APM1110 - FA 3 - Dacanay

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### 2) A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0.95, and a transmitted 1 is correctly received with probability 0.75. Also, 70% of all messages are transmitted as a 0. If a signal is sent, determine the probability that:

### (a) a 1 was received;

#### First, let t1 be the event that a one is transmitted. Second, let r1\_t1 be the event that a one is received given that one is transmitted. Thirdly, let t0 be the event that a zero is transmitted. Lastly, let r1\_t0 be the event that a zero is transmitted given that zero is transmitted.

t1 <- 0.3  
r1\_t1 <- 0.75  
t0 <- 0.7  
r1\_t0 <- 0.05  
  
p\_r1 <- (t1 \* r1\_t1) + (t0 \* r1\_t0)

#### ANSWER

print(paste0("The probability that a 1 was received is ", p\_r1, " or ", p\_r1 \* 100, "%."))

## [1] "The probability that a 1 was received is 0.26 or 26%."

### (b) a 1 was transmitted given that a 1 was received.

#### In this problem, we’ll use all the data provided in Section 2.a. Additionally, we’ll include the ‘p\_r1’, which represents probability that a 1 was received.

p\_t1\_r1 <- (t1 \* r1\_t1) / p\_r1  
p\_t1\_r1\_rounded <- round(p\_t1\_r1, 4)

#### ANSWER

print(paste0("The probability that a 1 was transmitted given that a 1 was received is ", p\_t1\_r1\_rounded, " or ", p\_t1\_r1\_rounded \* 100, "%."))

## [1] "The probability that a 1 was transmitted given that a 1 was received is 0.8654 or 86.54%."

### 7) There are three employees working at an IT company: Jane, Amy, and Ava, doing 10%, 30%, and 60% of the programming, respectively. 8% of Jane’s work, 5% of Amy’s work, and just 1% of Ava‘s work is in error.

### (a) What is the overall percentage of error?

#### Let j be the event of Jane’s work percentage, am be the event of Amy’s work percentage, and av be the event of Ava’s work percentage. Additionally, let j\_e be the event of error in Jane’s work, am\_e be the event of error in Amy’s work, and av\_e be the event of error in Ava’s work.

j <- 0.1  
am <- 0.3  
av <- 0.6  
  
j\_e <- 0.08  
am\_e <- 0.05  
av\_e <- 0.01  
  
  
p\_e <- (j \* j\_e) + (am \* am\_e) + (av \* av\_e)  
p\_e\_percentage <- p\_e \* 100

#### ANSWER

print(paste0("The overall percentage of error is ", p\_e\_percentage, "%."))

## [1] "The overall percentage of error is 2.9%."

### (b) If a program is found with an error, who is the most likely person to have written it?

#### In this problem, we’ll use all the data provided in Section 7.a. Additionally, we’ll include the ‘p\_e\_percentage’, which represents the overall percentage of error, as part of our given information.

#### Let’s find out first the percentage of each employee’s error given that an error has occurred.

# Jane  
p\_je\_oe <- (j \* j\_e) / p\_e  
p\_je\_oe\_rounded <- round(p\_je\_oe, 3)  
p\_je\_oe\_rounded\_percentage <- p\_je\_oe\_rounded \* 100  
  
print(paste0("Jane's error given that an error occurred: ", p\_je\_oe\_rounded, " or ", p\_je\_oe\_rounded\_percentage, "%"))

## [1] "Jane's error given that an error occurred: 0.276 or 27.6%"

# Amy  
p\_ame\_oe <- (am \* am\_e) / p\_e  
p\_ame\_oe\_rounded <- round(p\_ame\_oe, 3)  
p\_ame\_oe\_rounded\_percentage <- p\_ame\_oe\_rounded \* 100  
print(paste0("Amy's error given that an error occurred: ", p\_ame\_oe\_rounded, " or ", p\_ame\_oe\_rounded\_percentage, "%"))

## [1] "Amy's error given that an error occurred: 0.517 or 51.7%"

# Ava  
p\_ave\_oe <- (av \* av\_e) / p\_e  
p\_ave\_oe\_rounded <- round(p\_ave\_oe, 3)  
p\_ave\_oe\_rounded\_percentage <- p\_ave\_oe\_rounded \* 100  
print(paste0("Ava's error given that an error occurred: ", p\_ave\_oe\_rounded, " or ", p\_ave\_oe\_rounded\_percentage, "%"))

## [1] "Ava's error given that an error occurred: 0.207 or 20.7%"

#### ANSWER

# Compare probabilities and print the most likely person  
if (p\_je\_oe\_rounded\_percentage > p\_ame\_oe\_rounded\_percentage && p\_je\_oe\_rounded\_percentage > p\_ave\_oe\_rounded\_percentage) {  
 print(paste0("The most likely person to have written a program with an error, given that a program is found with an error, is Jane with ", p\_je\_oe\_rounded\_percentage, "%."))  
} else if (p\_ame\_oe\_rounded\_percentage > p\_je\_oe\_rounded\_percentage && p\_ame\_oe\_rounded\_percentage > p\_ave\_oe\_rounded\_percentage) {  
 print(paste0("The most likely person to have written a program with an error, given that a program is found with an error, is Amy with ", p\_ame\_oe\_rounded\_percentage, "%."))  
} else if (p\_ave\_oe\_rounded\_percentage > p\_je\_oe\_rounded\_percentage && p\_ave\_oe\_rounded\_percentage > p\_ame\_oe\_rounded\_percentage) {  
 print(paste0("The most likely person to have written a program with an error, given that a program is found with an error, is Ava with ", p\_ave\_oe\_rounded\_percentage, "%."))  
} else {  
 print(paste0("There is a tie in the error probabilities."))  
}

## [1] "The most likely person to have written a program with an error, given that a program is found with an error, is Amy with 51.7%."