Another view of VAE (max $\log p(x)$)

$$p(x) = \int p(x,z)\,dz$$
$$= \int p(x|z)\cdot p(z)\,dz$$
$$= \int \frac{q_\lambda(z|x)\cdot p(z)\cdot p(x|z)}{q_\lambda(z|x)}\,dz$$

$\longleftrightarrow$ Computing the "log" version of this formular is intractable. (we don't like log sum.)

$$\log p(x) = \log \int \frac{q_\lambda(z|x)\cdot p(z)\cdot p(x|z)}{q_\lambda(z|x)}\,dz$$

* applying Jensen's inequality

$$\log p(x) \geq \int q_\lambda(z|x)\cdot \log \frac{p(z)\cdot p(x|z)}{q_\lambda(z|x)}\,dz$$
$$= \int q_\lambda(z|x)\log p(x|z)\,dz + \int q_\lambda(z|x)\log\frac{p(z)}{q_\lambda(z|x)}\,dz$$
$$= \underbrace{E_{q_\lambda}(\log p(x|z)) - KL(q_\lambda(z|x)\,//\,p(z))}_{\text{ELBO} \longleftrightarrow \text{Evidence lower bound.}}$$

For VAE, we want to maximize the log likelihood.

i.e, $\log p(x)$, we could maximize the ELBO

i.e., $\max\left(E_q(\log p(x|z)) - KL(q_\lambda(z|x)//p(z))\right)$.

i.e minimize its negative. w.r.t its parameters.

$$\min\ - E_q(\log p(x|z)) + KL(q_\lambda(z|x)\,||\,p(z))$$

This is the objective function of VAE