STA4020 Final Project Report

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Introduction

This project explores the practical application of asset allocation methods to portfolio construction and evaluation, including mean-variance optimization (MVO), equal-weight (1/N) portfolios, and Black-Litterman models incorporating perspectives. The project selected five assets and analysis using two years of three-month daily data. The first two years of data were used as the in-sample data of the model, and the remaining data were used for out-of-sample analysis. First, the MVE portfolio without short constraints is constructed by numerical optimization method, and the average return, volatility and annualized Sharpe ratio are compared with the equal-weight portfolio in the sample. Then, based on analysis reports and historical data, the Black-Litterman model is generated to derive a posteriori distribution of asset returns and optimize weights. Finally, the in-sample weights of MVE, 1/N, and Black-Litterman are applied to the out-of-sample data to evaluate the average return, volatility, and Sharpe ratio, so as to summarize and compare the advantages and disadvantages of the three methods.

Data Collection Process

I have chosen the following stocks for the investment portfolio: Luzhou Laojiao (000568), Gree Electric Appliances (000651), Wuliangye (000858), Hengrui Medicine (600276), and Kweichow Moutai (600519) in the Chinese A-shares market. I chose them because they are top companies in their fields. They make good products and have a strong customer base. These companies are in different areas like drinks, home goods, and medicine, which helps to spread risk. They also have the potential to grow, making them a good choice for investing.

After selecting the assets, the first step is to preprocess and split the daily frequency stock return data. After loading the return data from August 1, 2022, to October 31, 2024 (a total of 2 years and 3 months), the *pivot* method is used to restructure the raw data into a matrix format, with trade dates as the index, stock codes as columns, and the return changes as the values. Then, the *isna* method is applied to check for missing data frame values. It is verified that there are no empty values in the data frame. Finally, the data is split into two parts: the period from August 1, 2022, to July 31, 2024, is designated as the in-sample data, used for model training; and the period from August 1, 2024, to October 31, 2024, is designated as the out-of-sample data, used for model validation.

Methodology

1. Mean-Variance Optimization Process

To construct the Mean-Variance Efficient (MVE) portfolio, we adopt an optimization way that maximizes the portfolio's Sharpe ratio, subject to constraints ensuring full investment and no short-selling. Given the assumption of a zero risk-free rate, the optimization problem is as follows:

$$\min_{\omega} -\frac{\omega^T \mu}{\sqrt{\omega^T \Sigma\, \omega}},$$
$$s.t. \sum_{i=1}^{N} \omega_i = 1\,, \omega_i > 0,$$

where ω is the vector of portfolio weights, μ is the vector of expected returns for the assets, and Σ is the covariance matrix of asset returns.

2. Black-Litterman Model

The Black-Litterman model, developed by Fischer Black and Robert Litterman, provides a way to incorporate both market equilibrium data and subjective investor views into the asset allocation process. The Black-Litterman model starts with a prior distribution of returns and then incorporates investor views about certain assets, either in the form of absolute return expectations or relative return expectations compared to other assets. These views are combined with the prior to generate a posterior distribution of expected returns. The posterior distribution is as follows:

$$\mu^e | q \sim N([(\tau\Sigma)^{-1} + P^T \Omega^{-1} P]^{-1}((\tau\Sigma)^{-1}\pi + P^T \Omega^{-1} q), [(\tau\Sigma)^{-1} + P^T \Omega^{-1} P]^{-1})$$

where π is the prior vector of expected returns, P is the matrix of views (each row corresponds to a specific view on the asset returns), q is the vector of views (each element represents the magnitude of an investor's view), Ω is the variance matrix of the views (diagonal elements indicate the uncertainty of each view), and τ is a scalar parameter that controls the relative importance of the prior covariance matrix versus the view.

The Views for the Black-Litterman Model

Luzhou Laojiao (000568) is slightly underperforming compared to the CSI 300 Index, while Wuliangye (000858) is nearly on par with the CSI 300 in terms of stock price. The overall yield of Wuliangye (000858) has shown an upward trend. These support the market's expectation that Wuliangye is likely to outperform Luzhou Laojiao by 0.5% daily, with a 1% uncertainty.

Wuliangye (000858) is expected to slightly outperform Hengrui Medicine (600276) by 0.1% daily with a 1% uncertainty, primarily due to Wuliangye's strong brand value, optimized product portfolio, particularly its high-end products, and increased revenue from distribution channels. In contrast, Hengrui Medicine may face

challenges from external environmental changes and product price reductions, leading to market expectations that Wuliangye's growth momentum is relatively stronger.

Hengrui Medicine (600276) is expected to slightly outperform Kweichow Moutai (600519) by 0.02% daily, with an uncertainty of 0.5%. The primary reason for this expectation is Hengrui Medicine's continuous increase in R&D investment for innovative drugs and its active expansion in both domestic and international markets. In contrast, although Kweichow Moutai has a strong brand, it is more conservative in terms of internationalization and innovation, which may result in a slightly weaker growth momentum compared to Hengrui Medicine.

So finally I constructed a total of three views as follows:

1. Wuliangye (000858) is expected to outperform Luzhou Laojiao (000568) by 0.5% daily; uncertainty is 1%.
2. Wuliangye (000858) is expected to outperform Hengrui Medicine (600276) by 0.1% daily; uncertainty is 1%.
3. Hengrui Medicine (600276) is expected to outperform Kweichow Moutai (600519) by 0.02% daily; uncertainty is 0.5%.

Performance

1. Comparison of the In-Sample Performance of the 1/N and MVE Portfolio

The Minimum Variance Equilibrium (MVE) portfolio demonstrated superior in-sample performance compared to the Equal-Weighted (1/N) portfolio, with an average return of 0.000697 against the Equal-Weighted's -0.000047, indicating a positive gain versus a slight loss. The MVE portfolio also exhibited a lower standard deviation of 0.013099, suggesting less return volatility.

Furthermore, the MVE portfolio's annualized Sharpe Ratio of 0.844635 highlights its effective risk-adjusted return, significantly outperforming the Equal-Weighted portfolio's Sharpe Ratio of -0.054005, which implies a lack of adequate risk compensation. Figure 1 corroborates these findings, showing the MVE portfolio's cumulative return trending upwards and finishing higher, while the Equal-Weighted portfolio's return trends downward.

Portfolio	MVE	Equal-Weighted
Average Return	0.000697	-0.000047
Standard Deviation	0.013099	0.013796
Annualized Sharpe Ratio	0.844635	-0.054005

Table 1: Two Portfolios Performance (In-sample)

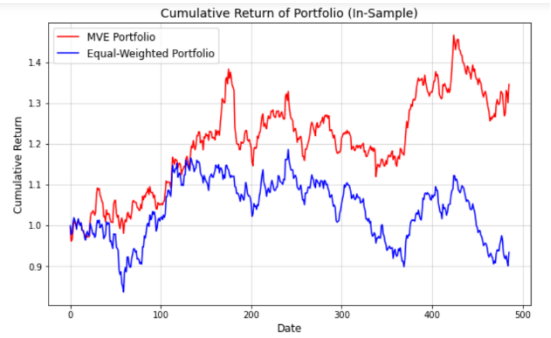

Figure 1: Cumulative Return of Portfolio (In-Sample)

2. Comparison of the Out-Of-Sample Performance of the 1/N, MVE, and Black-Litterman Portfolios.

The out-of-sample performance of the three portfolios—MVE, Equal-Weighted, and Black-Litterman Model—demonstrates distinct differences in terms of average return and risk-adjusted returns. The MVE portfolio achieved an average return of 0.002466, closely followed by the Black-Litterman Model at 0.002651, and the Equal-Weighted portfolio at 0.002256. This suggests that all three strategies were effective in generating positive returns, with the Black-Litterman Model slightly edging out the others.

When considering risk-adjusted returns, as measured by the Sharpe Ratio, the Black-Litterman Model portfolio leads with a ratio of 1.830033, indicating a superior balance between return and risk. The MVE portfolio also shows a strong performance with a Sharpe Ratio of 1.759865, while the Equal-Weighted portfolio has a lower, yet still positive, Sharpe Ratio of 1.381586. The lower standard deviation of the MVE and Black-Litterman portfolios compared to the Equal-Weighted portfolio suggests that these two strategies were better at managing volatility, contributing to their higher Sharpe Ratios. Figure 2 visually supports these findings, with the MVE and Black-Litterman portfolios showing more consistent growth and less volatility, particularly in the later part of the out-of-sample period.

Portfolio	MVE	Equal-Weighted	Black-Litterman
Average Return	0.002466	0.002256	0.002651
Standard Deviation	0.022245	0.025918	0.022995
Annualized Sharpe Ratio	1.759865	1.381586	1.830033

Table 2: Three Portfolios Performance (Out-of-sample)

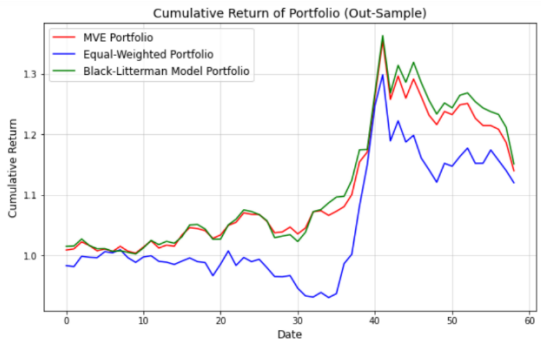

Figure 2: Cumulative Return of Portfolio (Out-Sample)

The comparison of weights for the three portfolios is shown in Figure 3.

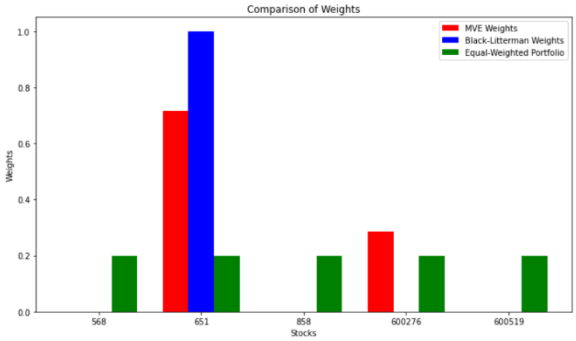

Figure 3: The comparison of weights for the Three Portfolios

Discussion

The minimum variance equilibrium (MVE) strategy minimizes portfolio risk by optimizing asset weights, and its advantages lie in mathematical robustness and operational simplicity. However, this approach may rely too heavily on historical data and assume that correlations and volatility between assets will remain the same in the future, which may not always hold true in real markets.

The equal-weight (1/N) strategy, with its simplicity and equal distribution of assets, provides investors with a way to achieve diversification without in-depth analysis. The strength of this approach lies in its resilience to the underperformance of a single asset, but it ignores the differences in risk and return between assets.

Black-Litterman models combine market equilibrium returns with investors' subjective views, allowing investors to incorporate their own insights into asset allocation. The strength of this approach lies in its flexibility and ability to integrate market information, but it requires investors to have a deep understanding of the market.