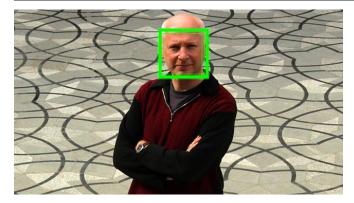
# Stable Marriage Problem

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The Secret Rules of Modern Living: Algorithms





### 안정적인 결혼이란?

• 바람이 나지 않는 커플



### 남자와 여자의 선호도

	1	2	3
철수	영희	미경	지영
영호	미경	영희	지영
광수	영희	미경	지영

	1	2	3
영희	영호	철수	광수
미경	철수	영호	광수
지영	철수	영호	광수

### 매칭 예1

	1	2	3
철수	영희	미경	지영
영호	미경	영희	지영
광수	영희	미경	지영

	1	2	3
영희	영호	철수	광수
미경	철수	영호	광수
지영	철수	영호	광수

Stable matching? 즉, 바람나는 커플이 있을까? 없을까?

## 매칭 예2

	1	2	3
철수	영희	미경	지영
영호	미경	영희	지영
광수	영희	미경	지영

	1	2	3
영희	영호	철수	광수
미경	철수	영호	광수
지영	철수	영호	광수

#### 매칭 문제?

- N명의 남자와 N명의 여자를 연결시키는 방법?
  - 손쉽게 생각할 수 있는 방법은 아무렇게나 짝을 지은 다음 서로 다른 커플 사이에 눈이 맞는 남녀가 있으면 그 두 커플의 파트너를 서로 바꾸어 주는 과정을 반복
    - 더 이상 깨지는 커플이 없으면 문제가 풀렸다
    - 끝날까?
- SMP 알고리즘: 남자가 여자에게 프로포즈 가정
  - 남성 쪽에서 마음에 드는 여성에게 구애를 하고 여성 쪽에서 구애를 받아 들임
  - 물론 여성 쪽에서 구애를 거절하면, 남성은 다른 여성을 찾아야 함
  - 현실에서는 거절하지 않는 경우도 있지만, 모든 남녀가 정직하게 반응한다고 가정
  - 또, 최종 과정에서 "차라리 솔로로 살래."라는 결론을 내리는 경우도 있지만, 우리가 다루고 있는 상황에서는 남녀 모두 최선을 다해 커플을 만든다고 가정
  - "마음에 안 들지만 남은 남성/여성이 저 사람밖에 없으니 할 수 없지"라는 긍정적인 (?) 사고 방식

### Stable Marriage Problem

- Algorithms for finding solutions to the stable marriage problem have applications in a variety of real-world situations, perhaps the best known of these being in
  - the assignment of graduating medical students to their first hospital appointments.[1]
- In 2012, the <u>Nobel Prize in Economics</u> was awarded to <u>Lloyd S. Shapley</u> and <u>Alvin E. Roth</u> "for the theory of stable allocations and the practice of market design"

# The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012



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Alvin E. Roth

Prize share: 1/2



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Lloyd S. Shapley

Prize share: 1/2

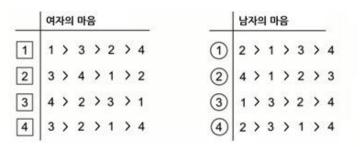
The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012 was awarded jointly to Alvin E. Roth and Lloyd S. Shapley "for the theory of stable allocations and the practice of market

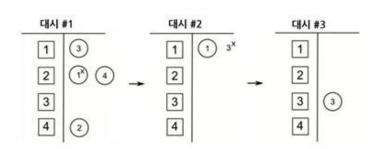
### SMP Algorithm

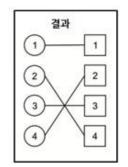
- 1. 매일 아침 남자는 가장 상위에 있는 여성을 찾아간다.
  - 1. 라운드
- 2. 여성은 자신을 찾아온 남성 가운데, 자신의 목록에서
  - 1. 가장 상위에 있는 사람에게는 "내일 다시 오세요."라고 답하고(아직),
  - 2. 나머지 사람에게는 "다른 여성을 찾아보세요."라고 거절한다.
- 3. 거절당한 남성이 없으면 매칭 종료
- 4. 거절당한 남성은 저녁에 (술집에) 모여 (울면서)
  - 1. 자신을 거절한 여성을 목록에서 삭제
- 5. 다시 1번부터 반복한다.

#### 남자의 프로포즈

- 1번 남자 2번 여자에게 거절
- 3번 남자 1번 여자에게 거절
- 1번 여자-1번 남자 • 1순위-2순위
- 2번 여자-4번 남자 • 2순위-1순위
- 3번 여자-3번 남자 • 3순위-2순위
- 4번 여자-2번 남자 • 2순위-1순위







#### 안정적일까?

- 1. 여자 A가 최종커플로 남자 A와 매칭되었으나, 여자 A가 남자 B를 더 좋아할 수 있지 않을까?
  - 남자 B는 여자 A를 찾아오지 않았다!
  - 즉, 남자 B는 다른 여자와 커플이 되었다!
- 2. 남자 A가 현재 커플인 여자 A보다 여자 B를 더 좋아할 수 있 지 않을까?
  - 남자 A는 이미 여자 B를 찾아가서 프로포즈를 했다. 하지만, 거절됨. 왜? 여자 B는 더 좋아하는 남자와 짝이 되었으니까.
  - 그러므로 여자 B 커플은 깨어질 수가 없음

#### 응용: 대학교 입학

COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE
D. Gale and L. S. Shapley

#### Introduction.

The problem which we shall be concerned with relates to the following typical situation: A college is considering a set of n applicants of which it can admit a quota of only q. Having evaluated all their qualifications, the admissions office must decide which ones to admit. The procedure of offering admission only to the q best-qualified applicants will not generally be satisfactory, for it cannot be assumed that all who are offered admission will accept. The reason is of course that many applicants will apply to and be admitted by more than one college and hence will accept only their first choice. Accordingly, in order for a college to receive q acceptances, it will generally have to offer to admit more than q applicants.

#### 응용: CDN 서버 선정

#### **Algorithmic Nuggets in Content Delivery**

Bruce M. Maggs Duke and Akamai bmm@cs.duke.edu Ramesh K. Sitaraman UMass, Amherst and Akamai ramesh@cs.umass.edu

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The authors take full responsibility for this article's technical content. Comments can be posted through CCR Online.

#### **ABSTRACT**

This paper "peeks under the covers" at the subsystems that provide the basic functionality of a leading content delivery network. Based on our experiences in building one of the largest distributed systems in the world, we illustrate how sophisticated algorithmic research has been adapted to balance the load between and within server clusters, manage the caches on servers, select paths through an overlay routing network, and elect leaders in various contexts. In each instance, we first explain the theory underlying the algorithms, then introduce practical considerations not captured by the theoretical models, and finally describe what is implemented in practice. Through these examples, we highlight the role of algorithmic research in the design of complex networked systems. The paper also illustrates the close synergy that exists between research and industry where research ideas cross over into products and product requirements drive future research.

#### 1. INTRODUCTION

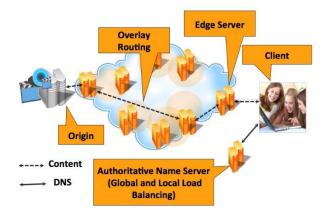


Figure 1: A CDN serves content in response to a client's request.

### 응용: 신장 기증

#### A Kidney Exchange Clearinghouse in New England

By Alvin E. Roth, Tayfun Sönmez, and M. Utku Ünver\*

In September, 2004, the Renal Transplant Oversight Committee of New England approved the establishment of a clearinghouse for kidney exchange, proposed by Francis Delmonico, Susan Saidman, and the three authors of this paper. We outline here the potential gains from kidney exchange and discuss practical constraints encountered as we begin designing and implementing a matching mechanism.

#### I. Background

Lainie Friedman Ross et al. (1997) discussed the possibility of exchange between incompatible patient—donor pairs. Not only have a few such two-way exchanges been performed, but two three-way exchanges (in which the donor kidney from one pair is transplanted into the patient in a second pair, whose donor kidney goes to a third pair, whose donor kidney goes to the first pair) have been performed at Johns Hopkins. There have also been a number of "list exchanges" in which an incompatible patient—donor pair makes a denotion to company on the

### 응용: 고등학교 배정

PRACTICAL MARKET DESIGN: FOUR MATCHES<sup>†</sup>

#### The New York City High School Match

By Atila Abdulkadiroğlu, Parag A. Pathak, and Alvin E. Roth\*

We assisted the New York City Department of Education (NYCDOE) in designing a mechanism to match over 90,000 entering students to public high schools each year. This paper makes a very preliminary report on the design process and the first year of operation, in academic year 2003–2004, for students entering high school in fall 2004. In the first year, only about 3,000 students had to be assigned to a school for which they had not indicated a preference, which is only 10 percent of the number of such assignments the previous year.

#### I. The Prior (2002–2003) New York City Matching Procedure

There are seven specialized high schools in New York City whose places are allocated by entrance exam (one by auditions). Rising high-school students (mostly 8th-graders, but some 9th-graders) could also apply to up to *five* other programs, by ranking them on a preference list. (Different high-school programs, with separate applications and admissions, are referred to here, interchangeably, as schools or programs.

### 참고자료

 http://www.claimcare.co.kr/bbs/board.php?bo\_table=claim\_req uest&wr\_id=3748&page=321