Master_Theorem

CT: $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3, \quad b = 2$

$k = \log_b a = \log_2 3$

$f(n) = n^2$

So sánh: n^k và $f(n)$

$\Rightarrow n^{\log_2 3} < n^2$

$\Rightarrow O(n) = n^2$

2) $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4, \quad b = 2$

$k = \log_b a = \log_2 4 = 2$

$f(n) = n^2$

So sánh: $n^{\log_2 4}$ và n^2

$\Rightarrow n^2 = n^2$

$\Rightarrow O(n) = n^2 \log n$

2.1) $T(n) = 3T\left(\frac{n}{3}\right) + n\log n$

$a = 3$, $b = 3$

$h = \log_3 3 = 1$

$f(n) = n\log n$

So sánh: $n^{\log_3 3} = n < n\log^1 n$

$\Rightarrow O(n) = n\log^2 n$

3) $T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a = 1$, $b = 2$

$h = \log_2 1 = 0$

$f(n) = 2^n$

So sánh : $1 < 2^n$

$\Rightarrow O(n) = 2^n$

4) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 2$, $b = 2$

$h = \log_2 2 = 1$

$f(n) = \frac{n}{\log n}$

So sánh $n^1 > \frac{n}{\log n}$

$O(n) = n^1$

8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0,51}$

$a = 2, \quad b = 4$

$k = \log_4 2 = \frac{1}{2}$

$f(n) = n^{0,51}$

$\Rightarrow n^{0,5} < n^{0,51}$

$\Rightarrow O(n) = n^{0,51}$

10) $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a = 16, \quad b = 4$

$k = \log_4 16 = 2$

$f(n) = n!$

So sánh: n^2 và $n!$

Xét $\lim_{n \to +\infty} \frac{n^2}{n!} = \lim_{n \to +\infty} \frac{n-1+1}{(n-1)!}$

$= \lim_{n \to +\infty} \left[\frac{1}{(n-2)!} + \frac{1}{(n-1)!} \right]$

$= 0$

$\Rightarrow O(n) = n!$

16) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3 \quad , b = 3$

$h = \log_3 3 = 1$

$f(n) = \frac{n}{2}$

So sánh : $n^1 = \frac{n^1}{2}$

$\Rightarrow O(n) = n \log n$