54 + 1 = 143
- `A(10)` = `A(9)` + `A(8)` + 1 = 143 + 88 + 1 = 232

Therefore, an AVL tree of height h = 10 must contain at least 232 nodes.