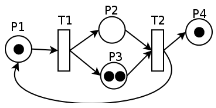
ADT : Proofs (again)



October 22rd 2015

David Lawrence Edmundo Lopez Alexis Marechal

This session will continue with the exercises started in the last session, explaining the notions of Σ-algebra, model and the proofs by induction (which include the equational proofs).

Consider the following AlPiNA ADT for the natural numbers :

```
ADT nat
Sorts
  nat;
Generators
  zero: nat;
  suc : nat -> nat;
Operations
  + : nat, nat -> nat;
Axioms
  x + zero = x;
  x + suc(y) = suc(x+y);
Variables
  x : nat;
  y : nat;
```

In the last session, we proved the following theorems :

```
x + suc(zero) = suc(x) (equational proof)
suc(zero) + x = suc(x) (induction)
```

Exercise 1 : Another inductive proof

Consider the following theorem :

```
suc(x) + y = suc(x + y)
```

1. *(Finitely generated by the generators algebra)* Make a detailed proof of this theorem in $\mathbb{N}$ by taking into account that $\mathbb{N}$ is finitely generated by the generators of the AADT.
2. *(Finitely generated algebra)* How would you do this proof if $\mathbb{N}$ was only finitely generated ?

Exercise 2 : Another algebra

Consider the set with only one item, $\{0\}$, that trivially defines the $+$ and suc operations ($0+0=0$ and $\mathrm{suc}(0) = 0$).

1. Is this a Σ-algebra is Σ is the signature defined by the listing shown previously ?
2. Is it a model of the specification ?
3. Are the two theorems from the last session verified by this set ?