GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 12: DECISION TREE CLASSIFIERS

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DAT5 SF // April 16, 2014

LAST TIME:

- RECOMMENDATION SYSTEMS
- RECOMMENDATION SYSTEMS IN PYTHON

QUESTIONS?

I. DECISION TREES
II. BUILDING DECISION TREES
III. OPTIMIZATION FUNCTIONS
IV. PREVENTING OVERFITTING
V. (BRIEF INTRO TO) RANDOM FORESTS

EXERCISE:
V. IMPLEMENTING DECISION TREES WITH SCIKIT-LEARN

I. DECISION TREES

	continuous	*categorical*
supervised	???	???
unsupervised	???	???

	continuous	categorical
supervised	regression	classification
unsupervised	dimension reduction	clustering

Q: What is a decision tree?

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non-parametric*: no parameters, no distribution assumptions*

hierarchical*: consists of a sequence of questions which yield a class label when applied to any record*

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A: Using a configuration of **nodes** *and* **edges***.*

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A: Using a configuration of **nodes** *and* **edges**.

More concretely, as a multiway tree, which is a type of (directed acyclic) **graph**.

*In a decision tree, the nodes represent questions (***test conditions***) and the edges are the answers to these questions.*

The top node of the tree is called the **root node**. *This node has 0 incoming edges, and 2+ outgoing edges.*

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TYPES OF NODES

The top node of the tree is called the **root node***. This node has 0 incoming edges, and 2+ outgoing edges.*

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NOTE

The nodes in our tree are connected by *directed edges*.

These directed edges lead from *parent nodes* to *child nodes.*

EXAMPLE – DATA

Table 4.1. The vertebrate data set.

Name	Body Temperature	Skin Cover	Gives Birth	Aquatic Creature	Aerial Creature	Has Legs	Hiber-nates	Class Label
human	warm-blooded	hair	yes	no	no	yes	no	mammal
python	cold-blooded	scales	no	no	no	no	yes	reptile
salmon	cold-blooded	scales	no	yes	no	no	no	fish
whale	warm-blooded	hair	yes	yes	no	no	no	mammal
frog	cold-blooded	none	no	semi	no	yes	yes	amphibian
komodo dragon	cold-blooded	scales	no	no	no	yes	no	reptile
bat	warm-blooded	hair	yes	no	yes	yes	yes	mammal
pigeon	warm-blooded	feathers	no	no	yes	yes	no	bird
cat	warm-blooded	fur	yes	no	no	yes	no	mammal
leopard shark	cold-blooded	scales	yes	yes	no	no	no	fish
turtle	cold-blooded	scales	no	semi	no	yes	no	reptile
penguin	warm-blooded	feathers	no	semi	no	yes	no	bird
porcupine	warm-blooded	quills	yes	no	no	yes	yes	mammal
eel	cold-blooded	scales	no	yes	no	no	no	fish
salamander	cold-blooded	none	no	semi	no	yes	yes	amphibian

source: http://www-users.cs.umn.edu/~kumar/dmbook/ch4.pdf

EXAMPLE – DECISION TREE

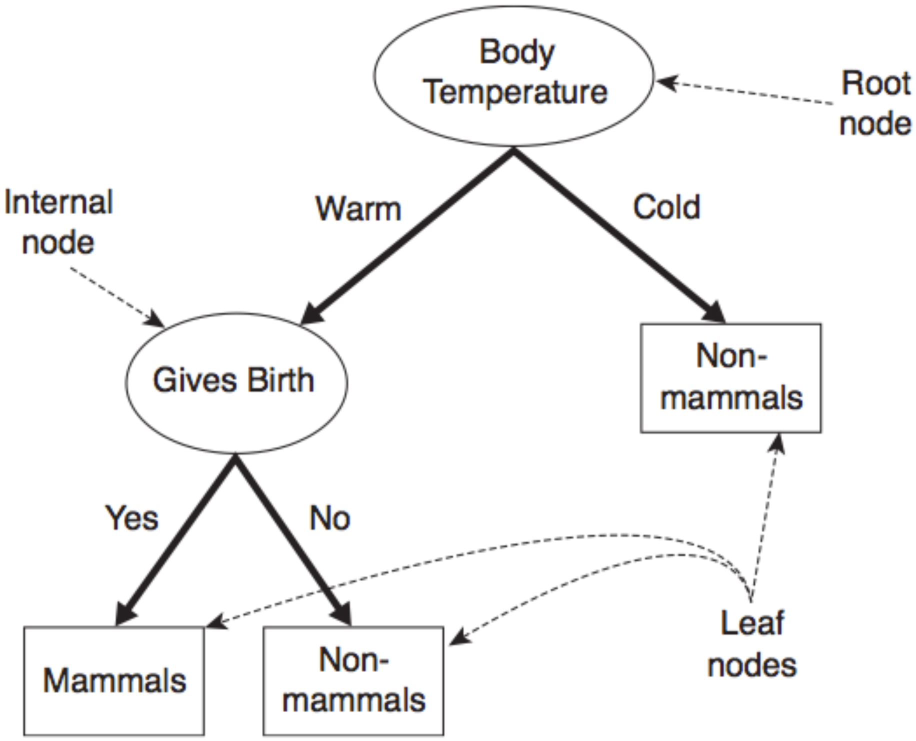

Figure 4.4. A decision tree for the mammal classification problem.

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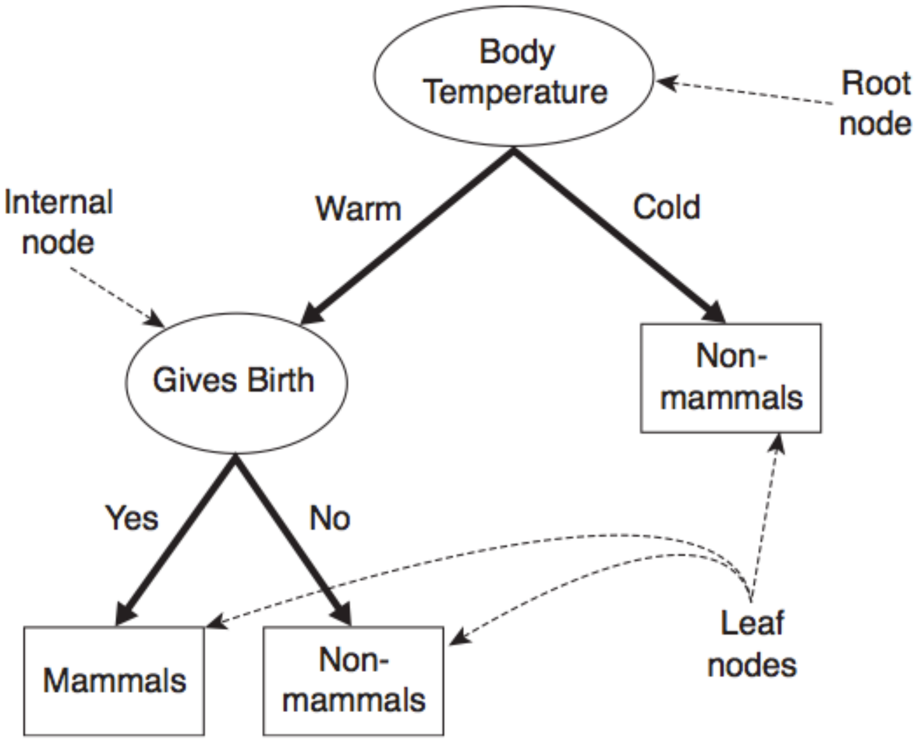

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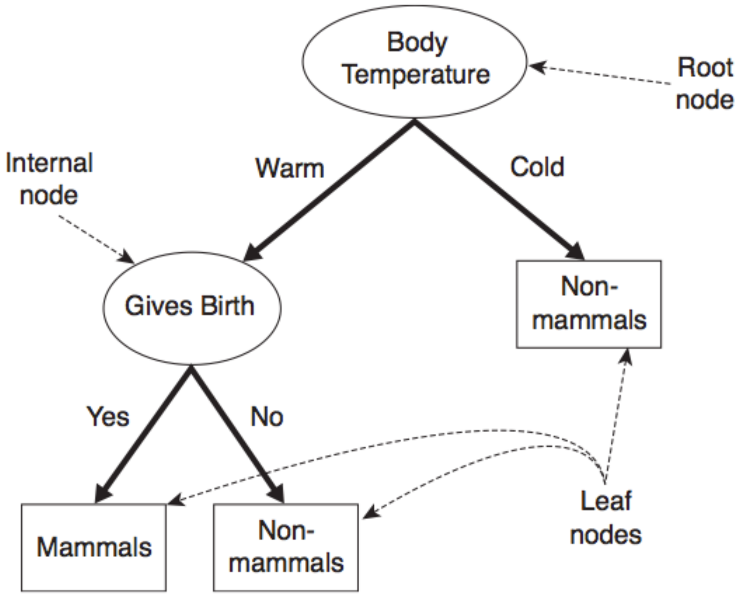
NOTE

Internal nodes represent test conditions which *partition the records* at that node.

EXAMPLE – DECISION TREE

Name	Body Temperature	Skin Cover	Gives Birth	Aquatic Creature	Aerial Creature	Has Legs	Hiber-nates	Class Label
gila monster	cold-blooded	scales	no	no	no	yes	yes	?

Now, let's try an example...



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EXAMPLE – DECISION TREE

Unlabeled data	Name	Body temperature	Gives Birth	...	Class
	Flamingo	Warm	No	...	?

And another example…

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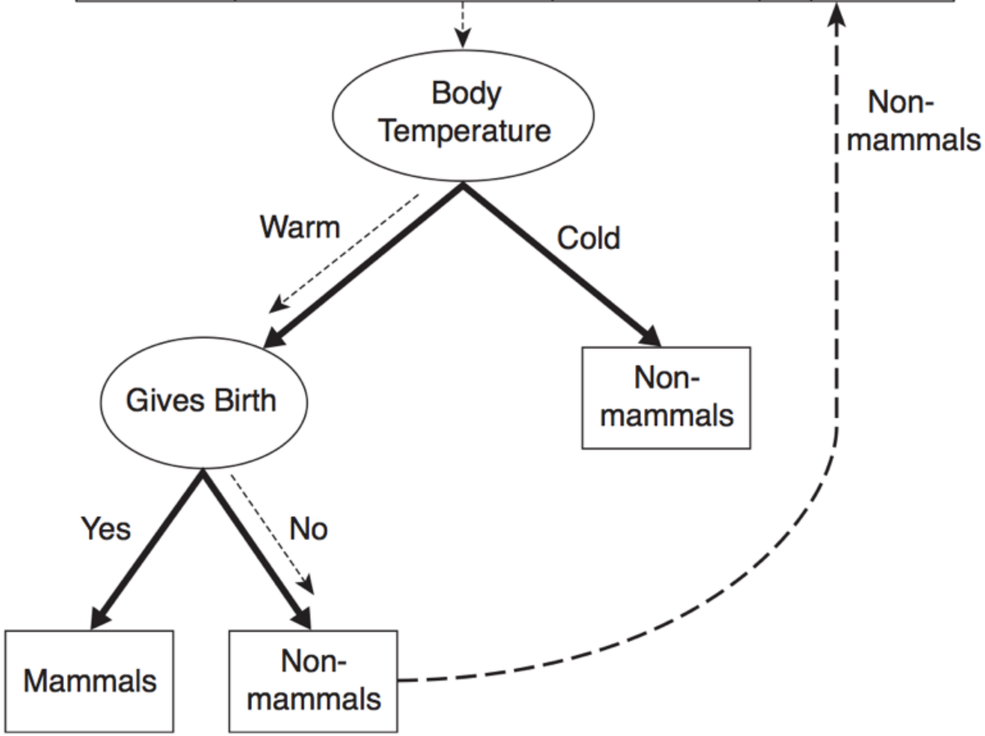
EXAMPLE – DECISION TREE

And another example…

Unlabeled data

Name	Body temperature	Gives Birth	...	Class
Flamingo	Warm	No	...	?



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II. BUILDING DECISION TREES

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But this is generally too complex to be practical → $O(2^n)$.

Q: How do we find a practical solution that works?

A: Use a **heuristic** *algorithm.*

The basic method used to build (or "grow") a decision tree is **Hunt's algorithm.**

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This is a **greedy recursive** *algorithm that leads to a* **local optimum***.*

greedy – *algorithm makes locally optimal decision at each step*
recursive – *splits task into subtasks, solves each the same way*
local optimum – *solution for a given neighborhood of points*

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The partitioning decision is made at each node according to a metric called **purity**.

A partition is 100% pure when all of its records belong to a single class.

Consider a binary classification problem with classes X, Y. Given a set of records D_t at node t, Hunt's algorithm proceeds as follows:

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NOTE

This is the *base case* for the recursive algorithm.

Consider a binary classification problem with classes X, Y. Given a set of records D_t at node t, Hunt's algorithm proceeds as follows:

2) If D_t contains records from both classes, then a test condition is created to partition the records further. In this case, t is an internal node whose outgoing edges correspond to the possible outcomes of this test condition.

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2) If D_t contains records from both classes, then a test condition is created to partition the records further. In this case, t is an internal node whose outgoing edges correspond to the possible outcomes of this test condition.

These outgoing edges terminate in **child nodes**. *A record d in D_t is assigned to one of these child nodes based on the outcome of the test condition applied to d.*

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NOTE

Decision trees are easy to interpret, but the algorithms to create them are a bit complicated.

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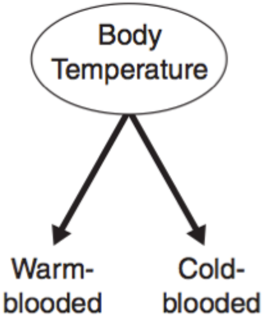
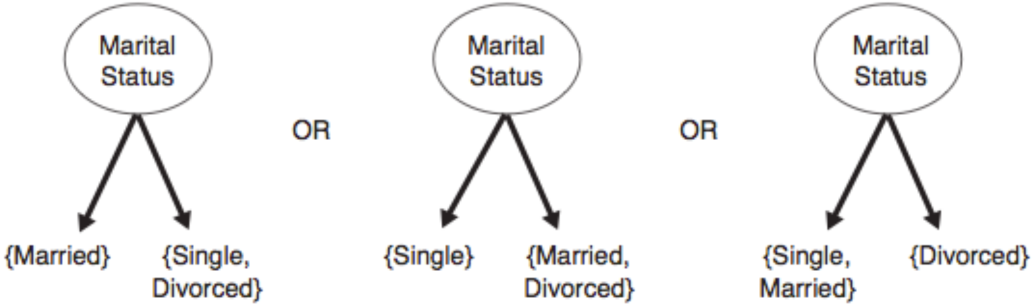


Figure 4.8. Test condition for binary attributes.

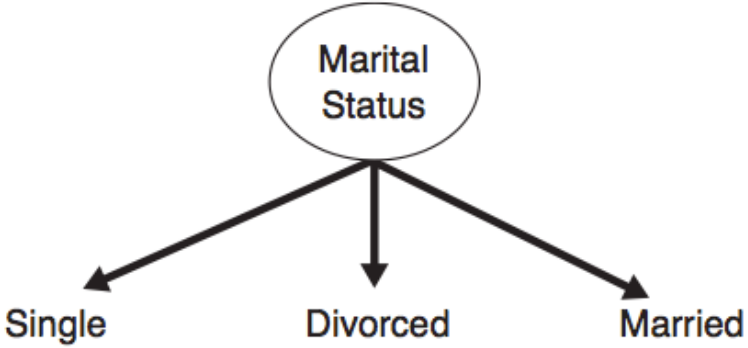


(b) Binary split {by grouping attribute values}

Q: How do we partition the training records?

A: There are a few ways to do this.

Alternatively, we can create **multiway splits***:*



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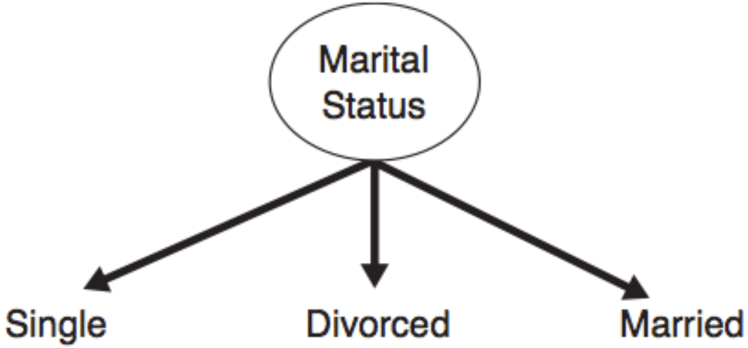


(a) Multiway split

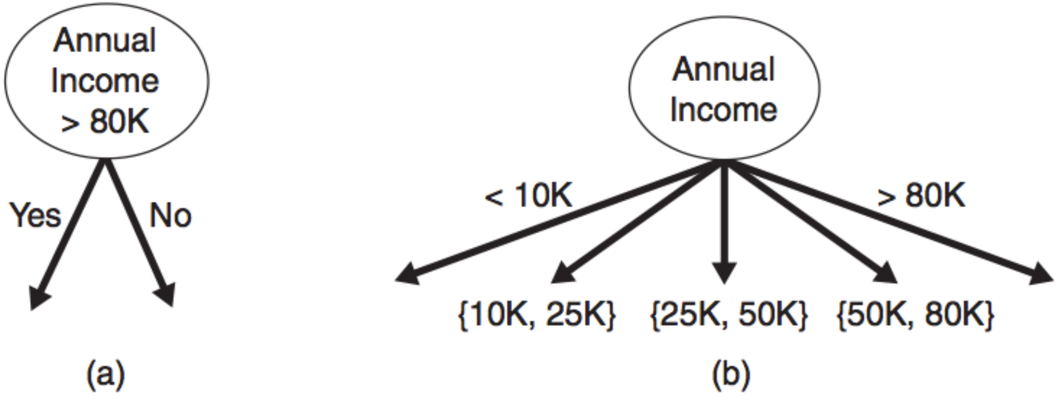
NOTE

Multiway splits can produce purer subsets, but may lead to overfitting!

Q: How do we partition the training records?

A: There are a few ways to do this.

For continuous features, we can use either method:



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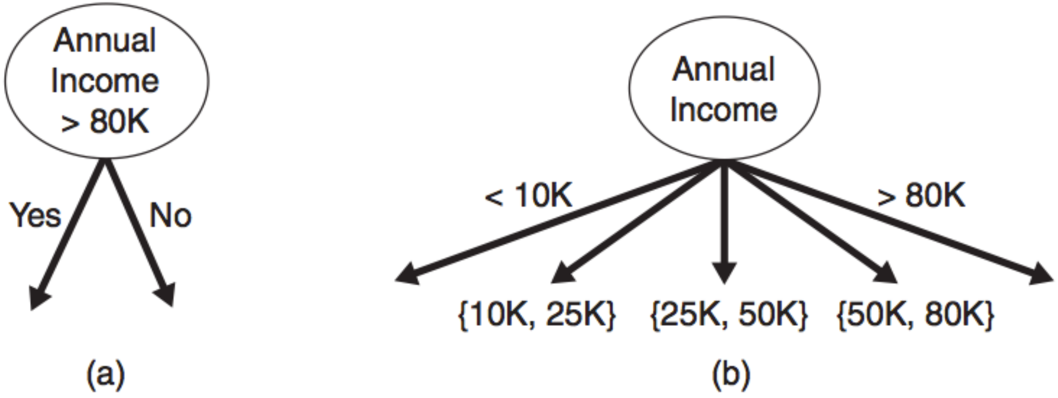

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NOTE

There are optimizations that can improve the naïve quadratic complexity of determining the optimum split point for continuous atrributes.

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Therefore we want each step to create the partition with the highest possible purity.

We need an objective function to optimize!

III. OPTIMIZATION FUNCTIONS

We want our objective function to measure the gain in purity from a particular split.

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For example, let $p(i \mid t)$ *be the probability of class* i *at node* t *(eg, the fraction of records labeled* i *at node* t*).*

Then for a binary (0/1) classification problem,

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The maximum purity partition is given (eg) by the distribution:

$$p(0|t) = 1 - p(1|t) = 1$$

Some measures of impurity include:

$$
\begin{aligned}
\text{Entropy}(t) &= -\sum_{i=0}^{c-1} p(i|t) \log_2 p(i|t), \\
\text{Gini}(t) &= 1 - \sum_{i=0}^{c-1} [p(i|t)]^2, \\
\text{Classification error}(t) &= 1 - \max_i [p(i|t)],
\end{aligned}
$$

CONSISTENCY OF OBJECTIVE FUNCTIONS

Note that each measure achieves its max at 0.5, min at 0 & 1.

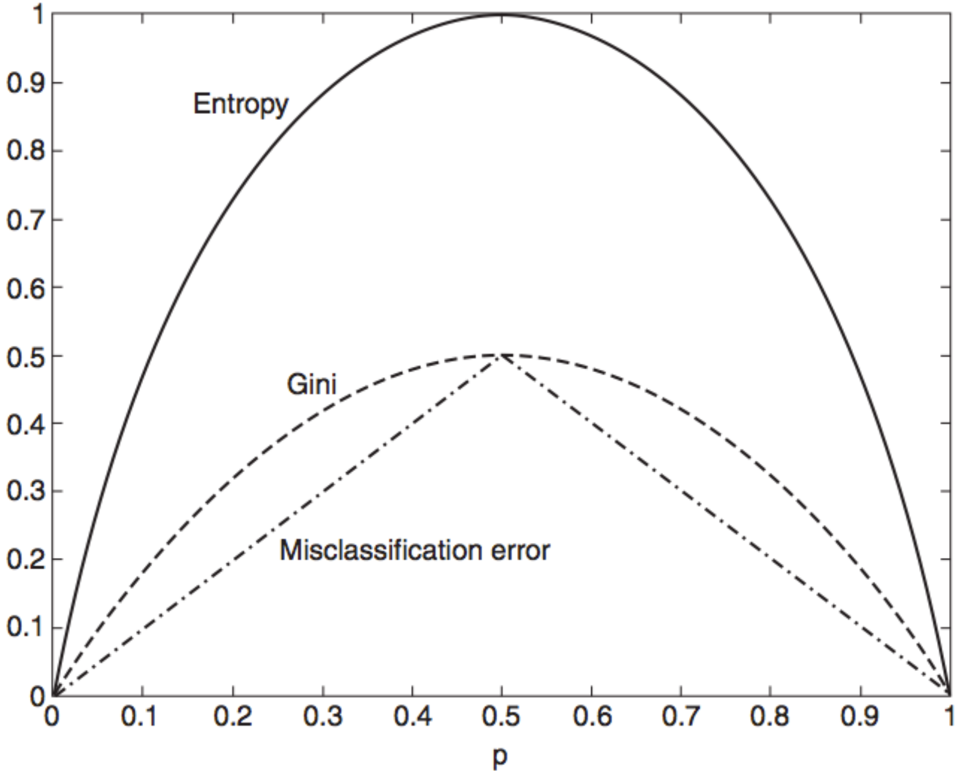

Figure 4.13. Comparison among the impurity measures for binary classification problems.

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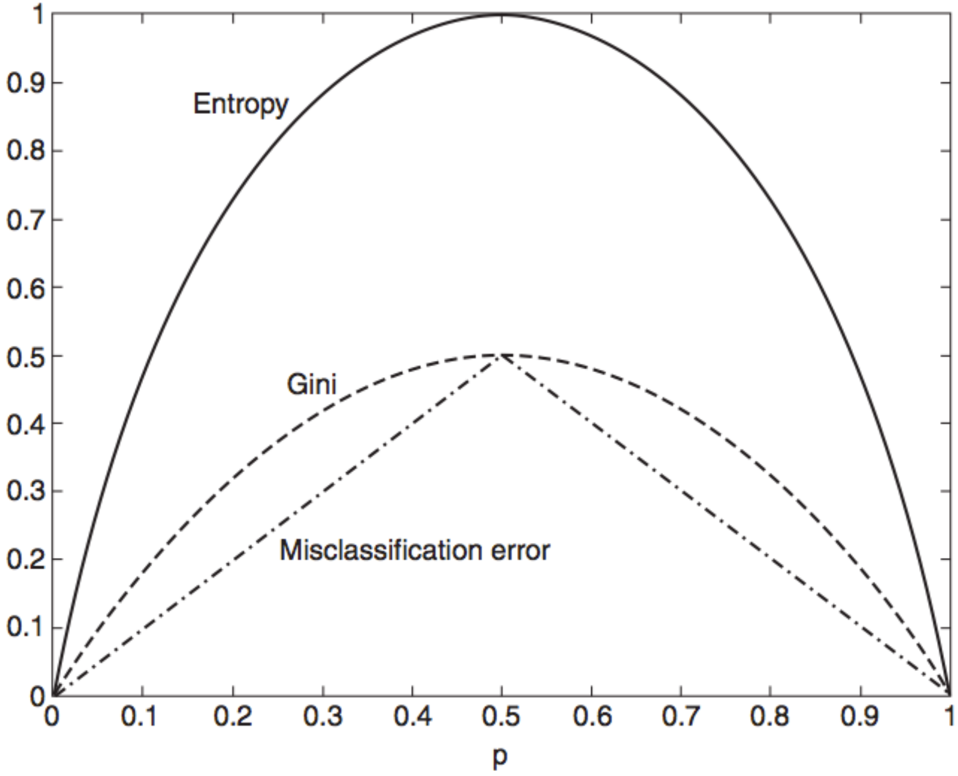
NOTE

Despite consistency, different measures may create different splits.



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Q: Why is this true?

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Q: Why is this true?

A: We still need to look at impurity before & after the split.

We can make this comparison using the **gain***:*

$$\Delta = I(\text{parent}) - \sum_{\text{children } j} \frac{N_j}{N} I(\text{child } j)$$

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(Here I is the impurity measure, N_j denotes the number of records at child node j, and N denotes the number of records at the parent node.)

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(Here I is the impurity measure, N_j denotes the number of records at child node j, and N denotes the number of records at the parent node.)

When I is the entropy, this quantity is called the **information gain***.*

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One way of dealing with this is to restrict the algorithm to binary splits only (CART).

Another way is to use a splitting criterion which explicitly penalizes the number of outcomes (C4.5)

We can use a function of the information gain called the **gain ratio** *to explicitly penalize high numbers of outcomes:*

$$\text{gain ratio} = \frac{\Delta_{info}}{-\sum p(v_i) log_2 p(v_i)}$$

(Where $p(v_i)$ refers to the probability of label i at node v)

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NOTE

This is a form of regularization!

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IV. PREVENTING OVERFITTING

In addition to determining splits, we also need a stopping criterion to tell us when we're done.

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This is correct in principle, but would likely lead to overfitting.

One possibility is **pre-pruning***, which involves setting a minimum threshold on the gain, and stopping when no split achieves a gain above this threshold.*

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This prevents overfitting, but is difficult to calibrate in practice (may preserve bias!)

Alternatively we could build the full tree, and then perform **pruning** *as a post-processing step.*

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To prune a tree, we examine the nodes from the bottom-up and simplify pieces of the tree (according to some criteria).

Complicated subtrees can be replaced either with a single node, or with a simpler (child) subtree.

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The first approach is called **subtree replacement**, *and the second is* **subtree raising**.

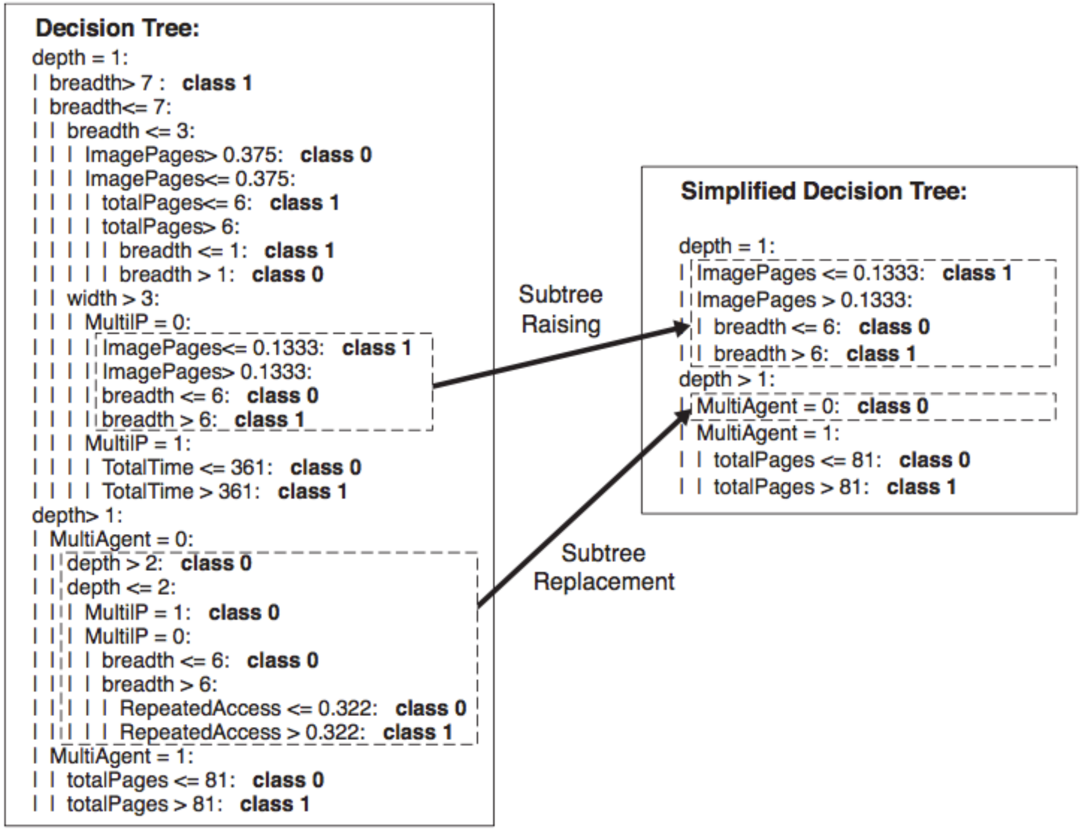

Figure 4.29. Post-pruning of the decision tree for Web robot detection.

V. RANDOM FORESTS

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One way to do this is to randomly choose one of the top k features to split each node.

For a small number of features, we can also create linear combinations of features and select splits from the enhanced feature set (Forest-RC).

Or, we can select splitting features completely at random (Forest-RI).

EX: DECISION TREES IN PYTHON