HW1

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April 16, 2020

Abstract

Write Theorem one page.

Theorem 1 (The renewal theorem)**.** *If F is nonarithmetic and h is directly Riemann integrable then as $t \to \infty$*

$$H(t) \to \frac{1}{\mu}\int_0^\infty h(s)ds$$

Intuitively, this holds since Theorem 2.6.9 implies

$$H(t) = \int_0^t h(t-s)dU(s)$$

and Theorem 2.6.4 implies $dU(s) \to ds/\mu$ as $s \to \infty$. We will define directly Riemann integrable in a minute. We will start doing the proof and then figure out what we neet to assume.

Proof. Suppose

$$h(s) = \sum_{k=0}^{\infty} a_k 1_{[k\delta,(k+1)\delta)}(s)$$

where $\sum_{k=0}^{\infty} |a_k| < \infty$. Since $U([t, t+\delta]) \le U([0,\delta]) < \infty$, it follows easily from Theorem 2.6.4 that

$$\int_0^t h(t-s)dU(s) = \sum_{k=0}^{\infty} a_k U((t-(t+1)\delta, t-k\delta]) \to \frac{1}{k}\sum_{k=0}^{\infty} a_k\delta$$

(Pick K so that $\sum_{k\ge K} |a_k| \le \epsilon/2U([0,\delta])$ and then T so that

$$|a_k| \cdot |U((t-(t+1)\delta, t-k\delta]) - \delta/\mu| \le \frac{\epsilon}{2K}$$

for $t \ge T$ and $0 \le k < K$.) If h is an arbitrary function on $[0,\infty)$, we let

$$I^\delta = \sum_{k=0}^{\infty} \delta sup\{h(x) : x \in [k\delta, (k+1)\delta)\}$$

$$I_\delta = \sum_{k=0}^{\infty} \delta inf\{h(x) : x \in [k\delta, (k+1)\delta)\}$$

be upper and lower Riemann sums approximating the integral of h over $[0,\infty)$. Comparing h with the obvious upper and lower bounds that are constant on $[k\delta,(k+1)\delta)$ and using the result for the special case,

$$\frac{I_\delta}{\mu} \le \liminf_{t\to\infty}\int_0^t h(t-s)dU(s) \le \limsup_{t\to\infty}\int_0^t h(t-s)dU(s) \le \frac{I^\delta}{\mu}$$

If I^δ and I_δ both approach the same finite I as $\delta \to 0$, then h is said to be **directly Riemann integrable, and it follows that**

$$\int_0^t h(t-s)dU(y) \to I/\mu$$

□