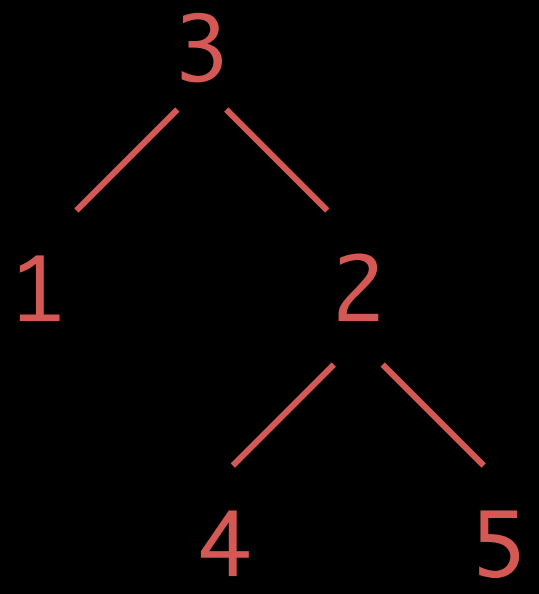
Trees

```
def tree(root_label, branches=[]):
    for branch in branches:
        assert is_tree(branch), 'branches must be trees'
        return [root_label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

def is_leaf(tree):
    return not(branches(tree))
```

Trees

```
>>> t = tree(3, [tree(1), tree(2, [tree(4), tree(5)])])
>>> t -> [3, [1], [2, [4], [5]]]

>>> label(t) -> 3

>>> branches(t) -> [[1], [2, [4], [5]]]

>>> label(branches(t)[0]) -> 1

>>> label(branches(t)[1]) -> 2

>>> r = branches(t)[1]
>>> label(branches(r)[0]) -> 4
>>> label(branches(r)[1]) -> 5
```



Trees (with objects)

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# Trees with objects (manual tree construction)

class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = branches

    def __repr__(self):
        if self.branches:
            return 'T[{0}, {1}]'.format(self.label, repr(self.branches))
        else:
            return 'T[{0}]'.format(repr(self.label))

t = Tree(3)
t.branches = [Tree(1),Tree(2)]
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print(t)
```

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T[3, [T[1], T[2, [T[4], T[5]]]]]
```

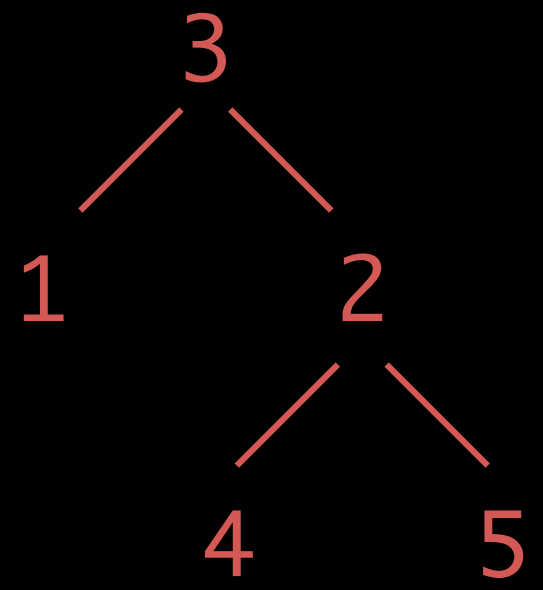

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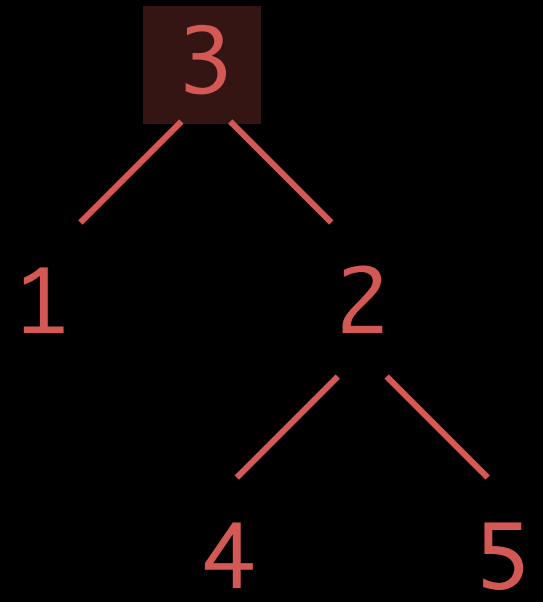

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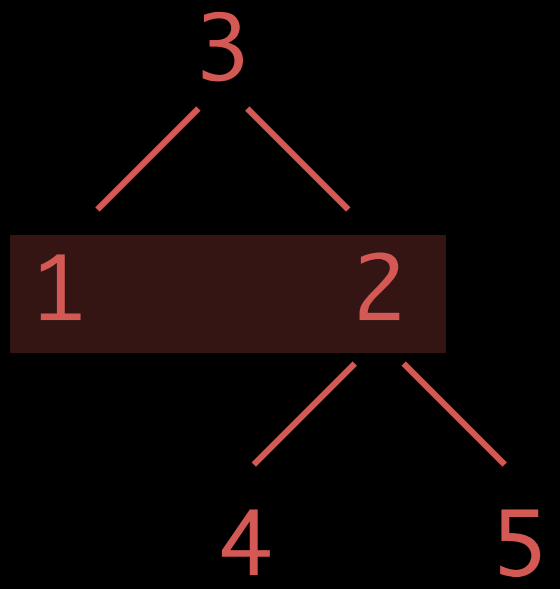

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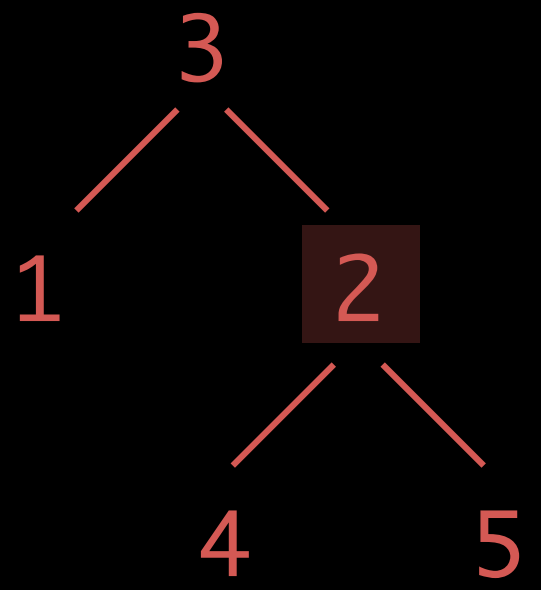

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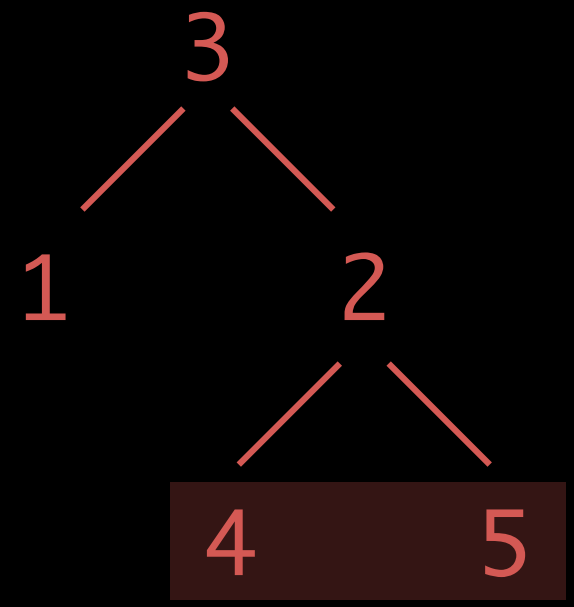

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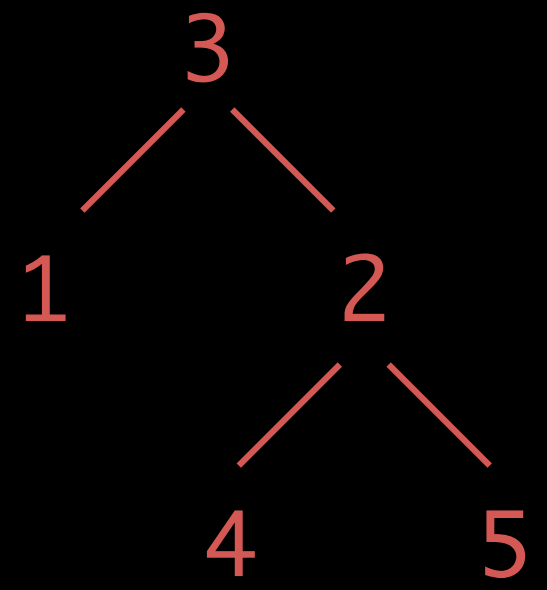
poor data abstraction



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9      def add_child(self,val):
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15     # return subtree in location num (0-indexed)
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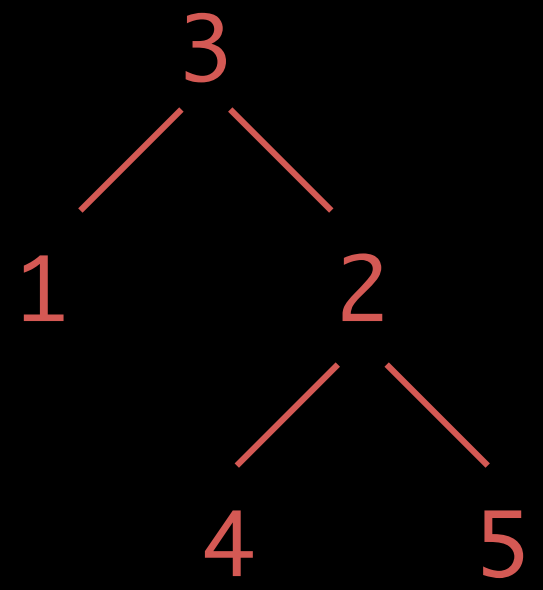

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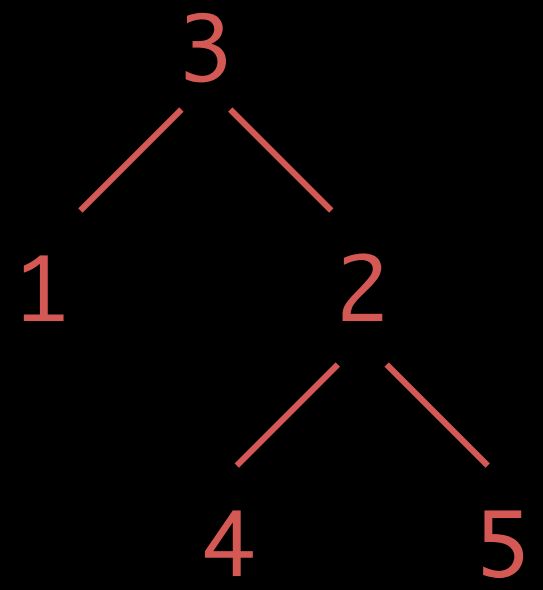

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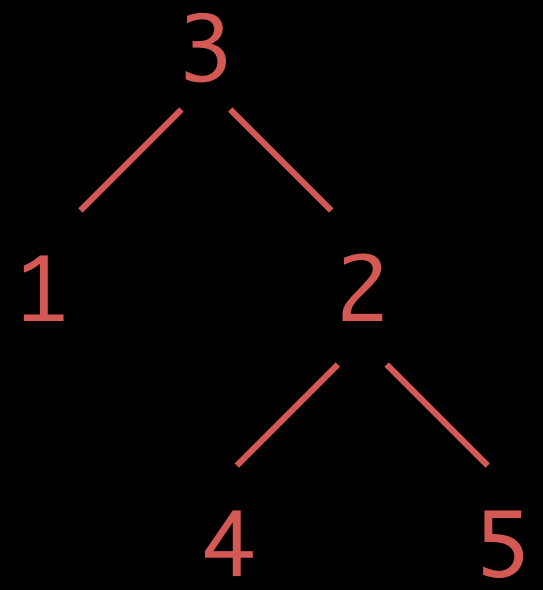

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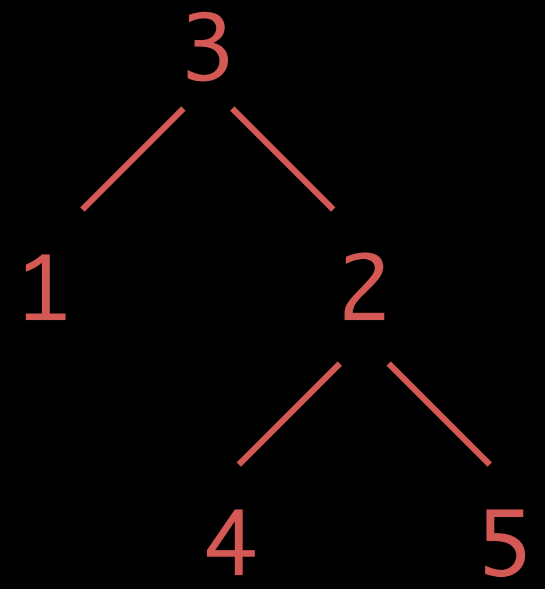

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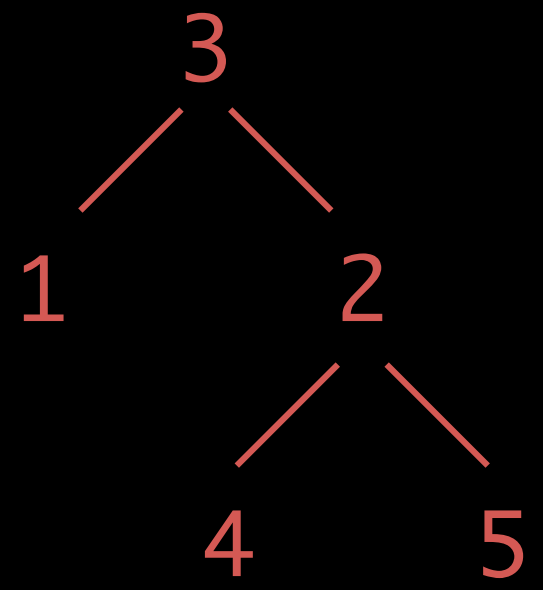
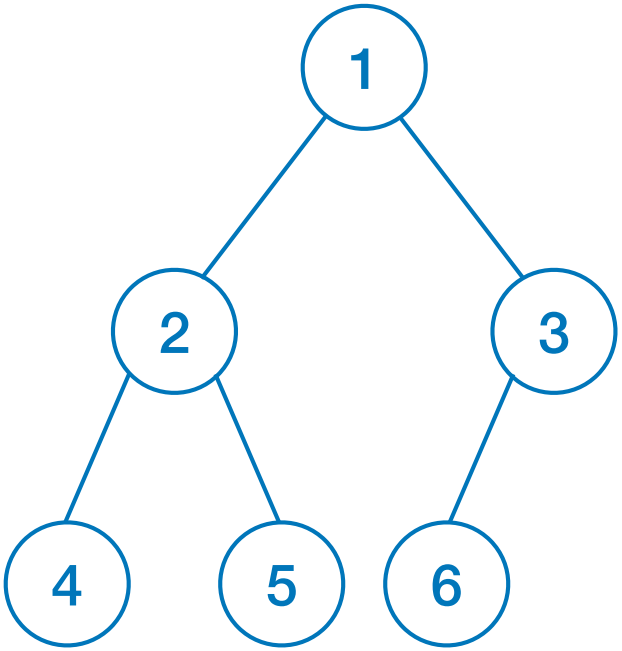


Binary Trees



Binary Trees (representation)

```python
# Binary Tree in Python
class Node:
    def __init__(self, data):
        self.label = data
        self.left  = None
        self.right = None
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root                     = Node(1)
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```
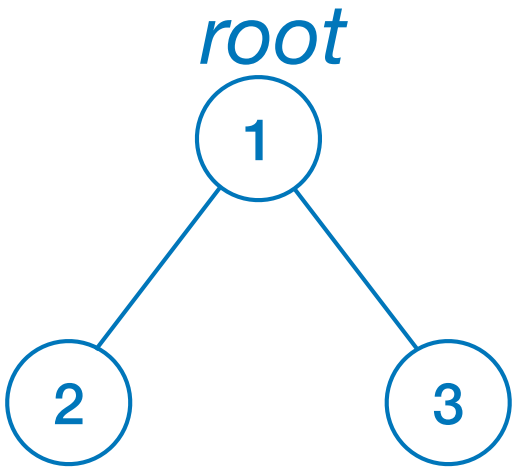

```python
root                = Node(1)
root.left           = Node(2)
root.right          = Node(3)
```



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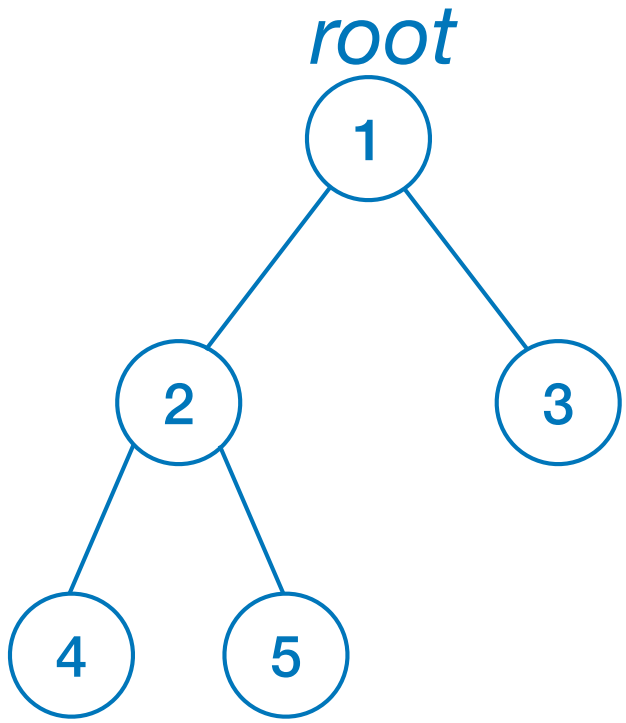
```



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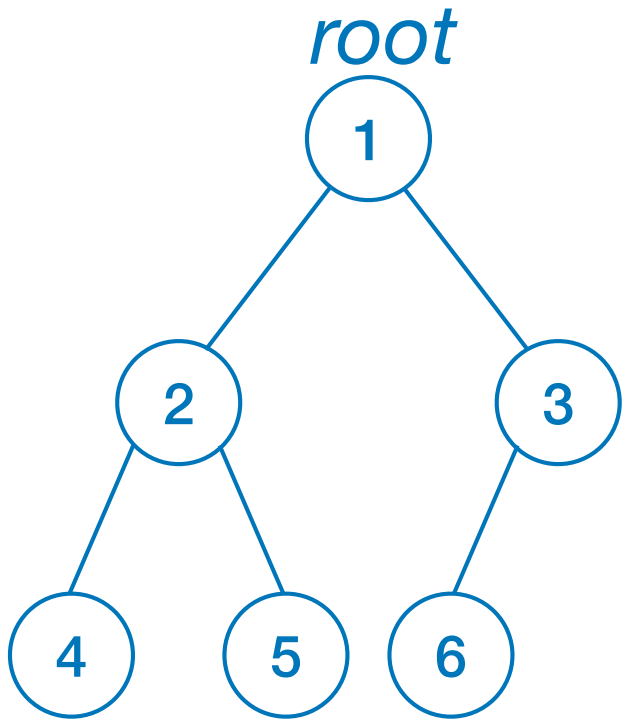
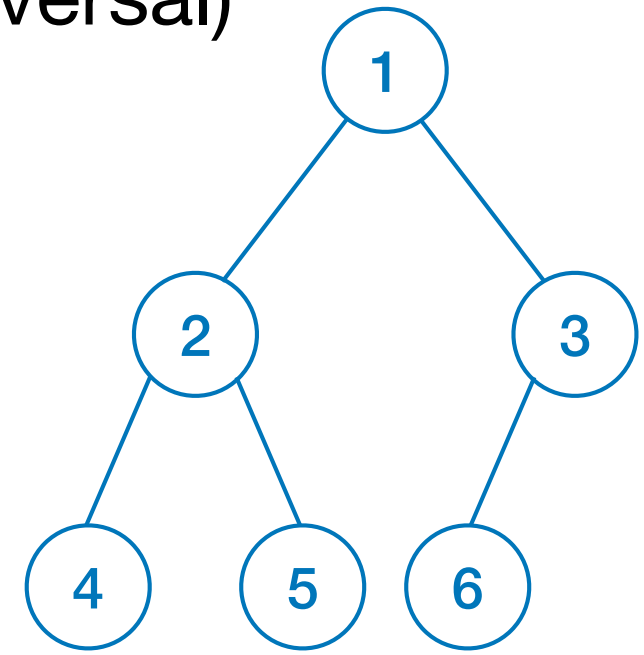
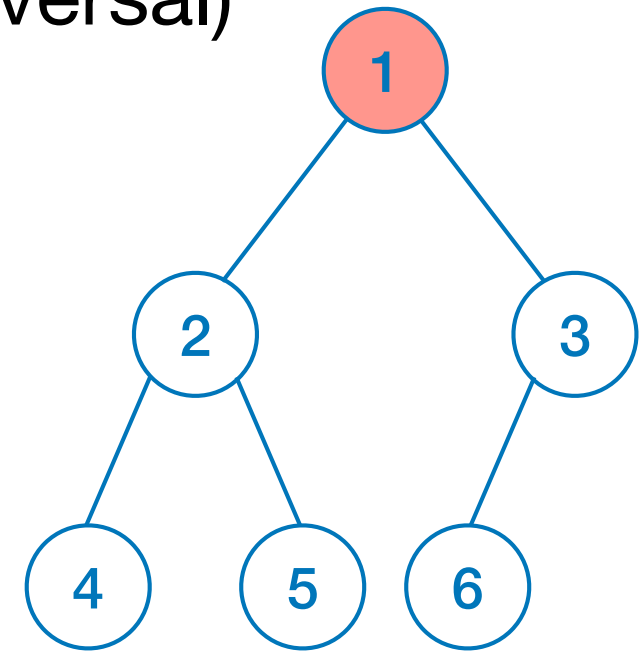


Binary Trees (traversal)



preorder: root, left subtree, right subtree

Binary Trees (traversal)



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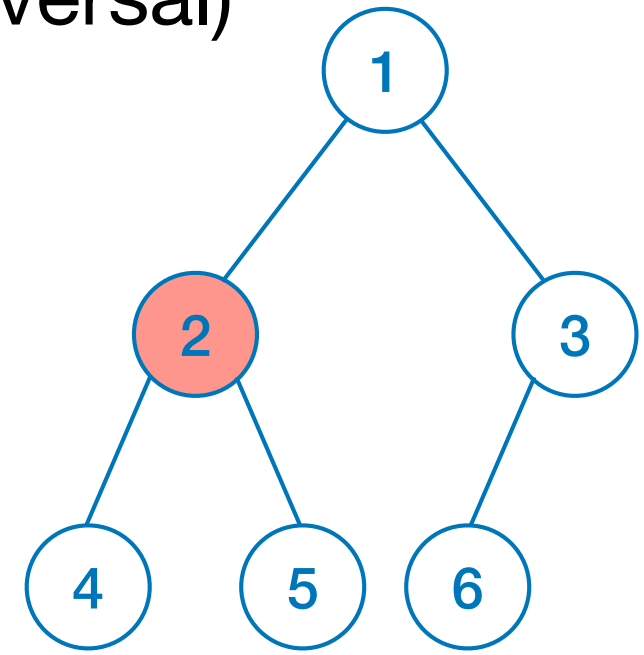

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1 2

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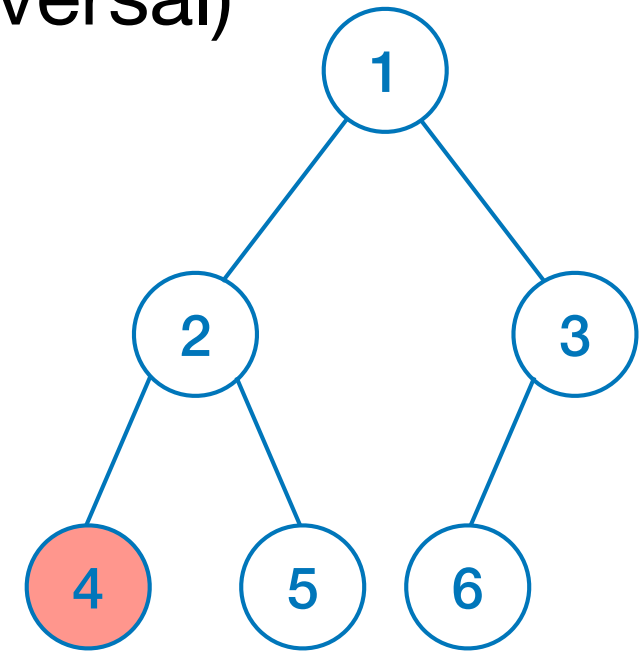

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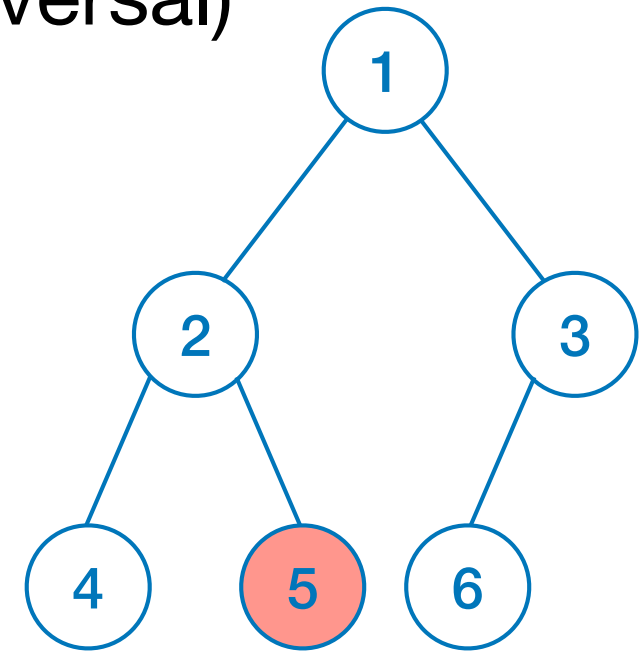
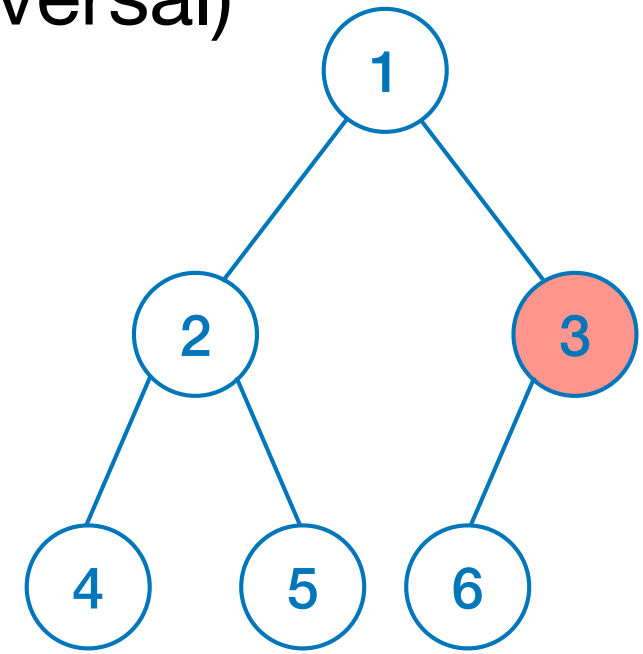


preorder: root, left subtree, right subtree

1 2 4 5

Binary Trees (traversal)



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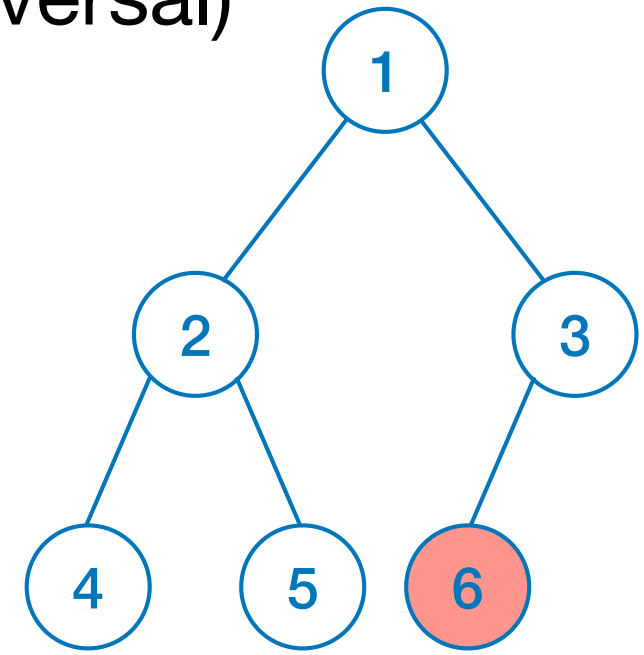
Binary Trees (traversal)



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1 2 4 5 3 6

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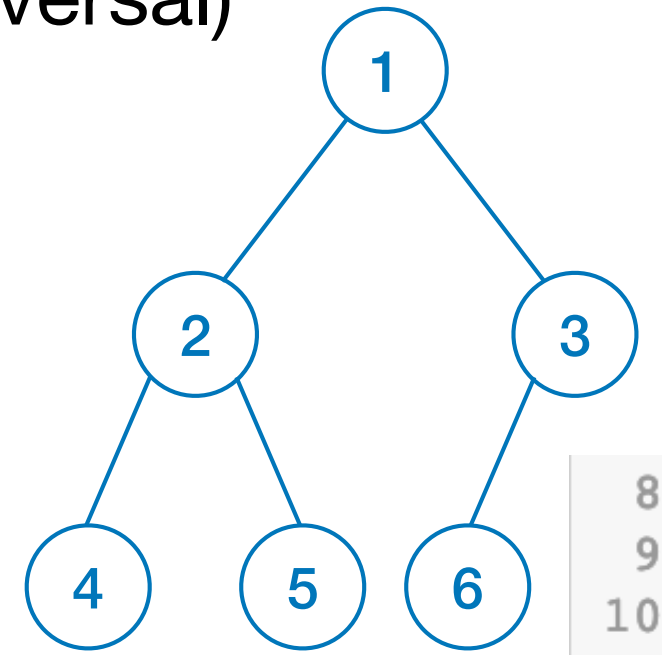
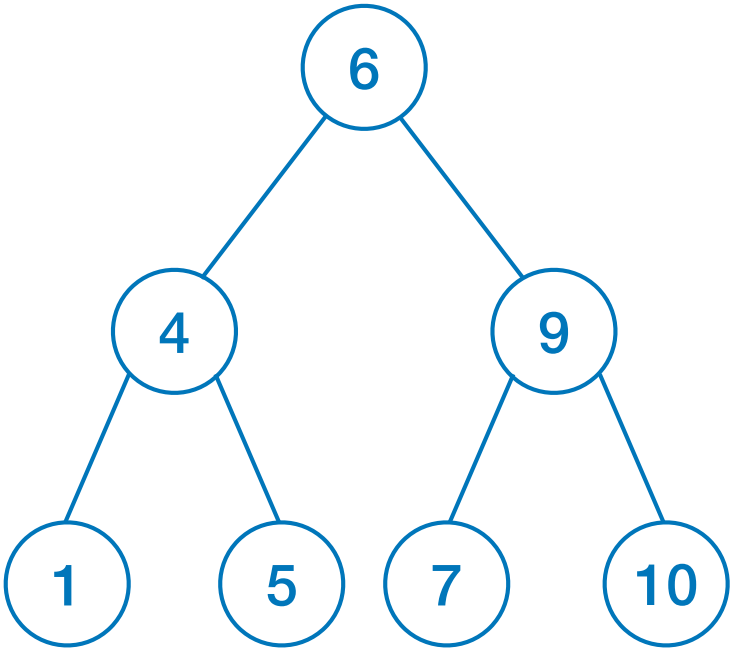
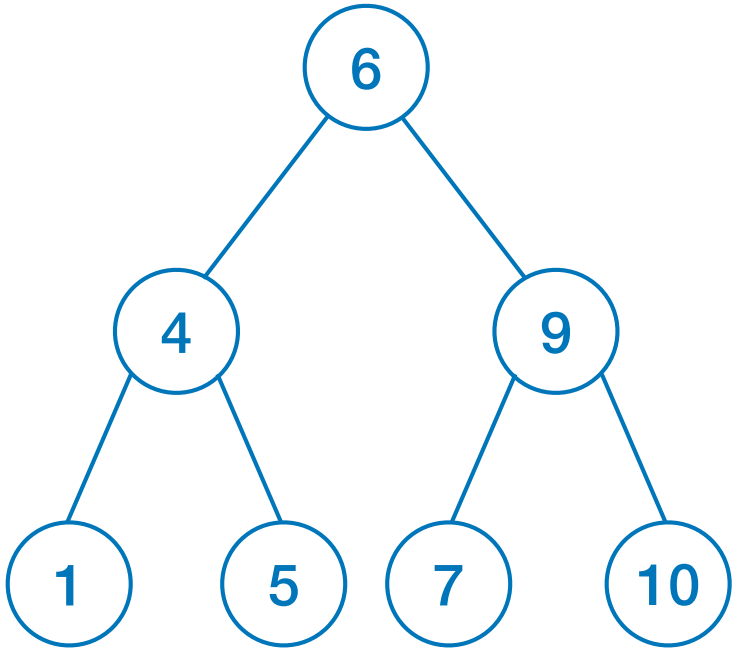



```
 8       # Traverse preorder
 9       def traversePreOrder(self):
10           print(self.label, end=' ')
11           if self.left:
12               self.left.traversePreOrder()
13           if self.right:
14               self.right.traversePreOrder()
```

Binary Search Trees

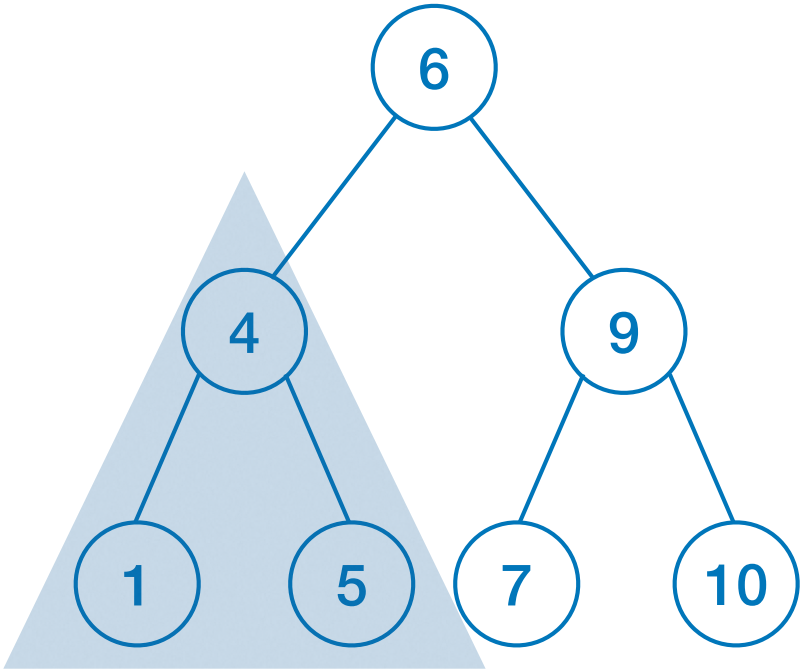


Binary Search Trees



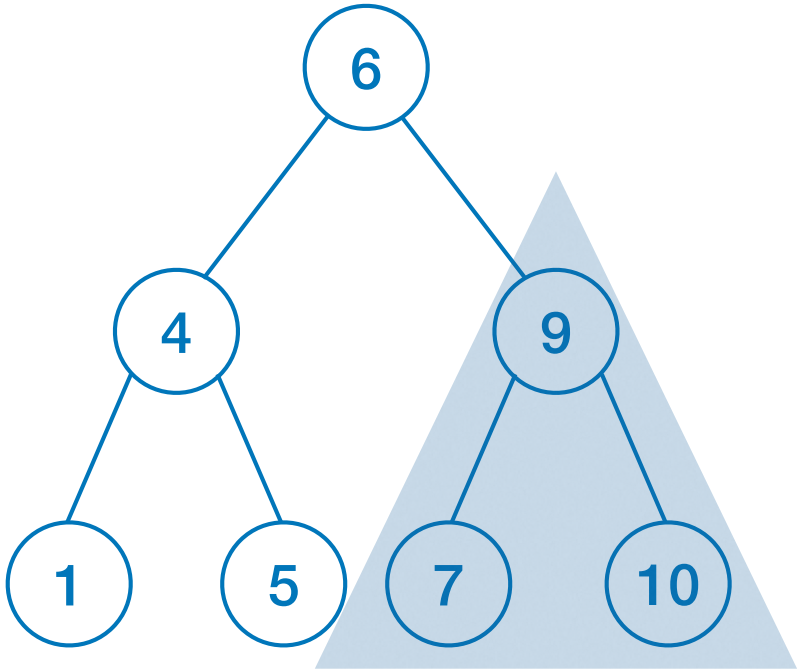
all nodes in left subtree are less than root
all nodes in right subtree are larger than root

Binary Search Trees



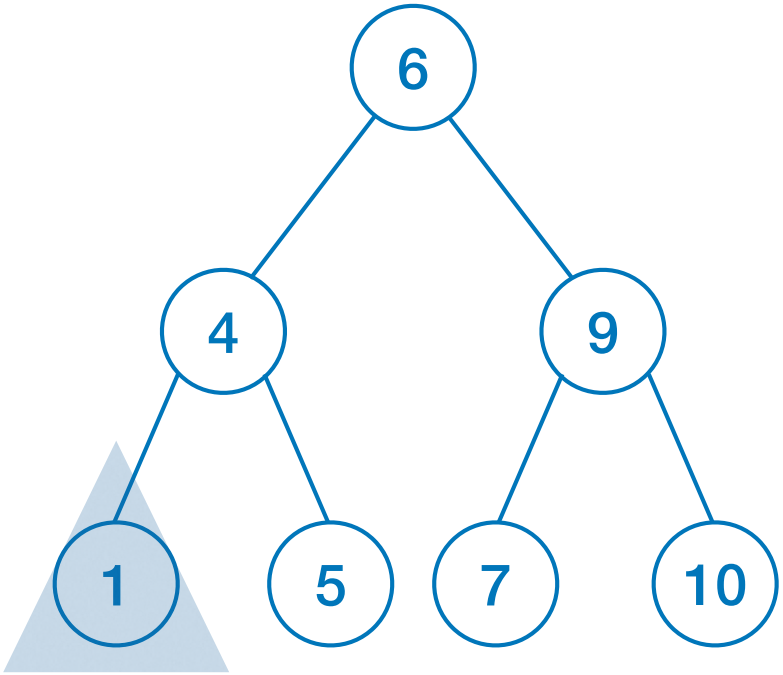
less than 6

Binary Search Trees



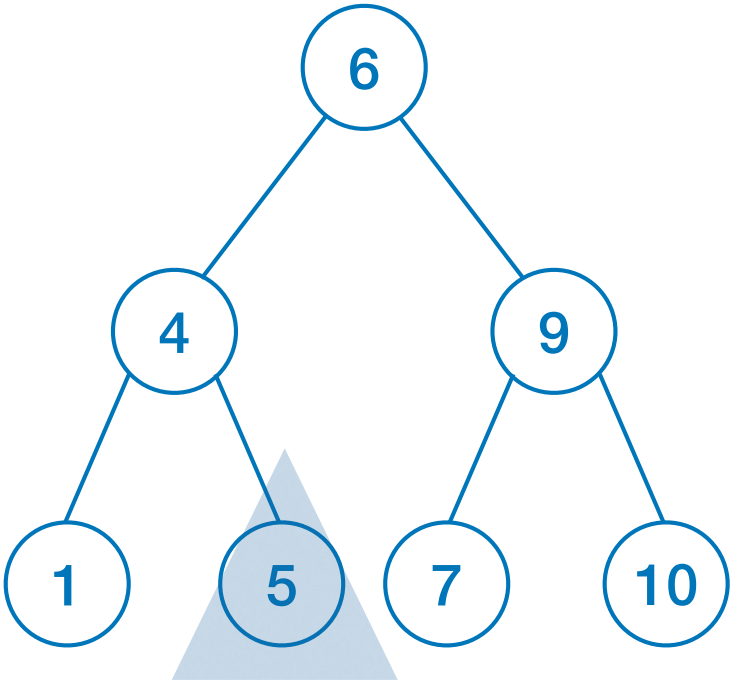
greater than 6

Binary Search Trees



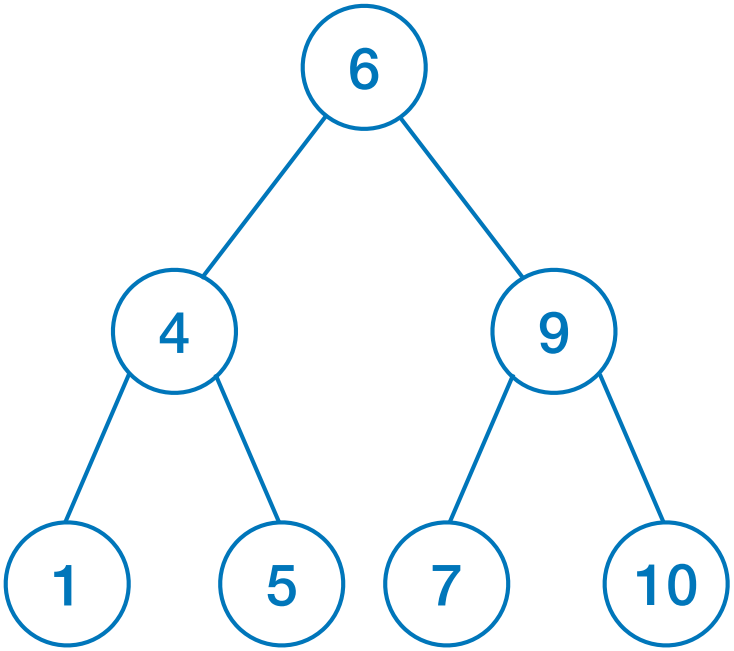
less than 4

Binary Search Trees



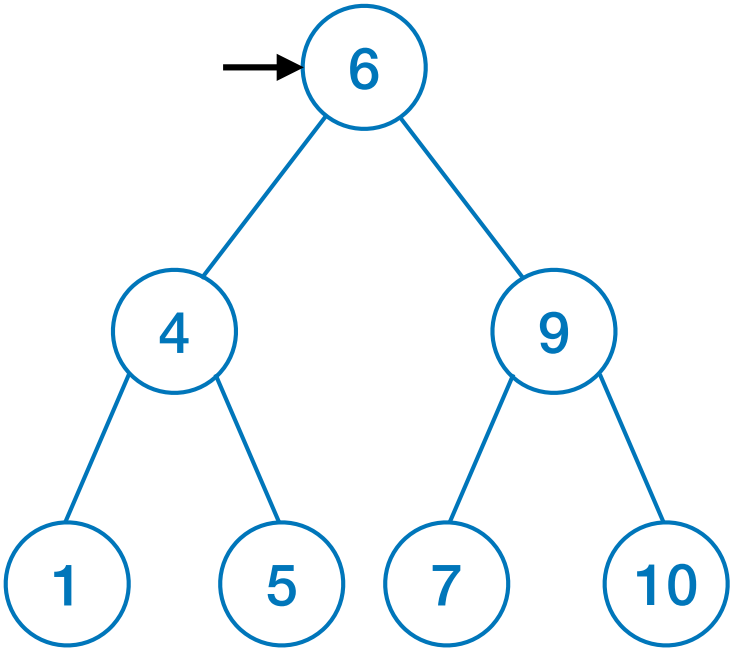
greater than 4

Binary Search Trees



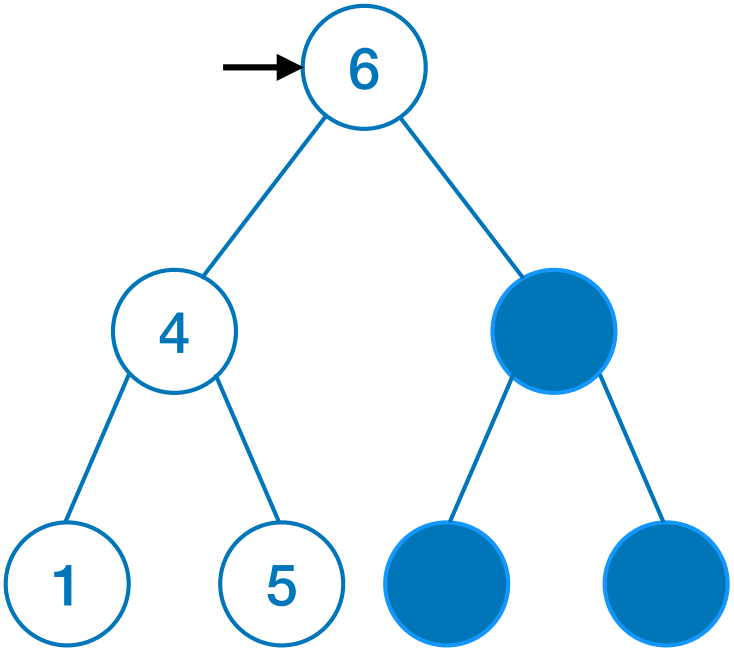
is 5 in this tree?
search in preorder (root, left, right)
n comparisons

Binary Search Trees



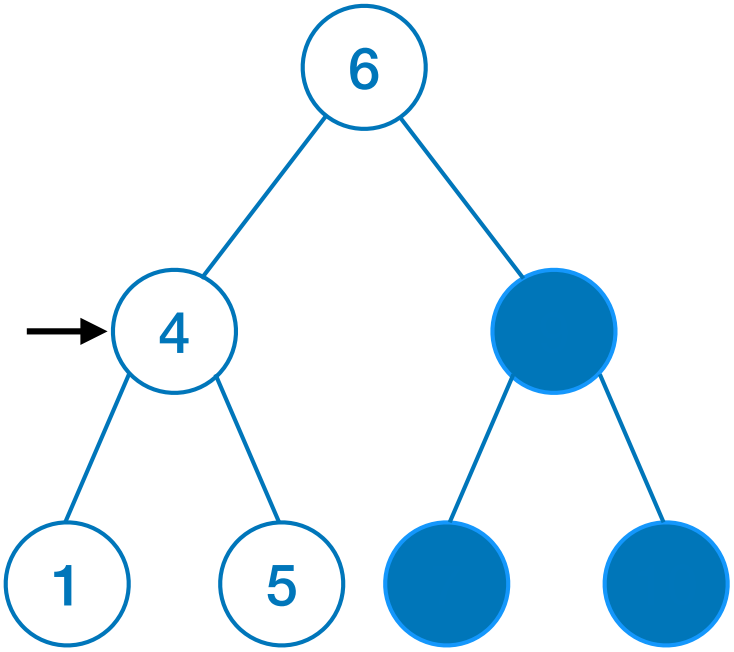
is 5 in this tree?

Binary Search Trees



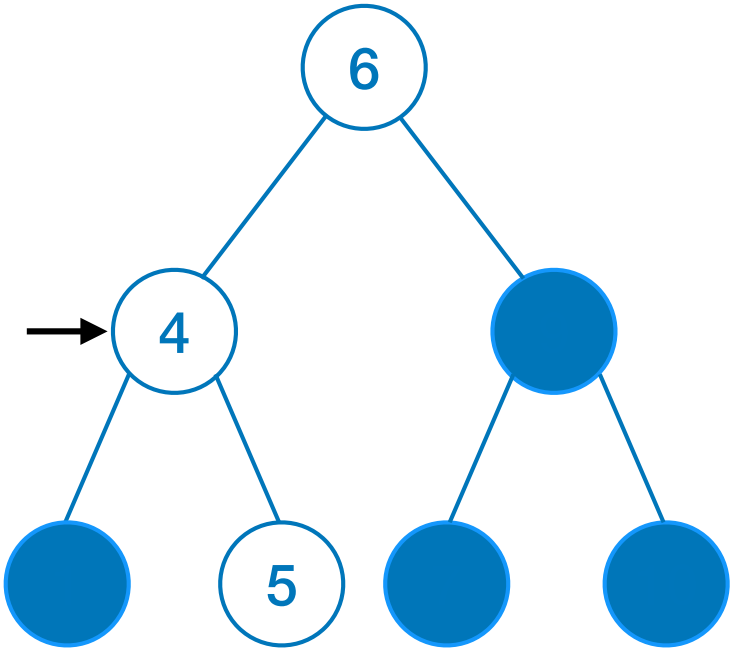
is 5 in this tree?
5 cannot be in right subtree

Binary Search Trees



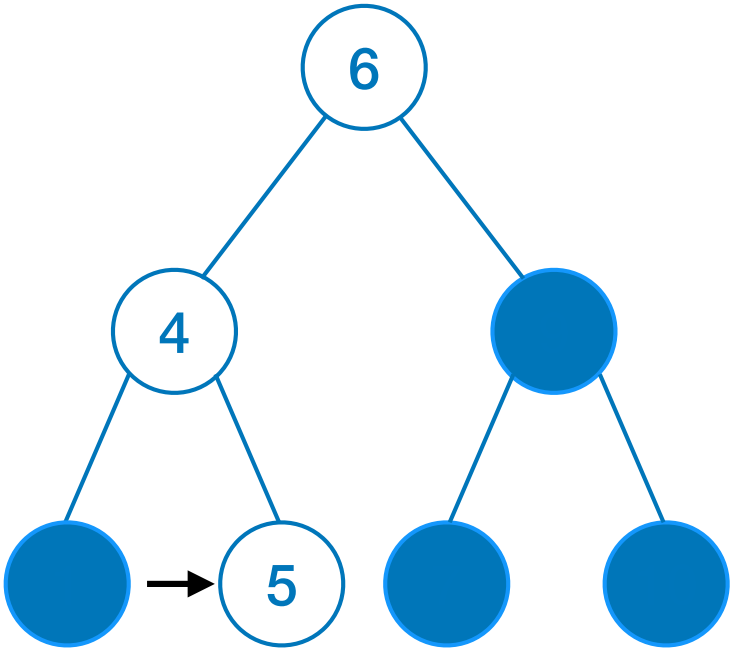
is 5 in this subtree?

Binary Search Trees



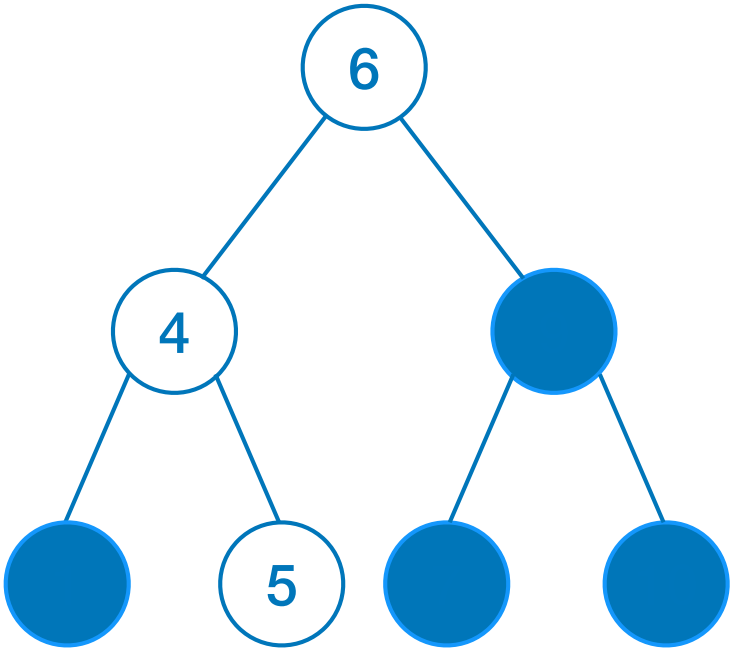
is 5 in this subtree?
5 cannot be in left subtree

Binary Search Trees



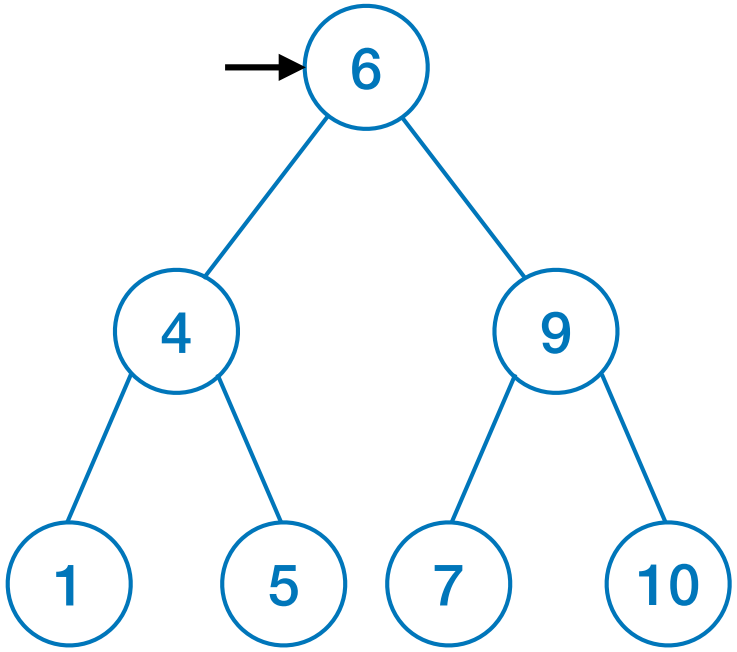
is 5 in this subtree?

Binary Search Trees



is 5 in this subtree?
height of tree is at most *log(n)*

Binary Search Trees




```python
def bst(node, val):
    if node == None:
        return False
    else:
        if val == node.label:
            return True
        elif val < node.label:
            return bst(node.left,val)
        elif val > node.label:
            return bst(node.right,val)

bst(root,5)
```