Artificial Intelligence

Lecture 5: Adversarial Search

Credit: Ansaf Salleb-Aouissi, and “Artificial Intelligence: A Modern Approach”, Stuart Russell and Peter Norvig, and “The Elements of Statistical Learning”, Trevor Hastie, Robert Tibshirani, and Jerome Friedman, and “Machine Learning”, Tom Mitchell.

Adversarial Search

- Adversarial search problems ≡ games
- They occur in multi-agent competitive environments
- There is an **opponent** we can' t control planning again us!
- Game vs. search: optimal solution is not a sequence of actions but a **strategy** (policy) If opponent does *a*, agent does *b*, else if opponent does *c*, agent does *d*, etc.
- Tedious and fragile if hard-coded (i.e., implemented with rules)
- Good news: Games are modeled as **search problems** and use **heuristic evaluation** functions.

Games: hard topic

- Games are a big deal in AI
- Games are interesting to AI because they are too hard to solve
  - Chess has a branching factor of 35, with 35^{100} nodes $\approx 10^{154}$
- Need to make some decision even when the optimal decision is infeasible

Adversarial Search

Checkers:

- Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994.
- Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Adversarial Search

Chess:

- In 1949, Claude E. Shannon in his paper "Programming a Computer for Playing Chess", suggested Chess as an AI problem for the community.
- Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997.
- In 2006, Vladmir Kramnik, the undisputed world champion, was defeated 4-2 by Deep Fritz.

Adversarial Search

Go: $b > 300$! Google Deep mind Project AlphaGo. In 2016, AlphaGo beat both Fan Hui, the European Go champion, and Lee Sedol, the world best player.

Othello: Several computer othello exists and human champions refuse to compete against computers, that are too good.

Types of games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

We are mostly interested in deterministic games, fully observable environments, zero-sum, where two agents act alternately.

Zero-sum Games

- Adversarial: Pure competition.
- Agents have different values on the outcomes.
- One agent maximizes one single value, while the other minimizes it.
- Each move by one of the players is called a “ply.”

One function: one agent maximizes it and one minimizes it!

Embedded thinking…

Embedded thinking or backward reasoning!

- One agent is trying to figure out what to do.
- How to decide? He thinks about the consequences of the possible actions.
- He needs to think about his opponent as well…
- The opponent is also thinking about what to do etc.
- Each will imagine what would be the response from the opponent to their actions.
- This entails an embedded thinking.

Formalization

- The **initial state**
- Player(s): defines which player has the move in state s. Usually taking turns.
- Actions(s): returns the set of legal moves in s
- **Transition** function: $S \times A \rightarrow S$ defines the result of a move
- Terminal test: True when the game is over, False otherwise. States where game ends are called **terminal states**
- *Utility*(s, p): **utility function** or objective function for a game that ends in terminal state s for player p. In Chess, the outcome is a win, loss, or draw with values +1, 0, 1/2. For tic-tac-toe we can use a utility of +1, -1, 0.

Single player…

Assume we have a tic-tac-toe with one player.

Let' s call him Max and have him play three moves only for the sake of the example.

Single player...

In the case of one player, nothing will prevent Max from winning (choose the path that leads to the desired utility of 1), unless there is another player who will do everything to make Max lose, let' s call him Min.

Adversarial search: minimax

- Two players: Max and Min
- Players alternate turns
- Max moves first
- Max maximizes results
- Min minimizes the results
- Compute each node' s minimax value that is the best achievable utility against an optimal adversary
- Minimax value ≡ best achievable payoff against best play

Minimax example

Adversarial search: minimax

- Find the optimal strategy for Max:
  - Depth-first search of the game tree
  - An optimal leaf node could appear at any depth of the tree
  - Minimax principle: compute the utility of being in a state assuming both players play optimally from there until the end of the game
  - Propagate minimax values up the tree once terminal nodes are discovered

Adversarial search: minimax

- If state is terminal node: Value is utility(state)
- If state is MAX node: Value is highest value of all successor node values (children)
- If state is MIN node: Value is lowest value of all successor node values (children)

Adversarial search: minimax

For a state s minimax(s) =

$$\begin{cases} Utility(s) & \text{if Terminal-test(s)} \\ max_{a \in Actions(s)} \text{minimax(Result(s,a))} & \text{if Player(s) = Max} \\ min_{a \in Actions(s)} \text{minimax(Result(s,a))} & \text{if Player(s) = Min} \end{cases}$$

The minimax algorithm

```
/* Find the child state with the lowest utility value */

function MINIMIZE(state)
    returns TUPLE of ⟨STATE, UTILITY⟩ :

    if TERMINAL-TEST(state):
        return ⟨NULL, EVAL(state)⟩

    ⟨minChild, minUtility⟩ = ⟨NULL, ∞⟩

    for child in state.children():
        ⟨ _, utility⟩ = MAXIMIZE(child)

        if utility < minUtility:
            ⟨minChild, minUtility⟩ = ⟨child, utility⟩

    return ⟨minChild, minUtility⟩
```

```
/* Find the child state with the highest utility value */

function MAXIMIZE(state)
    returns TUPLE of ⟨STATE, UTILITY⟩ :

    if TERMINAL-TEST(state):
        return ⟨NULL, EVAL(state)⟩

    ⟨maxChild, maxUtility⟩ = ⟨NULL, −∞⟩

    for child in state.children():
        ⟨ _, utility⟩ = MINIMIZE(child)

        if utility > maxUtility:
            ⟨maxChild, maxUtility⟩ = ⟨child, utility⟩

    return ⟨maxChild, maxUtility⟩
```

```
/* Find the child state with the highest utility value */

function DECISION(state)
    returns STATE :

    ⟨child, _⟩ = MAXIMIZE(state)

    return child
```

Minimax example

Minimax example

Minimax example

Minimax example

Minimax example

Properties of minimax

- Optimal (opponent plays optimally) and complete (finite tree)
- DFS time: $O(b^m)$
- DFS space: $O(bm)$
  - **Tic-Tac-Toe**
    - ≈ 5 legal moves on average, total of 9 moves (9 plies).
    - 5^9 = 1,953,125 •9! = 362,880 terminal nodes
  - **Chess**
    - $b \approx 35$ (average branching factor)
    - $d \approx 100$ (depth of game tree for a typical game)
    - $b^d \approx 35^{100} \approx 10^{154}$ nodes
  - **Go** branching factor starts at 361 (19X19 board)

Case of limited resources

- Problem: In real games, **we are limited in time, so we can' t search the leaves.**
- To be practical and run in a reasonable amount of time, minimax can only search to some depth.
- More plies make a big difference.
- **Solution:**
  1. Replace terminal utilities with an evaluation function for non-terminal positions.
  2. Use Iterative Deepening Search (IDS).
  3. Use pruning: eliminate large parts of the tree.

$\alpha - \beta$ pruning

A two-ply game tree.

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

Which values are necessary?

$\alpha - \beta$ pruning

$$
\begin{aligned}
Minimax(root) &= max(min(3,12,8), min(2,X,Y), min(14,5,2)) \\
&= max(3, min(2,X,Y), 2) \\
&= max(3, Z, 2) \qquad \text{where } Z = min(2,X,Y) \leq 2 \\
&= 3
\end{aligned}
$$

Minimax decisions are independent of the values of X and Y.

$\alpha - \beta$ pruning

- **Strategy:** Just like minimax, it performs a DFS.
- **Parameters**: Keep track of two bounds
  - α: the value of the best (i.e., highest-value) choice we have found so far at any choice point along the path for MAX.
  - β: the value of the best (i.e., lowest-value) choice we have found so far at any choice point along the path for MIN.
- **Initialization:** $\alpha = -\infty$, $\beta = \infty$
- **Propagation**: Send α, β values down during the search to be used for pruning.
  - Update α, β values by propagating upwards values of terminal nodes.
  - Update α only at Max nodes and update β only at Min nodes.
- **Pruning:** Prune any remaining branches whenever $\alpha \geq \beta$.

$\alpha - \beta$ pruning

- If α is better than a for Max, then Max will avoid it, that is prune that branch.
- If β is better than b for Min, then Min will avoid it, that is prune that branch.

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

$\alpha - \beta$ pruning

```
/* Find the child state with the lowest utility value */

function MINIMIZE(state, α, β)
    returns TUPLE of ⟨STATE, UTILITY⟩ :

    if TERMINAL-TEST(state):
        return ⟨NULL, EVAL(state)⟩

    ⟨minChild, minUtility⟩ = ⟨NULL, ∞⟩

    for child in state.children():
        ⟨_, utility⟩ = MAXIMIZE(child, α, β)

        if utility < minUtility:
            ⟨minChild, minUtility⟩ = ⟨child, utilit

        if minUtility ≤ α:
            break

        if minUtility < β:
            β = minUtility

    return ⟨minChild, minUtility⟩
```

```
/* Find the child state with the highest utility value */

function MAXIMIZE(state, α, β)
    returns TUPLE of ⟨STATE, UTILITY⟩ :

    if TERMINAL-TEST(state):
        return ⟨NULL, EVAL(state)⟩

    ⟨maxChild, maxUtility⟩ = ⟨NULL, −∞⟩

    for child in state.children():
        ⟨_, utility⟩ = MINIMIZE(child, α, β)

        if utility > maxUtility:
            ⟨maxChild, maxUtility⟩ = ⟨child, utility⟩

        if maxUtility ≥ β:
            break

        if maxUtility > α:
            α = maxUtility

    return ⟨maxChild, maxUtility⟩
```

```
/* Find the child state with the highest utility value */

function DECISION(state)
    returns STATE :

    ⟨child, _⟩ = MAXIMIZE(state, −∞, ∞)

    return child
```

Move ordering

- It does matter as it affects the effectiveness of $\alpha - \beta$ pruning.
- Example: We could not prune any successor of D because the worst successors for Min were generated first. If the third one (leaf 2) was generated first we would have pruned the two others (14 and 5).
- Idea of ordering: examine first successors that are likely best.

Move ordering

- **Worst ordering:** no pruning happens (best moves are on the right of the game tree). Complexity $O(b^m)$.
- **Ideal ordering:** lots of pruning happens (best moves are on the left of the game tree). Complexity $O(b^{m/2})$ (in practice).
- **How to find a good ordering?**
  - Remember the best moves from shallowest nodes.
  - Use domain knowledge.
  - Bookkeep the states, they may repeat!

Real-time decisions

- Minimax: generates the entire game search space
- $\alpha - \beta$ algorithm: prune large chunks of the trees
- BUT $\alpha - \beta$ still has to go all the way to the leaves
- Impractical in real-time (moves has to be done in a reasonable amount of time)
- Solution: bound the depth of search (cut off search) and **replace** *utility*(*s*) **with** *eval*(*s*), an evaluation function to **estimate** value of current board configurations

Real-time decisions

- ***eval*(*s*)** is a heuristic at state *s*
  - E.g., Othello: # white pieces - # black pieces
  - E.g., Chess: value of all white pieces - value of all black pieces
  - turn non-terminal nodes into terminal leaves!
- An ideal evaluation function would rank terminal states in the same way as the true utility function; but must be fast
- Typical to define features, make the function a linear weighted sum of the features
- Use domain knowledge to craft the best and useful features.

Real-time decisions

- How does it works?
  - Select useful features $f_1, \ldots, f_n$ e.g., Chess: # pieces on board, value of pieces (1 for pawn, 3 for bishop, etc.)
  - Weighted linear function:

$$eval(s) = \sum_{i=1}^{n} w_i f_i(s)$$

  - **Learn** w_i from the examples
  - Deep blue uses about 6,000 features!

Stochastic games

- Include a random element (e.g., throwing a dice).
- Include chance nodes.
- Backgammon: old board game combining skills and chance.
- The goal is that each player tries to move all of his pieces off the board before his opponent does.

Stochastic games

Partial game tree for Backgammon.

Stochastic games

Algorithm **Expectiminimax** generalized Minimax to handle chance nodes as follows:

- If state is a Max node then return the highest Expectiminimax-Value of Successors(state)
- If state is a Min node then return the lowest Expectiminimax-Value of Successors(state)
- If state is a chance node then return average of Expectiminimax-Value of Successors(state)

Stochastic games

Example with coin-flipping:

Expectiminimax

For a state *s*:

Expectiminimax(*s*) =

$$\begin{cases} Utility(s) & \text{if Terminal-test(s)} \\ max_{a \in Actions(s)} \text{ Expectiminimax(Result(s,a))} & \text{if Player(s) = Max} \\ min_{a \in Actions(s)} \text{ Expectiminimax(Result(s,a))} & \text{if Player(s) = Min} \\ \sum_r P(r) \text{ Expectiminimax(Result(s,r))} & \text{if Player(s) = Chance} \end{cases}$$

Where *r* represents all chance events (e.g., dice roll), and Result(*s*, *r*) is the same state as *s* with the result of the chance event is *r*.

Games: conclusion

- Games are modeled in AI as a search problem .
- Minimax algorithm choses the best move given an optimal play from the opponent.
- Minimax goes all the way down the tree which is not practical give game time constraints.
- Alpha-Beta pruning can reduce the game tree search which allow to go deeper in the tree within the time constraints.
- Pruning, bookkeeping, evaluation heuristics, node re-ordering and IDS are effective in practice.

Games: conclusion

- Games is an exciting and fun topic for AI.
- Devising adversarial search agents is challenging because of the huge state space.
- We have just scratched the surface of this topic.
- Further topics to explore include partially observable games (card games such as bridge, poker, etc.).
- Except for robot football (a.k.a. soccer), there was no much interest from AI in physical games. (see http://www.robocup.org/).

To be continued