Phys 250 Project Proposal

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I like dynamical systems, so for this project I wanted to pick one of the classical problems which resurrected interest in dynamics in the 20th century. One of the earliest such examples is Yoshisuke Ueda's nonlinear oscillator, a restricted version of the Duffing oscillator. The equation I will study is

$$\ddot{x} + \delta\dot{x} + x^3 = \gamma \cos t$$

where δ and γ are given real constants. Physically, this equation models a damped, periodically forced nonlinear oscillator, for example an elastic pendulum whose spring stiffness doesn't exactly adhere to Hooke's law. On the dynamics side, this is one of the simplest nonlinear dynamical systems I have looked at which exhibits a wide variety of interesting phenomena while being entirely deterministic. In particular, regular and chaotic motions, coexisting attractors, regular and fractal basin boundaries, and local and global bifurcations. All of this behavior is best understood visually, via simulation—the intention of the project—while the geometric theory of dynamical systems also bears out much of this phenomena in a rigorous manner—Hirsch, Smale, Devaney is a classic book which provides mathematical analysis of the behavior of damped and driven oscillators.

I plan to begin with the standard sort of visual analysis: plots of the systems evolution, phase maps, and Poincaré section. Time-permitting, I will try to also animate a flow map, which shows using color how points flow over a few periods of the oscillator. I would like to end by describing the transition to chaos. I guess bifurcation diagrams are used here. There are mathematical characterizations of how these occur that may be interesting to describe and provide visualizations to support—for instance, when $\gamma \gg \delta$, according to Ueda[1] successive period doubling is the most common in this scenario.

I have pretty good references for this: two original papers by Ueda and as well as some coverage in the popular textbook *Nonlinear dynamics and chaos* by Thompson and Stewart.

[1] see here

Intro.

Describe physical model and introduce equation

$$\ddot{x} + \delta \dot{x} + x^3 = \gamma \cos t$$

* Blank spaces are for graphics *

Exhibit simple and chaotic motion via

evolution, phase, and first-return plots

Sensitivity to initial conditions

Model using numerically computed Lyapunov exponents: graph this. Discuss

Transitions to Chaos

Look at 2-3 ways it happens. Describe the physical situation. Bifurcation diagrams

Conclusion + interpretation

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Link to github

References