CBDC and Card Payment Market

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Abstract

I looked into the impact of the introduction of CBDC in Korea on the payment market variables like card fees and social welfare by extending the static model in Li, McAndrews, and Wang(2020). The optimal CBDC remuneration rate, the concept of benefits paid to the transaction volume, was estimated at $59bp$ which is equal to the transaction cost of cash for consumers. But the optimal rate can be changed by specific designs such as accessibility and protection of privacy. The introduction of CBDC can improve social welfare through two channels. The first direct channel is that they can provide consumers with a better means of payment than cash. The second indirect channel comes from the reduction of card fees and the price of trading goods due to the reaction of the card network to defend their business area from CBDC. The introduction of CBDC lowers the hurdle for card usage, which has a positive effect on financial inclusion but can widen inequality due to the regressive-transfer structure of the benefits.

JEL Classification: E41, E42, G21
Key words: CBDC, payment market, card fee, optimal CBDC rate, financial inclusion, two-sided markets

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I thank Young Sik Kim who is my adviser professor, and also thank my colleagues for good comments on this paper. Nevertheless, any errors that may remain in this paper are my own responsibility. Go to `https://github.com/doyeongong/Do_public/tree/main/CBDC` to visit my github repository for matlab codes for this paper.

1 Introduction

Significant changes like the declining use of cash and emerging fast payment systems, are occurring in the payment and settlement environment due to the progress of digital technology. In this circumstance, the central banks could not ignore the need to discuss the introduction of Central Bank Digital Currency(CBDC). Unlike other digital currencies, it is clear that the introduction of CBDC unleashes many crucial changes in the existing payment market, especially for the card(debit or credit) payment market. But the magnitude of the effect is difficult to estimate and it will depend on the specific designs of CBDC such as interest compensation and privacy policies.

Previous studies for CBDC have been attempted to analyse the effect of introduction CBDC, but they have been still limited in some sub-topics due to a lack of data and unspecified introduction plans. Looking at the previous studies of CBDC in academia, most of the literature has focused on the bank run crisis triggered by flight-to-quality following the introduction of CBDC or the setting of the optimal CBDC interest rate considering that effect and monopolistic power of banks. For the first time, there were many concerns that CBDC could cause financial instability due to the withdrawal of deposits from commercial banks to the central bank.(Dyson and Hodgson(2016), Broadbent(2016)). However, recent papers have proven that social welfare can increase with the introduction of CBDC because the negative effect of monopolistic powers and absconding incentives of financial intermediaries were undermined by adopting a new public digital payment instrument. (Williamson(2022), Andolfatto(2019), Chiu et al.(2020)) Especially, Chiu et al.(2020) show that the socially optimal CBDC interest rate is 0.85% through their calibration model. Most of these kinds of papers are based on the Lagos and Wright(2007) model, as known as the DM-CM model, or overlapping generation(OLG) model.

On the other hand, the working groups of central banks and BIS focus more on payment and settlement issues. For example, Riks bank, one of the pioneer central banks in this field, introduced the e-krona and also investigates the relationship between the e-krona and the card payment market(Bergman (2020)). In addition, According to the results of the BIS(2019) survey, the most significant motivations for issuing CBDC to central banks are related to payment safety and efficiency. The monetary policy and financial stability are less significant than the payment issue at least in the introduction of CBDC. [1] So, there is a gap between the interested field of academic theorists and working groups. However, they primarily have published the survey outcomes and literature review rather than theoretical and quantitative model analysis. Until the end of 2021, no publicly disclosed report analyzes the impact of CBDC on the payment market with a specific quantitative model.

[1]See BIS(2019) p.9 graph 5

The preexisting studies on the payment market, where CBDC-related working groups are interested, have mainly focused on the behavior of card networks and estimating the appropriate card interchange fees. Many card interchange fee determination models deal with the trade-off between cash and card. For example, cash has a relatively low adoption cost in that it can be accessible to anyone and accepted anywhere. In contrast, the card has a high adoption cost due to the entrance fee and set-up cost for acceptance devices. On the other hand, cash arouses a carrying cost and interest opportunity cost. So, it has a relatively high transaction cost than the card. Hence, it is more advantageous in terms of the transaction cost. The card fee determination model (two-sided markets model[2]) reflects these trade-offs well. Rochet and Tirole(2002, 2003), and Rochet and Wright(2010) are examples of the study using this model and contributed to the introduction of the upper limit of card interchange fees. Recently, Li et al.(2020) expanded these methods and combined a macroeconomic topic such as the endogenous economic growth theory. They interpret card company profits as a motive and driver for technological innovation in the payment market.

In this paper, unlike previous studies that focused on the effects of CBDC on monetary policy and financial stability based on macroeconomic models, I analyzed the effects of CBDC introduction in the payment market and estimated the optimal remuneration rates(or benefit fees, rewarding points)[3] through two-sided markets model. My model is the extended version of the static model in Li et al.(2020). I modified their model to fit South Korea's current economy and introduced the CBDC in the model, and analyzed its effects and transmission mechanism in the payment market.

The most significant difference between Li et al.(2020) and my model is that the card network's profit-maximizing behaviors have changed according to reality. In the former model, the card network controls the card fees for consumers and merchants with no upper-bound constraints. It is not consistent with the fact that card fee cap regulations are implemented in the majority of advanced countries and that card fees for consumers are close to zero. So, in this paper, the card network charges the card fees only to merchants and receives only membership fees from consumers. Aside from these, I allow the negative membership fees in the model, in other words, benefits and rewards can be more than the fixed fee of the card for consumers, which can be easily observed in reality.

Second, I build the social planner problem for the central bank which wants to maximize social welfare by choosing the CBDC remuneration rate. The overall structure of

[2]The card market consists of the issuing market for consumers and the acquiring market for merchants. The card network as a financial intermediation institution does not care about one-specific-sided income from either merchants or consumers. They maximize profits by considering the total income from both sides. In this regard, many researchers consider the card market as a two-sided market. In other words, it is only possible to understand the behavior of card companies by looking at both markets.

[3]The CBDC remuneration referred to here is not the interest rate paid on the balance, but the concept of benefit fees (negative fee) charged on the transaction amount.

the model is the form of a Stackelberg game. After the central bank chooses the CBDC remuneration rate, the card network decides the card fee for merchants and membership fee for consumers given the CBDC policy. After seeing the actions of central banks and the card network, consumers and merchants decide whether they use card or cash in the transaction. In addition to finding the equilibrium, I also analyzed the transmission channel of the CBDC policy through the social welfare decomposition and derived political implications concerned with financial inclusion.

Lastly, Li et al.(2020) were not too concerned about the matching of the model parameters with the real economy such as the transaction cost of cash, but I devised a calibration strategy to match the parameter values to the real statistics of the Korean economy. Also, I conducted a sensitivity test to assumptions for other noncalibrated parameters.

The key result of this paper is that the optimal CBDC remuneration fee(or benefit fee) is estimated at 59*bp* assuming that the central bank maximizes social welfare, which consists of adding up consumer surplus, monopoly card network profit and excluding operating costs of CBDC. The value, 59*bp* means that it is optimal to offer 59 won rewarding points when the consumer pays 10,000 won. However, this figure was derived from the assumption that the CBDC cannot give the consumer any further utility compared to the cash with respect to the digital and privacy benefits. If the privacy benefits of cash and CBDC are not the same and the digital benefits of CBDC are non-negligible, then the optimal CBDC benefit fees would be determined by the digital and privacy benefit of CBDC. In this regard, the specific design of CBDC is essential to pin down the optimal CBDC benefit fees.

The introduction of CBDC can improve social welfare through two channels. The first direct channel is that they can provide consumers with a better means of payment than cash. The second indirect channel comes from the reduction of card fees and the price of trading goods due to the reaction of the card network to defend their business area from CBDC. Although the direct effect accounts for 62%, the magnitude of the indirect effect is also nonignorable (38%). So, the increase in consumer surplus due to the reduction of card service costs accounts for a significant portion of the overall welfare increase. At the socially optimal equilibrium, the share of CBDC trading is only 42%. These results suggest that CBDC can significantly improve social welfare even if it is not used much in payment transactions. In addition, as the card membership fee is reduced, it has a positive impact on the low-income class in terms of financial inclusion. However, as the benefits of the introduction of CBDC are more concentrated on consumers with high incomes (or expenditures), it plays a role like a regressive transfer, which may adversely affect income inequality.

The paper is organized as follows. Section 2 provides an overview of the background of the payment intermediation market and the selected issues of CBDC introduction which are necessary to understand the following structural model. Section 3 narrates the structural model and characterizes the equilibrium. In section 4, some quantitative results and decomposition of the social welfare were shown. Section 5 shows the outcome of the sensitivity analysis for the relaxation of some assumptions and how we interpret these results. In section 6, I summarized the results and suggests policy implications, limitations, and contributions of this paper.

2 Background

2.1 Payment service market

In this subsection, we first look at the trade-off between cash and card which is the essential core of the structural model. Furthermore, we briefly examine the meaning of the two-sided markets model, sometimes called the card fee determination model. Also we will overview the regulations for the monopoly power of the card network.

2.1.1 The trade-off between cash and card

The payment system can be roughly divided by the payment instruments such as cash and card. The payment cards consist of credit and debit cards, and the former is distinguished from the latter because it can give a chance to receive short-term loan services. However, in Korea, most credit card transactions(93.4%) are used only for simple payment purposes. Therefore, this paper does not differentiate between credit and debit cards.[4]

Table 1 describes the adoption and transaction cost of cash and card. Cash has a lower adoption cost than the card. This is because cash is a medium of exchange issued

[4] In the structural model, I did not distinguish between credit and debit cards since I excluded the credit functionality of the credit card. The reasons for that are as follows.

First, most credit card users use the payment function rather than the credit function (short-term loan services). According to the Bank of Korea BIS standard statistics in 2021, the fraction of purchasing purposes for goods and services in the credit card payment volume was 93.4% in Korea, however, short-term loan services were only 6.6%. Even the profit of the credit card network can mainly come from the short-term loan services, but their share in the overall payment market is negligible.

Second, when considering the credit function, there is a problem that the model becomes very complicated. One of the ways to consider the credit function in the model is to introduce the limit for using CBDC, which means that CBDC cannot give short-term loan services. I tried to solve the model by introducing this assumption, but the stability of the solution is not guaranteed. It implies we cannot find the solution to the nonlinear system of equations by numerical methods. Therefore, in order to consider the credit function of the credit card, other models should be adopted such as Rochet and Wright (2010).

In the previous studies that faced similar criticism, they have suggested alternative results by using only debit card statistics, excluding credit cards. In this paper, I present results using only debit card statistics in subsection 5.4.

		Cash	Card
Consumer	Adopting cost	• Owning or carrying cash (e.g. wallet)	• Entrance or membership fees • Requirements for issuing a card (e.g. credit score)
	Transaction cost	•The opportunity cost by interest paid on deposits • ATM fees • Forgoing benefits from cards	•Consumer account service fees
Merchant	Adopting cost	• Owning or leasing safes to store cash	• Card readers, PIN pads, and electronic signature capture devices • Internet service to request and receive authorization and clearing messages
	Transaction cost	• The opportunity cost by interest paid on deposits (Giving up the interest cost reduction of working capital) • ATM fees • Counting and sorting cash, preparing cash deposits • Checking for counterfeit	• Merchant account service fees • Interchange fees

Table 1: Costs of cash and card

Note: Common cost of card and cash such as set-up cost for points of sale (POS) terminals were dropped.
Source: Reorganizing Hayashi(2021) and Li et al.(2020)

by the central bank and is accessible to everyone. In contrast, to use a card system, merchants need to purchase the device for communication system to recognize whether the consumer's card is authorized. Also, consumers need to open a bank account for the settlement and keep their credit scores to be qualified conditions for holding a card. These facts imply the adoption cost of a card for economic agents is much higher than the adoption cost of cash.

On the other hand, cash has a higher transaction cost than a card. This is partly because there is the opportunity cost of the interest paid on the deposit and partly because we need to forgo the benefits from the financial institution when we use cash for trading. However, once the consumers or merchants become cardholders or card recipients, they only need to pay the card fees during the transaction. As we can see in Figure 1, banks pay the interest rate for demand deposits and the average interest rate is about 0.34% since 2004. In particular, financial institutions are providing benefits to merchants, such as deducting the working capital loan interest rate when they trade through cards. These facts imply the transaction cost of cash for economic agents is much higher than the transaction cost of cards. In this sense, there exists a trade-off between using cash and a card.

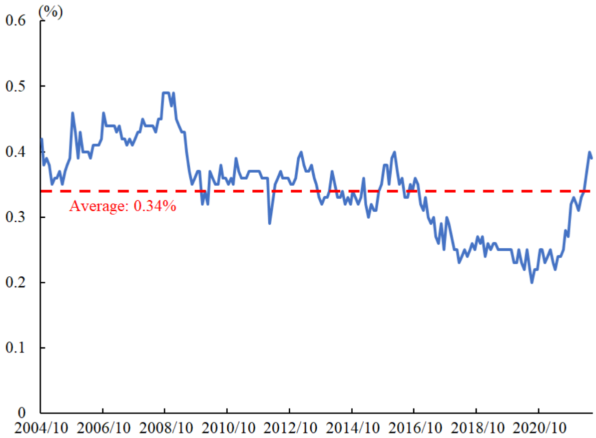


Figure 1: Demand deposit rate

Source: Bank of Korea

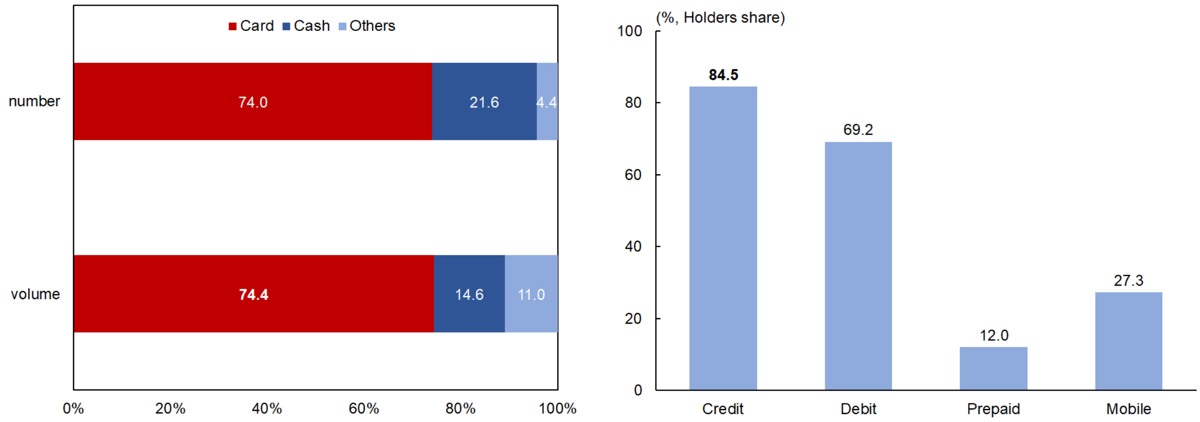
2.1.2 The statistics for card trading and holders

In this subsection, I would like to introduce crucial payment market statistics which will be used in calibration. Figure 2 shows the share of card transactions and cardholders in Korea in 2021. These figures refer to the report of the Bank of Korea's survey results on payment and mobile financial services in 2021. According to these survey results, the share of card transaction volume for consumer goods and services is 74.4%. This figure is high when compared to other countries such as Europe(about 50%). Therefore, it can be seen that the share of card trading volume in Korea's payment and settlement market is relatively high. Also, in the right panel of Figure 2, the share of credit card holders is 84.5%, which is also very high.

Since others in the left panel of Figure 2 include account transfers, department store gift certificates, and so on, there are various indicators to determine the share of card transactions. Some literature also uses the share of card transaction volume divided by the sum of card and cash transaction volume. In addition, these indicators can vary for each survey since it is not possible to know the exact transaction volume of cash. The proportion of card transactions surveyed by other departments of BOK was 58.3% (cash 21.6%, other 20.1%), which is somewhat different from the previous results. So I present other versions of the benchmark results in Appendix 3 with different assumptions for the share of the card trading volume.

2.1.3 Two-sided markets

The market of card payment is a kind of two-sided markets in that it consists of acquiring markets for merchants who are sellers of goods and services and issuing markets for



(a) Trading share (b) Holders share

Figure 2: The Statistics of payment market in 2021

Note: Others in the left panel include account transfers, department store gift certificates and so on.
Source: BOK(2022)

consumers who are buyers of goods and services. The financial intermediation companies get the card fee from two groups. So, the monopolistic card network only cares about the total revenue from these two sides and does not care about how much the share of the revenue is from one side. The previous literature on industrial organization such as Rochet and Tirole(2003) modelized these frameworks.

2.1.4 Market power and regulation

The financial intermediation industry has the economy of scale effect. This fact gives rise to the financial intermediation institutions naturally having market power and earning the economic rent in this market. In this regard, many countries have already been concerned about the market power of these payment service providers and introduced regulations like an upper limit on card fees[5]

For instance, in Korea, the Financial Services Commission released the upper limit on the interchange fees for debit and credit cards every three years. This regulation is planned to protect small merchants, so the upper limit of the card fee is set differently based on the merchant's annual sales volume. As we can see in Table 2, the upper limit of credit card fees in the highest section is 2.06% and that of debit cards is 1.47% since 2021. On the other hand, the Korea Credit Financial Association also announces the average card fee by the relevant laws. Their published figure is also 2.06%. Since the average card fee is the same as the upper limit rate in the highest revenue section, we can recognize the card network charges the fee according to the upper limit.

[5] Hayashi and Maniff summarized the regulation of the payment card market in many countries. See https://www.kansascityfed.org/research/interchange-fees/

<table>
<tr><th>Annual revenue(Billion Won)</th><th>2012</th><th>2015</th><th>2018</th><th>2021</th></tr>
<tr><td>below 0.2</td><td>1.50(1.00)</td><td>0.80(0.50)</td><td rowspan="2">0.80(0.50)</td><td rowspan="2">0.50(0.25)</td></tr>
<tr><td>0.2∼0.3</td><td rowspan="5">2.12(1.60)</td><td>1.30(1.10)</td></tr>
<tr><td>0.3∼0.5</td><td rowspan="4">2.09(1.60)</td><td>1.30(1.10)</td><td>1.10(0.85)</td></tr>
<tr><td>0.5∼1.0</td><td>1.40(1.10)</td><td>1.25(1.00)</td></tr>
<tr><td>1.0∼3.0</td><td>1.60(1.30)</td><td>1.50(1.25)</td></tr>
<tr><td>above 3.0</td><td>2.06(1.47)</td><td>2.06(1.47)</td></tr>
</table>

Table 2: The upper limit of card fees for merchants in Korea

Source: Financial Services Commission

2.2 The selected issues of CBDC

In this subsection, I confine the meaning of CBDC in this paper because there are many types of CBDC under consideration. Also, I would like to narrow down the design characteristics of CBDC related to following structural model. Finally, I will explain why CBDC can make the payment system more efficient.

2.2.1 CBDC classification

There are many different variations and definitions of CBDC. But here I want to talk about the account-based retail CBDC. The "account-bases" means that it is not token-based like a rechargeable prepaid card and is more similar to bank accounts. The most significant difference is that the account-based currency needs to recognize the identification of the owner, and the token-based currency to determine the authenticity. The "retail" means that the CBDC is not only for the primary dealer which refers to financial institutions that participate in traditional open market operations but also for many ordinary people.[6] These kind of CBDC is the most prominent type that many monetary authorities consider the introduction. The e-krona which is mentioned before also falls under this category of CBDC, and China's e-RMB, which is about to be introduced, also falls into this category.

2.2.2 CBDC design

The specific design of CBDC may have various components, but it can be broadly classified into interest payment, limit(or cap), and privacy protection functions.

If the central bank pays the interest to CBDC holders, then the transaction cost of trading CBDC will be reduced. However, much CBDC-related literature came up with the financial instability effect of interesting-bearing CBDC, which can give rise to the bank run crisis. These papers worried about the decrease in deposits of the commercial banks.

[6] For more detail, see BIS(2019).

Also, it can damage the profit of the card network in the payment market or distort the monetary policy transmission channels. So, setting up the CBDC rate inevitably face a trade-off problem. The central bank should set the optimal CBDC rate that does not harm the existing financial system while reducing transaction costs for economic agents. Recently, in order to avoid chaos in the financial market, many central banks are considering the introduction of noninterest-bearing CBDC.

Another widely mentioned CBDC design is the limit. If the introduction of CBDC can lead to sudden deposit withdrawals, the limit can be thought of as a means to prevent such bank runs. However, if the central bank sets the cap of CBDC, the private financial institution still has market power beyond the limit. Limit and rates are two main design points mentioned in BIS(2021) as the condition for avoiding disturbance in the financial system. In reality, there is a payment limit on cards, so CBDC also has no choice but to have a limit. This fact can be a reason why cash does not disappear even after the introduction of CBDC because cash has no such limit.

On the other hand, CBDC can supply a shelter for consumers who are worried about the data silo of the big-tech firms. These benefits are not related to the CBDC interest rate. In this regard, we need to consider other benefits excluding the pecuniary benefit and include these factors in the optimal CBDC rate model. In addition, Choi et al.(2022) conducted a survey in Korea, which suggested that the demand for CBDC may change depending on the level of privacy protection. Also, Willamson(2022) analyzed the effect of CBDC on social welfare by adding privacy shock to decentralized trading. Therefore, how the CBDC guarantees privacy is a significant factor in determining the CBDC remuneration.

2.2.3 Cost reduction by introducing CBDC

The reason why the introduction of CBDC is an innovation in the payment system is that it gives rise to the change in the cost structure of the payment market and enhances its market discipline in it. Since The settlement of payment is completed in the book of a central bank, so the direct connection between seller and buyers without the bank or card network can improve the efficiency of trading and payment. In this regard, the central bank can promote competition in the payment market and kick out the dead weight loss coming from the market power of private financial institutions. CBDC can provide consumers and merchants with other benefits. These are summarized in Table 3.

I would like to mention selected advantages of CBDC. First, for the consumers, CBDC can be an alternative payment instrument that avoids the data silo of big tech firms related to privacy issues. For merchants, since most of the card trading volume can be reused only after one month, they need to get a loan from the bank for the working capital. However, CBDC can directly increase the efficiency of the real economy because the trading volume can be immediately reused for purchasing intermediate inputs or paying to rent. Furthermore, many financial institutions force merchants to use their card services as a loan requirement. If CBDC is introduced, merchants will be freed from such tying and bundling, which could lead to more freedom in financing.

In addition, it is clear CBDC can compensate for the shortcomings of cash. Contrary to cash, CBDC has the advantage over cash in that it can be available online and reduce the interest opportunity costs.

	Consumer	Merchants
vs Card	• No user fee • Low possibility of default risk of FI • Universal acceptance • Privacy protect from data silo	• No user fee • Quick access to funds received. (available for immediate reuse) • Choose funding method freely
vs Cash	• Digital convenience(available online) • Compensating interest opportunity cost (Only for interest-bearing CBDC)	• Digital convenience(available online) • compensating interest opportunity cost (Only for interest-bearing CBDC) • Avoid the risk of counterfeit money

Table 3: The advantages of CBDC

Source: Khiaonarong and Humphrey(2022)

3 Structural model

3.1 Environment

In the model, there are consumers, merchants, the card network, and the central bank and they play the static Stackelberg game.

There exist consumers with a continuum of measure unity. Each consumer has a different income I which follows the exponential distribution, $F(I)$ with mean parameter $1/\lambda$. Each agent expends I completely, hence, the I indicates the expenditure volume of each consumer. Denote $x_{\alpha,I}$ as the demand quantity for α goods and I consumer. Consumers have a continuous type of Cobb-Douglas preference.

$$\ln U_I = \int_0^1 \frac{\alpha}{E(\alpha)} \ln x_{\alpha,I} dG(\alpha)$$

where α indicates the goods index and $G(\alpha)$ is assumed to be a uniform distribution on [0,1].[7] α is also the share parameter in consumer expenditure.

[7]According to Li et al.(2020), even if $G(\alpha)$ follows an exponential distribution rather than a uniform distribution, it does not affect the change in conclusions.

Also, there exists a continuum of merchants of measure one unit. Each merchant sells a distinct good and the transaction cost of production is given by μ.[8] The heterogeneity of merchants is represented by the size index or the consumer expenditure share α which is uniformly distributed on $[0, 1]$. Through this paper, the goods market is assumed to be a competitive market. So, all merchants earn zero profit in equilibrium.

The transaction cost structure of the two agents are summarized in Table 4. As mentioned in section 2, there exists a trade-off between cash and card. I normalized the adoption cost as the adoption cost of cash is equal to zero and the transaction cost as the transaction cost of the card is the same as the card fee. I set the bound for the parameters as follows.

$$K_c \geq 0, K_m \geq 0 \;\text{ and }\; \tau_c \leq f_c, \tau_m \leq f_m$$

These inequalities mean that the adoption cost of cash is less than the one of the card and the transaction cost of cash is greater than the one of the card. From these assumptions, there is the threshold income level(I_0) for cardholders in consumers and the threshold size(α_0) for card acceptance in merchants.

Agent	Cost	Cash	Card	CBDC
Consumers	adoption cost	0	K_c	J_c
	transaction cost	τ_c	f_c	y_c
Merchants	adoption cost	0	K_m	J_m
	transaction cost	τ_m	f_m	y_m

Table 4: The cost of payment instrument

The revenue of the card network comes from two sources. One is the card transaction fee (f_m, f_c) which is charged to consumers and merchants from the card transaction volume respectively. The other is the membership fee(K'_m, K'_c) which is the fixed fee from two agents. In other words, the adoption cost of the card consists of a fee for the card network and extra-opportunity costs.

$$k_c = k'_c + \kappa_c \;\; and \;\; k_m = k'_m + \kappa_m$$

where κ is the non-membership adoption cost of the card and $K = k \cdot (1/\lambda)$, that is the lowercase k means the percentage cost per average income. Let the marginal cost of providing card services be d. Therefore, the profit of the card network is as follows.

$$\max_{\{f_m, f_c, K'_m, K'_c\}} (f_m + f_c - d) \cdot (\text{card trading}) + K'_c \cdot (\text{card holder}) + K'_m \cdot (\text{card acceptor})$$

I need to remark the profit maximization problem of the card network is not an

[8] In this model, μ is only related to the level value of consumer surplus. Later, it is normalized to 1.

unconstrained one. The thresholds α_0 and I_0 are coming from the best response from consumers and merchants. So, the card network considers the best response correspondences of consumers and merchants.

The central bank set the CBDC rate(i^{CBDC}) and maximizes social welfare. I define social welfare as the aggregated value of the total sum of consumer surplus, the profit of the card network, and deducting the CBDC rate compensation expense.

$$(SW) = (CS) + (\text{profit}) - (\text{CBDC rate expense})$$

Some previous studies suggest consumer surplus is the only source of social welfare. However, this kind of consumer surplus only includes the short-run concept in that the exit of the card network causes the retreat of the payment service system. The profit of the card network can be interpreted as the incentives for innovation of payment technology.[9] Also, the CBDC rate act as a kind of negative tax in this model. So, we need to consider the expense of this transfer for calculating social welfare. This term can be interpreted as the opportunity cost in that the method of paying directly consumers in the form of lump-sum transfer rather than CBDC remuneration[10]. The central bank considers the best response correspondences of consumers, merchants, and the card network.

The timing of the game is as follows. First, Central bank sets the CBDC rate(i^{CBDC}) with considering the private agents' reactions. Second, given i^{CBDC}, the card network maximizes its profit by adjusting the card fee(f_m, f_c, K'_m, K'_c) with considering the consumers and merchants' reactions. Lastly, given i^{CBDC} and the card fee(f_m, f_c, K'_m, K'_c), consumers choose whether they hold cards and merchants choose whether they accept cards in their transactions.

3.2 Cash-Only Economy

Let's look at the cash-only economy first. As I assumed before, the preference of consumers is represented by the Cobb-Douglas utility function. The utility maximization problem of the consumer whose income level is equal to I is as follows.

$$\underset{\{x_{\alpha,I}\}_0^1}{max} \ln U_I = \int_0^1 \frac{\alpha}{E(\alpha)} \ln x_{\alpha,I} dG(\alpha) \qquad s.t \int_0^1 (1+\tau_c) P_{cash} \cdot x_{\alpha,I} dG(\alpha) = I$$

where the τ_c denoted the unit trading cost of cash for consumers. The Marshallian demand is

[9]Li et al.(2020) indicate the monopoly profit of the card network as the incentive for endogenous growth. Rochet and Wright(2010) also said that the card network's profit should be included in the long-run consumer surplus.

[10]Imagine the consumer utility function class of quasi=linear with respect to money. If the government gives consumer money directly, the utilities of consumer increase exactly by $i^{CBDC} \cdot (CBDC\ trading)$.

$$x_{\alpha,I} = \frac{\alpha I}{E(\alpha)(1+\tau_c)P_{cash}}$$

and Substituting this into the direct utility function again, the indirect utility function (consumer surplus) can be obtained as follows.

$$\ln V_I^{cash} = \int_0^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha I}{E(\alpha)(1+\tau_c)P_{cash}}\right) dG(\alpha) \tag{1}$$

The sales of index α merchants are the aggregate sum of total consumers demand.

$$\int_0^\infty P_{cash} x_{\alpha,I} dF(I) = \frac{\alpha E(I)}{E(\alpha)(1+\tau_c)}.$$

Since the size of sales is proportional to α, then we can call it is the size index of merchants. The market price (P_α) is determined by the free entry condition or zero profit condition.

$$[(1-\tau_m)P_{cash} - \mu] \int_0^\infty x_{\alpha,I} dF(I) = 0 \ \Rightarrow P_{cash} = \frac{\mu}{(1-\tau_m)}$$

Hence, the cash price is constant and not related to the size index, α. Plugging this price value into the indirect utility function and solving the integral, the consumer surplus is calculated as follows.

$$(CS) = \int_0^\infty V_I^{cash} dF(I) = \left(\frac{1}{\lambda}\right)\left(\frac{2(1-\tau_m)}{\mu(1+\tau_c)}\right) e^{-0.5}$$

Please refer to Appendix A.1 for the detailed derivation process. We can easily recognize that the consumer surplus decreases as the trading cost (τ_c or τ_m) increase.

3.3 Cash and Card Economy

Next, we will look at the model where there exists cash and card. It is a real economy before the introduction of CBDC and is the same as the static model of Li et al.(2020). In order to solve the Stackelberg game, I will use the backward induction. So, we need to inspect the consumers' and merchants' best response correspondence first.

Consumers' optimal decision is simple. A consumer holds the card if

$$\ln V_I^{card} \geq \ln V_I^{cash}.$$

Since the consumer surplus is proportional to the income level as seen in Equation (1), The above inequality is equivalent to $I \geq I_0$. Therefore, the consumer's best response is summarized as a consumer getting a card if the income level of the consumer is bigger than the threshold income level(I_0).

Merchants' optimal decision is more complicated than one of the consumers. Before solving the merchant's problem, we need to learn one assumption and one fact. First, the additional assumption is that price discrimination between different payment instruments cannot happen in this model. This assumption means that A single merchant must sell to cardholders or cash traders at the same price, or sell at a higher price for only the specific target consumer. Second, the cash price$\left(P_{cash} = \frac{\mu}{1-\tau_m}\right)$ is a constant, but the card price($P_{card}(\alpha)$) is a function that is inversely proportional to the size. This is because the larger the sales volume, the smaller the proportion of the adoption cost of the card in the profit calculation. Therefore, the card price decrease as the size index increase. From these facts, we can categorize the merchants as follows. Figure 3 shows the trading schedule in this economy.

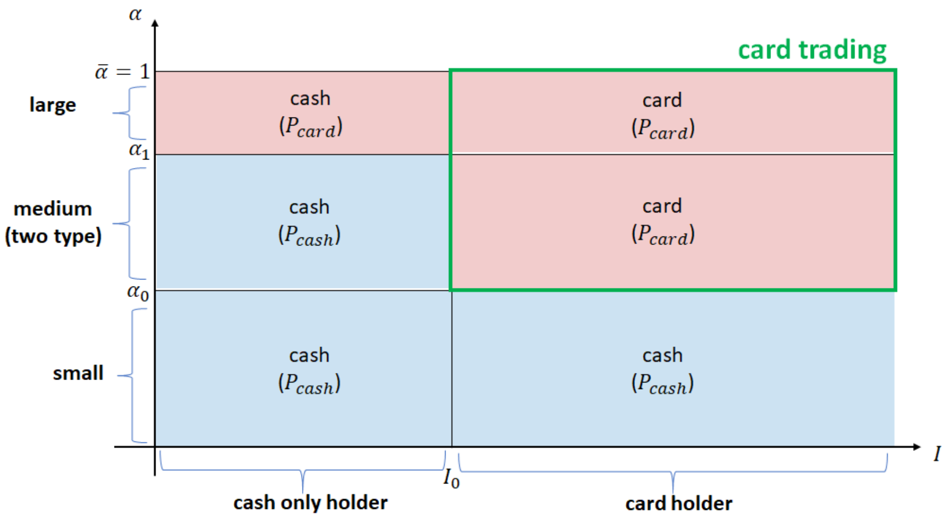


Figure 3: No CBDC trading plan

Note: The red section means that the merchants set up the card acceptance appliance. The blue section means the merchants have no such device.

The first case is $P_{card} \leq P_{cash}$. These merchants' sales are large enough that they trade at a lower price, (P_{card}) for both cardholders and cash traders. Let's denote the threshold size level as α_1. So, $\alpha \geq \alpha_1$ sell at P_{card} for all consumers. In this case, the zero profit condition is

$$(1 - f_m)P_{card}x_\alpha^{card} + (1 - \tau_m)P_{card}x_\alpha^{cash} = \mu(x_\alpha^{card} + x_\alpha^{cash}) + K_m$$

$$\text{where} \quad x_\alpha^{card} = \int_{I_0}^{\infty} \frac{\alpha(I - K_c)}{E(\alpha)(1 + f_c)P_{card}} dF(I), \quad x_\alpha^{cash} = \int_0^{I_0} \frac{\alpha I}{E(\alpha)(1 + \tau_c)P_{card}} dF(I)$$

and P_{card} can be derived as follows.

$$P_{card} = \frac{\mu \left[\frac{\alpha \int_{I_0}^{\infty} (I - K_c) dF(I)}{(1+f_c)} + \frac{\alpha \int_0^{I_0} I dF(I)}{(1+\tau_c)} \right]}{\left(\frac{1-f_m}{1+f_c} \right) \alpha \int_{I_0}^{\infty} (I - K_c) dF(I) + \left(\frac{1-\tau_m}{1+\tau_c} \right) \alpha \int_0^{I_0} I dF(I) - K_m E[\alpha]}$$

Using $P_{cash} = \frac{\mu}{1-\tau_m}$, the inequality $P_{card} \leq P_{cash}$ is equivalent to

$$\alpha \geq \frac{k_m}{2e^{-\lambda I_0}(\lambda I_0 + 1 - k_c) \left(\frac{\tau_m - f_m}{1+f_c} \right)} (\equiv \alpha_1)$$
$$\text{where } k_m = K_m/(1/\lambda), \quad k_c = K_c/(1/\lambda)$$

The second case is $P_{card} > P_{cash}$ and $(1 + f_c)P_{card} \leq (1 + \tau_c)P_{cash}$. The last inequality indicates the buying price using card is still cheaper than the buying price using cash for a cardholder considering the transaction cost of trading. In this case, the merchant has two choices. One way is to sell at the price P_{card} for only card holders. The other way is to sell at the price P_{cash} for only cash trading consumers. Let's denote the threshold size level as α_0 which is satisfied with $(1 + f_c)P_{card} = (1 + \tau_c)P_{cash}$. So, a merchant in $\alpha_0 \leq \alpha < \alpha_1$ sell at P_{card} for only cardholders. In this case, the zero profit condition is

$$(1 - f_m)P_{card} x_{\alpha}^{card} = \mu \text{ where } x_{\alpha}^{card} + K_m$$

and P_{card} can be derived as follows.

$$P_{card} = \frac{\mu \alpha \int_{I_0}^{\infty} (I - K_c) dF(I)}{(1 - f_m) \alpha \int_{I_0}^{\infty} (I - K_c) dF(I) - K_m E[\alpha](1 + f_c)}$$

Hence, the inequality $(1 + \tau_c)P_{cash} \geq (1 + f_c)P_{card}$ is equivalent to

$$\alpha \geq \frac{k_m}{2e^{-\lambda I_0}(\lambda I_0 + 1 - k_c) \left(\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+\tau_c} \right)} (\equiv \alpha_0)$$

The third case is $(1+f_c)P_{card} > (1+\tau_c)P_{cash}$. In that case, a merchant sell at the price P_{cash} for all consumers. So, If $\alpha < \alpha_0$, the merchant does not set up the card acceptance appliance. Even a cardholder must use cash when they want to trade with this category of merchants.

Returning to the consumers' problem, we can reorganize the indirect utility functions of consumers as follows.

$$\ln V_I^{cash} = \int_0^{min\{\alpha_1,1\}} \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha I}{E(\alpha)(1+\tau_c)P_{cash}}\right) dG(\alpha) + \int_{min\{\alpha_1,1\}}^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha I}{E(\alpha)(1+\tau_c)P_{card}}\right) dG(\alpha)$$

$$\ln V_I^{card} = \int_0^{\alpha_0} \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha(I-K_c)}{E(\alpha)(1+\tau_c)P_{cash}}\right) dG(\alpha) + \int_{\alpha_0}^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha(I-K_c)}{E(\alpha)(1+f_c)P_{card}}\right) dG(\alpha)$$

To derive the threshold income level, I_0, we can calculate the following inequality.

$$\ln V_I^{cash} - \ln V_I^{card} \leq 0$$

$$\Leftrightarrow \ln\left(\frac{I}{I-K_c}\right) \leq \int_{\alpha_0}^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{1+\tau_c}{1+f_c}\right) dG(\alpha) + \int_{\alpha_0}^{min\{\alpha_1,1\}} \frac{\alpha}{E(\alpha)} \ln \frac{P_{card}}{P_{cash}} dG(\alpha)$$

$$\Leftrightarrow I \geq \frac{\left(\frac{1+\tau_c}{1+f_c}\right)^{1-\alpha_0^2} \times k_c}{\left(\frac{1+\tau_c}{1+f_c}\right)^{1-\alpha_0^2} - exp\left\{\int_{\alpha_0}^{min\{\alpha_1,1\}} \alpha \ln\left(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha-(1+f_c)Z_0\alpha_0}\right) dG(\alpha)\right\}} (\equiv I_0)$$

$$\text{where} \quad Z_0 = \left(\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+\tau_c}\right)$$

There are two facts we should be aware of here. First, α_1 can be easily found by multiplying constant to α_0.

$$\alpha_1 = \frac{Z_0}{\left(\frac{\tau_m - f_m}{1+f_c}\right)} \times \alpha_0$$

Second, The α_0 and I_0 can be get from solving the system of nonlinear equations and the numerical algorithm can be applied because the solution of the system is stable.[11]. So, we can summarize the consumers' and merchants' best response correspondence as simultaneous equations.

In the next step, we can deploy the monopoly card service provider's profit maximization problem as follows. the card network decides fee (f_c, f_m, K_c', K_m') given the marginal operating cost of (d) and consumers and merchants best response correspondence (α_0,

[11] For more detail, please see Li et al.(2020)

α_1, and I_0).

$$\Pi(\tau_c, \tau_m, \kappa_m, \kappa_c, d) = \max_{\{f_m, f_c, K'_m, K'_c\}} \int_{\alpha_0}^{1} \int_{I_0}^{\infty} (f_m + f_c - d) P_{card} \cdot x_{\alpha,I} \, dF(I) \, dG(\alpha) \tag{2}$$
$$+ \int_{I_0}^{\infty} K'_c dF(I) + \int_{\alpha_1}^{1} K'_m dG(\alpha) + \int_{\alpha_0}^{\alpha_1} \int_{I_0}^{\infty} K'_m dG(\alpha) dF(I)$$
$$s.t. \begin{cases} \alpha_0 = \alpha_0(f_m, f_c, K'_m, K'_c, I_0) \\ I_0 = I_0(f_m, f_c, K'_m, K'_c, \alpha_0) \\ \alpha_1 = \frac{Z_0}{\left(\frac{\tau_m - f_m}{1+f_c}\right)} \times \alpha_0 \end{cases}$$

Hereafter, I want to add the reality restriction

$$0 \leq f_m \leq \bar{f}_m, \quad f_c = 0, \quad k'_m = 0, \quad -\kappa_c \leq k'_c$$

into (2). This is because many countries set caps on credit card fees for merchants($\bar{f}_m$) as seen in section 2 and because the observed fees for consumers and membership fees for merchants are close to zero in Korea.[12] Also, I allow the $k'_c < 0$. The negative membership fee means that the payback services or rewarding points can be higher than membership fee, which is often found in reality at some card services.

The consumer surplus in this economy is the sum of the consumer surplus of cash-only traders and cardholders. The analytical functional form with assumption $\mu = 1$ is

$$\begin{aligned}(CS) &= \int_0^{I_0} V_I^{No\ card} dF(I) + \int_{I_0}^{\infty} V_I^{card} dF(I) \\ &= \frac{2(1-\tau_m)}{(1+\tau_c)} \frac{e^{\Phi_1 - 0.5}}{\lambda} (1 - e^{-\lambda I_0} - \lambda I_0 e^{-\lambda I_0}) \\ &+ \frac{2(1-\tau_m)}{(1+\tau_c)} \left(\frac{1+\tau_c}{1+f_c}\right)^{(1-\alpha_0^2)} \frac{e^{\Phi_2 - 0.5 - \lambda I_0}}{\lambda} (1 + \lambda I_0 - k_c)\end{aligned}$$

where Φ_1 and Φ_2 are defined in Appendix A.1. However, it is difficult to recognize intuitively how the consumer surplus changes according to the change of the parameters(τ, f) by using the above formula. This is because the threshold values(α_0, I_0) are the function that depends on those parameter values.

3.4 The Economy with CBDC

The effect of CBDC in this model highly depends on the assumption of J and y. I assume that $J = 0$, so the adoption cost of CBDC for agents are same as cash. Of course,

[12] The annual fee for merchants is mainly paid to telecommunication companies, not to card companies or banks.

sector	cost	cash	card	CBDC
pecuniary	trading fee(+)	0	f	0
	interest benefit(-)	$-i$	0	$i^{CBDC} - i$
non-pecuniary	digital cost(+)	θ	0	0
	privacy benefit(-)	η_1	0	η_2

Table 5: The factors in the transaction cost of trading

Note: Interest benefits include the negative fee or rewarding points such as air mileages and cash-backs. Remark that these are only included the benefits which are proportional to the trading volume since we are not argue the adopting costs but the transaction costs.

digitarization costs for CBDC may occur, so it cannot be said that the adoption costs of cash and CBDC are necessarily the same. In particular, these discussions are crucial in topics of financial inclusion. However, I introduce this assumption because I want to narrow down the topic to the trade-off between private and central bank payment instruments. Consequently, CBDC is a superior substitute to cash in this model.

To clarify the transaction cost of CBDC (y) in trading, I divided the factors that determine the transaction cost as Table 5. The values in Table 5 are normalized as the total card transaction cost is the same as the card transaction fee. All factors of card cost excluding the transaction fee are normalized as zero. So, the transaction fee is zero for public means of payment. The interest benefits exist only on cards(i) and CBDC(i^{CBDC}). θ indicates the digital inconvenience cost of cash. Lastly, the public means of payment give consumers privacy benefits, and these are higher than the private payment instrument. Then, we can rewrite the transaction cost of each payment instrument.

$$
\begin{aligned}
f =& f \\
\tau =& 0 + i + \theta - \eta_1 \\
y =& 0 + (i - i^{CBDC}) - \eta_2
\end{aligned}
$$

I would like to introduce the term CBDC remuneration (ρ) here. The parameter ρ indicates how much the relatively improved transaction cost of CBDC is compared to cash. In mathematically,

$$
\rho(\equiv \tau - y) = i^{CBDC} + \theta + (\eta_2 - \eta_1) \tag{3}
$$

Before getting into the main topic, I will construct the proposition concerned with the conditions for the cashless economy with CBDC.

Proposition 1. *If $J_c = J_m \leq 0$, $\rho_m > 0$, $\rho_c > 0$ and $\bar{L}$(payment limit of CBDC)$\rightarrow \infty$, then CBDC is the dominant payment instrument and the economy with CBDC becomes a cashless economy.*

Proof. Since $J_c = J_m \leq 0$ $\tau_c > y_c$, and $\tau_m > y_m$, cash is dominated by CBDC as a means of payment. Therefore, the agents do not use cash anymore in this condition. □

The proposition 1 said that if one of the conditions fails, a cashless economy will not be guaranteed. It has two implications. First, even if the assumptions, $J \leq 0$ and $\rho > 0$, are realized, the cash will not go away as long as the central bank sets a settlement limit for CBDC in reality. Second, under that assumptions, the agents in the model do not use cash in their trading by the proposition 1. So the introduction of CBDC implies that all cash is replaced by CBDC in the model.

Hereafter I assume that the adoption cost of CBDC is assumed to be equal to 0 as cash. This is not a weird assumption because it implies that the adoption cost of CBDC is equal to cash rather than literally zero. Remark that the adoption cost of cash is normalized to zero. In other words, this means that the central bank will make the accessibility of CBDC equivalent to cash. If the adoption cost of CBDC is greater than cash, we can think about two cases. First, the adoption cost of CBDC is greater than the card. In that case, nobody wants to use the CBDC since it is dominated by the card. Second, the adoption cost of CBDC is less than the card. In that case, as the area where cash is used is divided into CBDC and cash, the model will be more complicated. The quantitative results are only slightly different but most of qualitative results are the same in the case of the assuming zero adoption cost. So, this assumption is mostly harmless in this analysis.

Since the adoption cost of CBDC is assumed to be equal to zero as cash, so whether it is the most dominant payment instrument depends on how much the transaction cost of CBDC is. If $y_c < f_c$, then CBDC is the most dominant payment instrument for consumers. On the contrary, $y_m < f_m$, then CBDC is the most dominant payment instrument for merchants. According to the calibration in section 4, the former case, $y_c < f_c$, is more likely to occur as the CBDC remuneration rate increases, so the latter case is excluded in this analysis. Proposition 2 characterizes the equilibrium when CBDC becomes the superior means of payment for consumers.

Proposition 2. *If $y_c < f_c = 0$ or $\rho > \tau_c$, then CBDC become the most dominant payment instrument for consumers and the following conditions should hold in the equilibrium.*

$$\begin{aligned} &I_0 = 0, \quad k_c' = -\kappa_c, \quad \alpha_1 < \alpha_0 \\ &f_m \textit{ satisfies } -2\kappa_m^2(f_m - d) + 4(y_m - f_m)^3 - \kappa_m^2(y_m - f_m) = 0 \end{aligned} \tag{4}$$

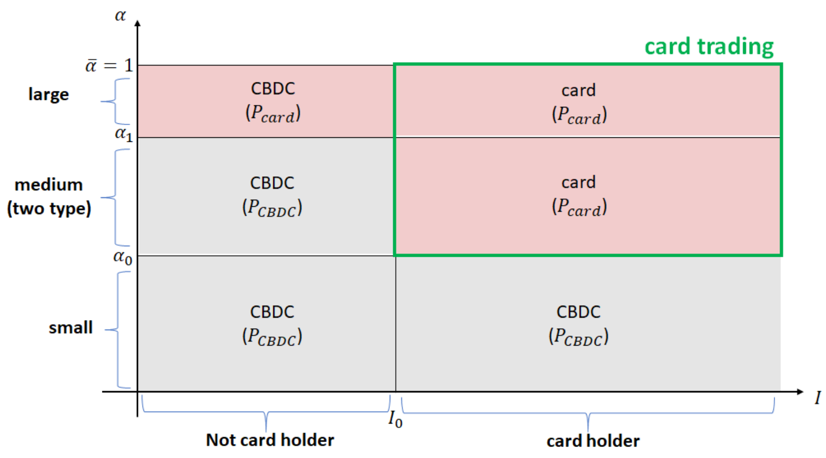
Proof. (sketch for proof) For the consumer, if both the adoption cost and the transaction cost of CBDC are cheaper than one of the card, then no one will use the card. Therefore, the card network adjusts the membership fee(k_c') so that the adoption cost for consumers is equal to zero. It implies there is no hurdle to issuing a card. ($I_0 = 0,\ k_c' = -\kappa_c$) The inequality $\alpha_1 < \alpha_0$ holds due to $y_c < f_c$. According to it, there exist only one threshold (α_1) for merchants because α_0 is meaningless in this case. Hence, the card network profit maximization problem becomes simpler and we can get the first order condition that $-\kappa_m^2(f_m - d) + 2(\tau_m - f_m)^3 - \kappa_m(\tau_m - f_m)^2 = 0$. For more specific deriving, please see Appendix A.2. □

From propositions 1 and 2, we can divide the case into three categories according to the level of CBDC remuneration rates. In the first case, CBDC still has higher transaction costs for consumers than the card. ($y_c > f_c$) This is shown in the upper panel of Figure 4. In this case, it is similar to the cash and card economy, and cash is perfectly replaced only with CBDC. So, the calculation in the CBDC economy is the same as the cash and card economy except that we need to replace τ_m and τ_c with y_m and y_c. In the second case, CBDC becomes the most dominant payment instrument. ($y_c < f_c$) This is shown in the middle panel of Figure 4. in this case, all consumers hold the card and pay by CBDC for small merchants and by card for large merchants. The calculation for the equilibrium is easily done by using Proposition 2. The last one is the case where the profit of the card network becomes negative and is expelled. In this case, CBDC becomes the only means of payment. Therefore, it is similar to the cash-only economy, but the transaction cost is reduced. These three cases occur sequentially as the CBDC remuneration increases.

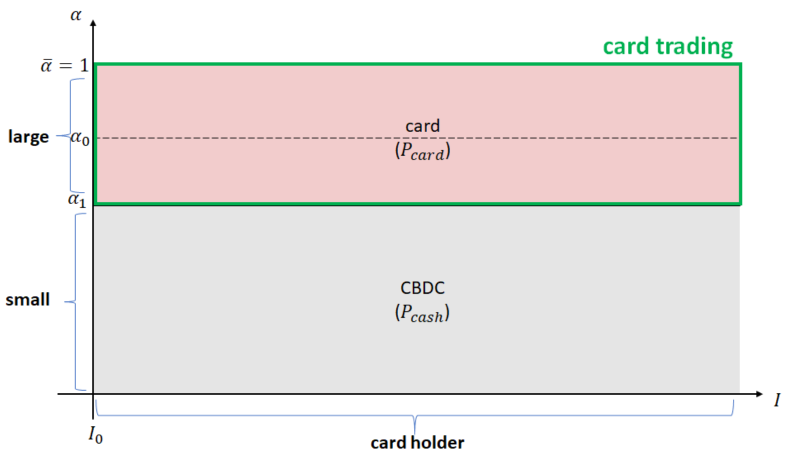
Before closing the model explanation section, I construct the social planner problem as follows.

$$\begin{aligned}\max_{\{i_m^{CBDC}, i_c^{CBDC}\}} \quad & \int_0^{I_0} \left[V_I^{No\ card} dF(I) + \int_{I_0}^{\infty} V_I^{card} dF(I) \right] \\ & + \Pi(\tau_c - \rho_c, \tau_m - \rho_m, \kappa_m, \kappa_c, d) \\ & - (i_m^{CBDC} + i_c^{CBDC}) \left[\int_0^{\alpha_0} \int_0^{\infty} P_{CBDC} x_{\alpha,I}\, dF(I)\, dG(\alpha) \right. \\ & \qquad \left. + \int_{\alpha_0}^{\alpha_1} \int_0^{I_0} P_{CBDC} \cdot x_{\alpha,I}\, dF(I)\, dG(\alpha) \right]\end{aligned} \tag{5}$$

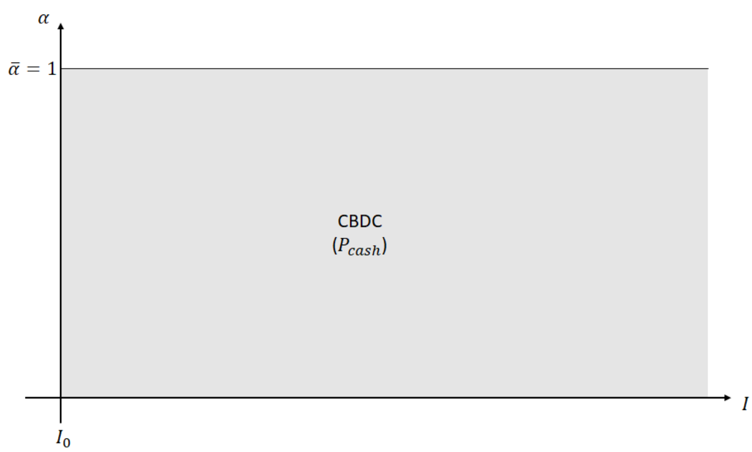
with considering the best response correspondences of the private agents. The first term means the total sum of the consumer surplus, so the central bank desire to increase the consumer surplus. The next term means the profit function of the card network. This term is significant in two respects. First, the profit of the monopolist can be interpreted as producer surplus. In addition, the monopoly profit of financial companies is an incentive for the development of payment and settlement technology, so it has a positive effect on consumer welfare in the long run. Second, if the card service does not exist, social



(a) $y_c > f_c$ case ($\rho < \tau_c$)



(b) $y_c \leq f_c$ case ($\rho \geq \tau_c$)



(c) the card network exist case ($\Pi = 0,\ \rho \geq \rho^{exit}$)

Figure 4: Three cases of the CBDC economy

efficiency will decrease.[13] Therefore, this term should be included in order to take into account the inefficiency of exit. The last one means the cost of paying benefit fees on CBDC. This operating cost can also be interpreted as the cost of managing and operating the payment and settlement system. It can also be understood as an opportunity cost that can be given in the form of a lump-sum transfer rather than being used for CBDC benefit payment. In this paper, the optimal CBDC rate is defined as the solution of (5).

4 Quantitative Results

4.1 Calibration

Notation	Definition	Value	Source / Target
τ_m	Merchant transaction cost of cash	3.02%	share of card holders
τ_c	Consumer transaction cost of cash	0.59%	& card trading
d	Card service marginal cost	0.50%	
κ_m	Merchant non-fee adoption cost of card	1.50%	assumptions
κ_c	Consumer non-fee adoption cost of card	0.05%	
f_m	Merchant transaction cost of card	2.06%	KCFA
f_c	consumer transaction cost of card	0.00%	observed in reality
μ	Merchant marginal cost of production	1	normalizing
$1/\lambda$	Average annual income(Million Won)	61.25	SFLC in 2021

Table 6: Parameter Values for Model CBDC Calibration

Note: The adoption costs are the relative ratio to the mean income. The merchants' marginal cost only affects the level of consumers' utility values.
Source: Bank of Korea, Korea Statistics, KCFA

Table 6 summarized the calibration strategies. The most important parameters in this model are τ_m and τ_c. However, since they are unobservable in reality, I matched them with the momentum conditions, the share of card holders, 84.5% and the credit card holder ratio, 74.4%. These two values can be seen in Figure 2. In the model, the share of the cardholder is calculated as

$$\text{(share of cardholders)} = \int_{I_0}^{\infty} I \; dF(I) = \exp(-\lambda I_0)$$

[13] Since $\tau_m + \tau_c \geq d$, the card economy is more efficient than cash only economy.

and the share of card trading is calculated as

$$
\begin{aligned}
(\text{share of card trading}) &= \frac{(\text{card trading})}{(\text{cash trading}) + (\text{card trading})} \\
&= \frac{\int_{\alpha_0}^{1}\int_{I_0}^{\infty}\frac{\alpha(I-K_c)}{E[\alpha](1+f_c)}dFdG}{\int_0^1\int_0^{I_0}\frac{\alpha I}{E[\alpha](1+\tau_c)}dFdG + \int_0^{\alpha_0}\int_{I_0}^{\infty}\frac{\alpha(I-K_c)}{E[\alpha](1+\tau_c)}dFdG + \int_{\alpha_0}^{1}\int_{I_0}^{\infty}\frac{\alpha(I-K_c)}{E[\alpha](1+f_c)}dFdG} \\
&= \frac{\frac{(1-\alpha_0^2)}{(1+f_c)}\frac{e^{-\lambda I_0}}{\lambda}(1+\lambda_0 I_0 - k_c)}{\frac{1}{(1+\tau_c)}\left[\frac{1}{\lambda} - \frac{e^{-\lambda I_0}}{\lambda}\{(1-\alpha_0^2)(1+\lambda I_0) + \alpha_0^2 k_c\}\right] + \frac{(1-\alpha_0^2)}{(1+f_c)}\frac{e^{-\lambda I_0}}{\lambda}(1+\lambda_0 I_0 - k_c)}.
\end{aligned}
$$

The matching performance is shown in Table 7.[14]

	Target	Model
share of card holders(%)	84.50	84.50
share of card trading(%)	74.40	74.44

Table 7: Matching performance

d, κ_m, and κ_c are kind of ad hoc assumptions. In section 5, I will relax these assumptions and test the sensitivity analysis of the results. So, these benchmark number is not significant to interpret the qualitative results. f_m can be observed in Korea Credit Financial Association. Since the actual average card fee is the same as the regulatory upper limit, I have checked whether the constraint $f_m \leq \bar{f}_m$ holds with equality. f_c is set to zero as I mentioned before. μ and $1/\lambda$ are concerned with the level values. μ is normalized for simplifying the calculation which is the same as Li et al.(2020). The source of $1/\lambda$ is the Survey of Household Finances and Living Conditions.

In equilibrium, $k_c' = 0.0102\% \;\Leftrightarrow\; K_c' = 6,248\ Won$. In the reality, there are many benefits such as returning the annual fee to consumers. Therefore, a low level of k reflects the reality well in that the average low annual fee is closed to zero. α_0 was calculated as 0.4972 which is similar to the one of Li et al.(2020). I_0 was 10.3526 in equilibrium. This means that a person whose average annual income was higher than 10 million won owns a credit card. Overall, these endogenous values are within reasonable ranges.

4.2 Main results

To simplify the quantitative analysis, I introduce the following assumption 1 in this subsection.

Assumption 1. *(a) θ_m and θ_c are negligible and η_1=η_2 ($\rho \approx i^{CBDC}$), (b) $\rho_m = \rho_c$*

[14]Unfortunately the model is parsimonious, I cannot suggest the untargeted momentum which can be compared to real statistics.

For (a), if we assume that the digital inconvenience cost is small enough to ignore ($\theta \to 0$) and that the privacy benefit of CBDC is same with cash.[15], then the CBDC remuneration can be approximated by the CBDC rate($\rho \approx i^{CBDC}$) by the equation (3). We will look at the case where θ is not equal to 0 later. The condition (b) is sort of the feasibility condition. This is because it is practically difficult for the central bank to differentiate between individuals and corporations for giving the CBDC remuneration rate. However, I suggest the outcome where the two rates are different when we examine the channel of the CBDC remuneration rate on social utility in the following subsection.

Figure 5 shows the equilibrium results according to the increase in CBDC remuneration rate. The optimal CBDC remuneration (ρ^*) is equal to $59bp$ which is the same as the transaction cost of cash τ_c and the exiting-trigger CBDC remuneration is equal to 1.46%. I would like to remark two things here.

First, the social welfare function has a kink point on the $\rho^* = \tau_c$. It implies that when the central bank pays the CBDC remuneration above the transaction cost of cash for consumers, the marginal cost of raising the CBDC remuneration increases more than the marginal benefit. This suggests that it is necessary for the central bank to empirically and accurately estimate the transaction cost of cash versus the card at the current level before designing the remuneration of CBDC.

Second, I would like to examine the social characteristics in optimal equilibrium. At the optimal CBDC remuneration level, $k_c = k_c' + \kappa_c = 0$. So, the opportunity cost of card issuance does not exist, and all consumers can easily access card financial services. In light of the definition of financial inclusion, the availability and equality of opportunities to access financial services, the introduction of CBDC has a positive effect on financial inclusion.[16]

In addition, the proportion of transactions in CBDC (42.5%) is not very large, but rather, the proportion of transactions in cards is above half. Social welfare increases as a direct effect of the introduction of CBDC, but changes in card service also increase social welfare, so the card network is not expelled or the share of card transactions is not greatly shrunk in the optimal equilibrium. In other words, if the share of CBDC transactions exceeds half, it means that they are paying too much CBDC compensation and it can also be interpreted as a signal that may cause a crisis of card network's exit. In the model, the share of CBDC trading at the time the card network exits is 70.7%.

[15] Most of the preceding studies presume that the privacy benefits of CBDC and cash are the same.(e.g. Williamson(2022)) However, the two values may be different. In this case, you can interpret the conclusion of this paper as putting this difference into $\rho - i^{CBDC}$.

[16] These results should be interpreted only as a result of positive analysis, and whether this is desirable or not is needed to be discussed independently. This is because, in this model, there is no case in which the consumer fails to repay the credit card bills. The reason why most card networks restrict the issuance of credit cards to the low-income class is due to the risk of defaults. Therefore, it is difficult to examine in this model whether these kinds of financial inclusion are desirable.

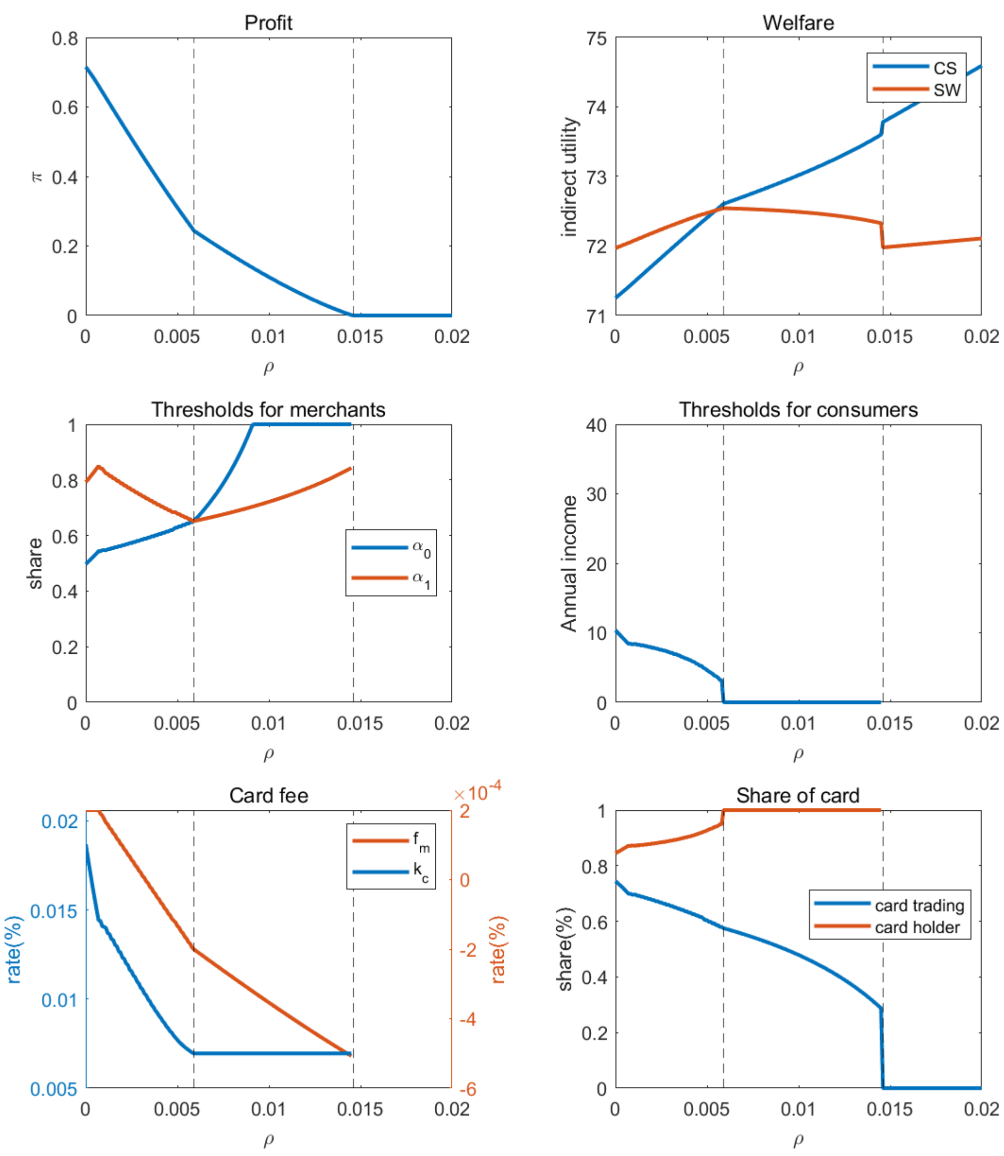


Figure 5: CBDC remuneration simulation results

Note: The left dashed line indicates $\rho = \tau_c$ and the right dashed line indicates exit rate of the card network.

4.3 Decomposition

In this subsection, I decompose the variation of social welfare into various channel. From this analysis, We will get a better idea of how the mechanism works. How does an increase in CBDC remuneration affect social welfare? Social welfare consists of the consumer surplus, profit of the card network, and the CBDC expense.($SW = CS + \Pi - expense$) The consumer surplus increases with the CBDC remuneration, while the profit and expense decrease. The CBDC costs consist of a price factor and a quantity factor. The former is affected by the CBDC remuneration rate more directly and the latter is affected through the increase of the share of the CBDC transaction.

To break down the increase of the consumer surplus($CS(y_m, y_c, f_m, k_c', I_0, \alpha_0, \alpha_1)$), I exploited the following total differential formula.

$$
\begin{aligned}
\frac{dCS}{d\rho} = & \underbrace{\frac{\partial CS}{\partial y_m}\frac{\partial y_m}{\partial \rho} + \frac{\partial CS}{\partial y_c}\frac{\partial y_c}{\partial \rho}}_{\text{central bank direct effect}} \\
& + \underbrace{\frac{\partial CS}{\partial f_m}\frac{\partial f_m}{\partial \rho} + \frac{\partial CS}{\partial k_c'}\frac{\partial k_c'}{\partial \rho}}_{\text{card network indirect effect}} + \underbrace{\frac{\partial CS}{\partial I_0}\frac{\partial I_0}{\partial \rho}}_{\text{consumer indirect effect}} + \underbrace{\frac{\partial CS}{\partial \alpha_0}\frac{\partial \alpha_0}{\partial \rho} + \frac{\partial CS}{\partial \alpha_1}\frac{\partial \alpha_1}{\partial \rho}}_{\text{merchant indirect effect}}
\end{aligned}
\tag{6}
$$

The first curly bracket is primarily a direct effect of cash being replaced by a better means of payment, CBDC. So, these are concerned with the central bank's choice variable, y_m, and y_c. The last three parts are indirect effects of the introduction of CBDC because these effects are increases in consumer welfare according to the reactions of private economic agents. The second curly bracket is coming from reducing the card fee for merchants and membership fee for consumers. The card network offers better benefits to consumers and merchants, giving up a few of its monopoly profits to defend their business from the threat of alternative payment methods. Also, the change of the threshold for issuing a card contributes to the increase in total consumer surplus, which is stated in the third curly bracket. It can be seen as a kind of financial inclusion effect for consumers because those who have the card get a higher level of utility than those who do not. Lastly, merchants' actions also contribute to improving consumer surplus through the goods market.

The actual calculation of (6) is implemented by the following equation.

$$\frac{\triangle CS}{CS_0} = \underbrace{\frac{CS(y_m, y_c, f_m, k_c', \alpha_0, \alpha_1, I_0) - CS(y_m^0, y_c^0, f_m, k_c', \alpha_0, \alpha_1, I_0)}{CS_0}}_{\text{central bank direct effect}} \tag{7}$$
$$+ \underbrace{\frac{CS(y_m, y_c, f_m, k_c', \alpha_0, \alpha_1, I_0) - CS(\cdots, f_m^0, k_c'^0, \cdots)}{CS_0}}_{\text{card network indirect effect}} +$$
$$+ \underbrace{\frac{CS(y_m, y_c, f_m, k_c', \alpha_0, \alpha_1, I_0) - CS(\cdots, I_0^0, \cdots)}{CS_0}}_{\text{consumer indirect effect}}$$
$$+ \underbrace{\frac{CS(y_m, y_c, f_m, k_c', \alpha_0, \alpha_1, I_0)) - CS(\cdots, \alpha_0^0, \alpha_1^0)}{CS_0}}_{\text{merchant indirect effect}} + (residual)$$

Table 8 suggests the decomposition results and Figure 6 is a graphical representation of the consumer surplus component values in Table 8. Overall, the consumer surplus(CS) growth rate as CBDC rate increases. As we can see in Figure 6, the main driving force of increasing the consumer surplus is the direct effect, the blue bars. The contribution rate of this effect for the consumer surplus is above 62%. The next most important factor is the card fee. The quantitative effect of other factors is not conspicuous, and in particular, the quantitative effect of financial inclusion on consumer surplus from the effect of lowering the barriers to entry to consumers was found to be very insignificant.

The reason for the decrease in social welfare above the optimal remuneration rate is mainly due to the increase in the cost of CBDC and the decrease in the use of cards by merchants compared to the slowdown in the consumer surplus increasing factors. An excessively high CBDC remuneration rate means that the cost burden on the central bank grows high because the trading share of CBDC increases rapidly. So, it becomes inefficient comparing to the directly transferring money to consumers. Also, as the CBDC

	SW	CS						Π	expense
			y_m, y_c	f_m, k_c'	I_0	α_0, α_1	res		
20bp	0.303	0.641	0.399	0.179	0.005	0.069	-0.012	-0.229	-0.110
40bp	0.583	1.289	0.800	0.421	0.008	0.108	-0.048	-0.455	-0.251
60bp	0.800	1.895	1.202	0.649	0.009	0.036		-0.660	-0.436
80bp	0.772	2.164	1.606	0.761	0.009	-0.212		-0.754	-0.638
100bp	0.752	2.307	1.809	0.816	0.009	-0.327		-0.798	-0.757

Table 8: The contribution decomposition for the social welfare

Note: The unit in this table is the percent point(%p) increase in social welfare. When the CBDC remuneration exceeds 59bp, it is difficult to calculate the contribution as in (7) due to the regime change. Therefore, the contribution of residual and α are combined.

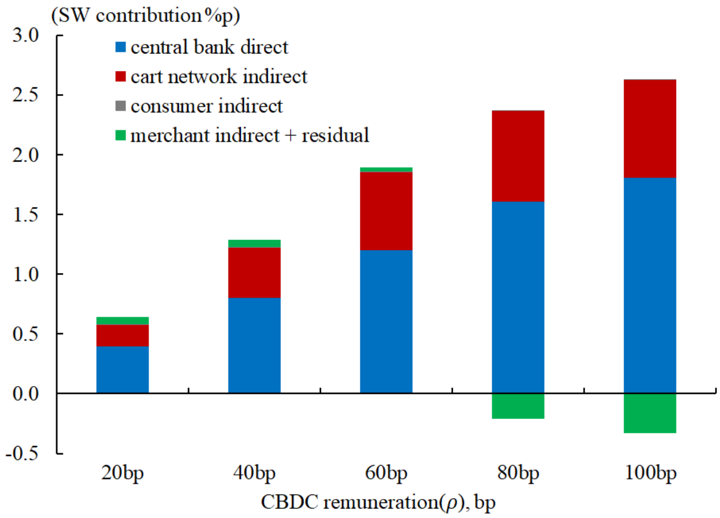


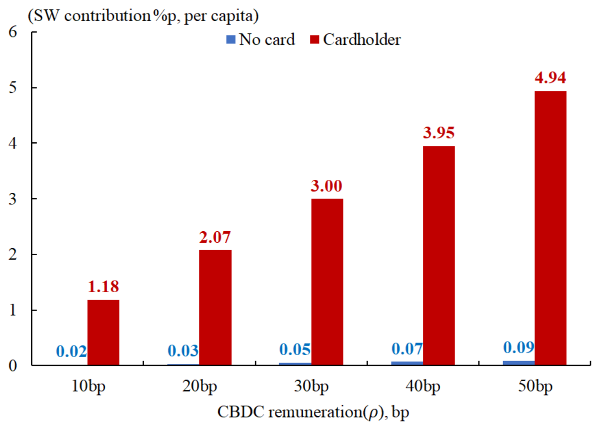
Figure 6: The contribution of CS components

remuneration rate increases, α_1 decreases but α_0 continues to rise. Below the optimal remuneration rate, the increase in the consumer surplus from these effects due to the downward effect of α_1 was larger than one of α_0. However, when the optimal remuneration rate is exceeded, α_1, which ultimately determines whether to use the card or not for merchants, continues to rise at a level higher than the initial α_0 value, so it acts as a consumer surplus reduction factor.

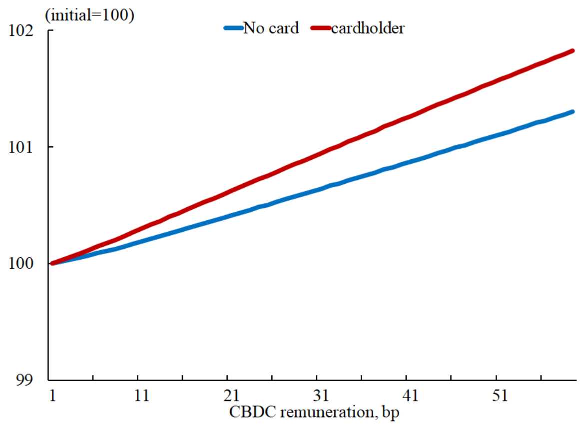
As a consequence of the result of consumer surplus variations, two important implications are derived for the introduction of CBDC. First, CBDC can unleash financial inclusion for consumers. With the introduction of CBDC, the card network eventually lowers the adoption cost of the card for consumers by subsidizing the membership benefit, which drags down the threshold income level for card issuance and increases the number of card users.

Second, the main source of growing consumer surplus is the CBDC and card usage benefits from the central bank and card network, so the benefits are concentrated on the high-income class with a large transaction volume. It implies that the introduction of CBDC would act like a reverse tax. The left panel of the Figure 7 shows this effect. Furthermore, the fact that the utility improvement for cardholders is much greater than no cardholders even after controlling for income difference makes it hard to deny the regress-taxation-like characteristics of CBDC. Since this model is assumed that the Cobb-Douglas utility function, the indirect utility function is unit elastic to income. Therefore, it is possible to decompose the consumer surplus into income-related parts and factors that determine utility per unit income($V(\cdot)$) as follows.

$$CS(I) = I \times V(\cdot) \quad \text{where} \quad V(\cdot) \text{ does not depend on } \quad I$$



(a) SW contribution



(b) the utility per income unit

Figure 7: No card vs Cardholders

Note: The figures in the left panel are recalculated so as to be compared in terms of per capita by adjusting the mass of each consumer ratio.

The right panel of Figure 7 shows the divergence of $V^{No\ card}(\cdot)$ and $V^{cardholder}(\cdot)$. This clearly shows that the welfare improvement due to the introduction of CBDC comes not only from CBDC but also from the improvement of card service.

Finally, by setting the case where the CBDC remuneration rate is paid asymmetrically, I will decompose the effect of the CBDC remuneration rate paid to consumers and merchants. Table 9 shows the percentage values of the model in which one side of the CBDC remuneration payment is shut down.

To begin with, looking at Social welfare, if the CBDC remuneration rate is not paid to the consumer, it can be seen that the increasing effect is shrunk. This is mainly due to the difference in consumer surplus. When the CBDC remuneration rate is paid to consumers, the CS rises significantly, but when it is paid only to merchants, the increased range of CS is limited. This is because, in the model of this paper, merchants are perfectly competitive, and the main source of social welfare is created through consumers.

On top of that, When the CBDC remuneration rate is paid only to one side, the response of the card network is also different. In the case of CBDC remuneration paying only to consumers, the membership cost of consumers(k_c') was greatly reduced. The effect of lowering the threshold at which consumers are issued cards also occurs only when the CBDC remuneration rate is paid to consumers. On the other hand, in the case of CBDC remuneration paid only to merchants, the card fee of merchants (f_c) was greatly lowered. As a result, if only one side of the CBDC remuneration is paid, the card network adjusts its action variables by using the two-sided market characteristics, so that the decision-making of merchants and consumers is asymmetrically reacted to one side.

However, there are also variables in which the two cases are similar. As mentioned

	SW	CS	Π	expense	f_m	k'_c	I_0	Card trading
total(50bp)	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
$\rho_c = 0$	10.3	30.9	50.7	41.0	47.7	35.7	16.1	49.8
$\rho_m = 0$	103.2	72.3	51.8	41.8	40.0	101.6	125.7	53.6
residual	-13.5	-3.2	-2.5	17.2	12.3	-37.3	-41.8	-3.4

Table 9: Asymmetric CBDC rate

Note: The values means the percentage for total variations.

earlier, the card network responds according to the characteristics of the two-sided market. Due to this characteristic, the profit of the card network and the share of card trading is relatively equivalent between the two cases. Consequently, as the share of the CBDC transaction volume is similar in both cases, the CBDC expense seems not to make much difference.

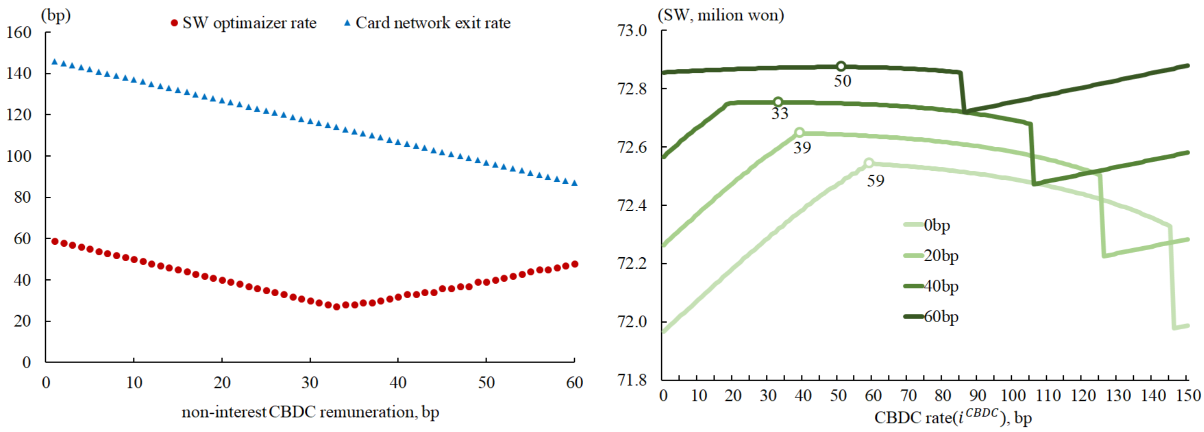
5 Sensitivity analysis

I will suggest the results of a sensitivity test by relaxing some assumptions. The method in this section has used a kind of perturbation method based on the benchmark model.

5.1 Nonpecuniary Benefits($\theta + (\eta_2 - \eta_1)$)

In this subsection, I detach the CBDC interest benefit or benefit fees(i^{CBDC}) from the CBDC remuneration(ρ) clearly. To do that, the assumption that digital benefits are negligible ($\theta \to 0$) and that the privacy benefits of cash and CBDC are the same($\eta_1 = \eta_2$) are relaxed. From now on, $\theta + (\eta_2 - \eta_1)$ will be named nonpecuniary CBDC remuneration. The most important difference between CBDC rate and nonpecuniary CBDC remuneration is that the former includes the direct effect on CBDC expense, but the latter excludes the cost and gives benefit only.

The left panel in Figure 8 shows the optimal CBDC rate when the nonpecuniary CBDC remuneration rises. If the nonpecuniary CBDC remuneration is under 32bp, then the optimal CBDC rate can be easily calculated by $\rho^* = \tau_c - [\theta + (\eta_2 - \eta_1)]$. That is, as digital and privacy benefits increase, the optimal CBDC rate declines. However, if the nonpecuniary CBDC remuneration exceeds 32bp, the kink point is no longer at the optimal level. Rather, the optimal CBDC rate gradually increases as the nonpecuniary CBDC remuneration rises. We can recognize the change in the social welfare function in the right panel of Figure 8. Each line shows the social welfare function when changing the non-pecuniary CBDC remuneration. The kink point gradually moves to the left, but as the nonpecuniary CBDC remuneration increases, there exists a local maximum point



(a) The optimal CBDC rate (b) The change of SW function

Figure 8: The change of the nonpecuniary CBDC remuneration

Note: The circles and figures in the right panel indicates the optimal CBDC rate given the non-pecuniary CBDC remuneration.

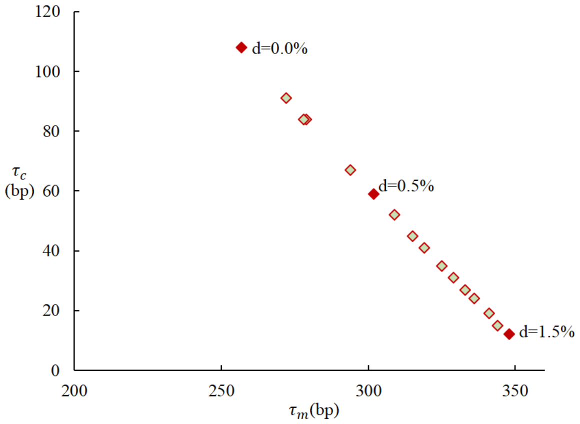
at the right side of the kink point.

These results show that the optimal CBDC rate decision may change if the benefits of noninterest-bearing CBDC are large. It implies that the accessibility and privacy policy for CBDC are crucial to measure the impact of CBDC on the payment system.

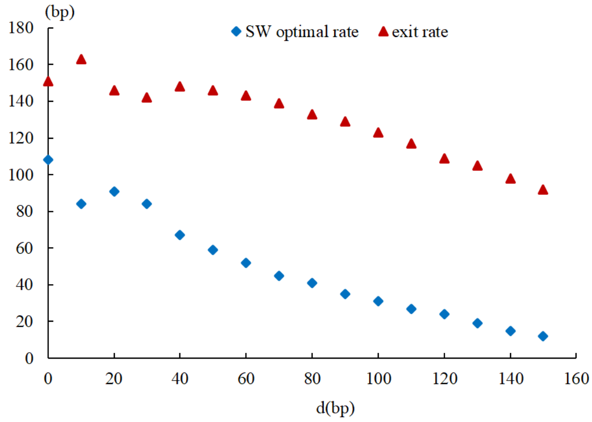
5.2 Marginal Cost of the Card Service(d)

Let's look at how the previous results have been changed when assumptions about the marginal cost of card services. Unlike θ and η, the variation of d can affect the value of τ_m and τ_c, which are parameters matched by the real momentum in calibration. Therefore, we need to recalibrate first, and then the optimal CBDC rate and exit rate can be calculated. I simulated the variation of d from 0.0% to 1.5% by 10bp increments.

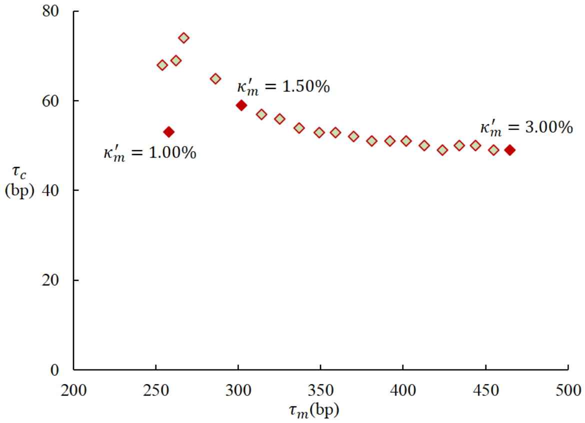
The upper panel of Figure 9 suggests the outcome of the variation of d. As the marginal cost of card service increases, the calibrated transaction cost of cash for consumers (τ_c) gets lower while the calibrated transaction cost of cash for merchants (τ_m) gets higher. The CBDC rate derived according to these results was found to decrease as d increases. Also, a similar effect appeared for the exit rate. The larger d means that the marginal cost of the payment service is larger, so the profit of the monopolist is not pretty much. Therefore, in this situation, even if a high CBDC rate is paid, social welfare does not improve significantly.



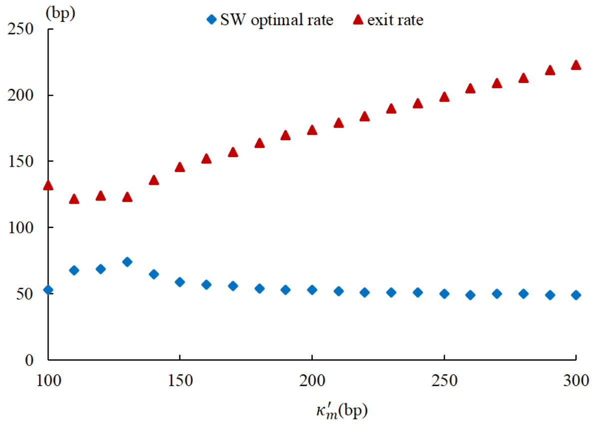
(a) Calibrated τ with variation d



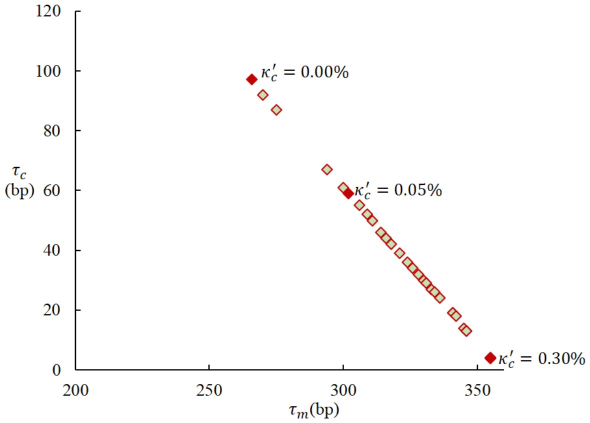
(b) Optimal CBDC rate with variation d



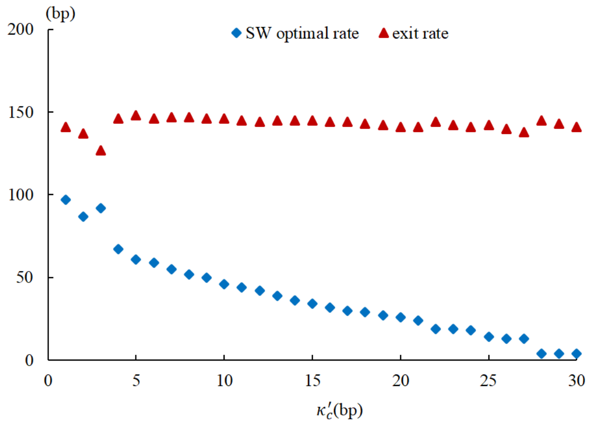
(c) Calibrated τ with variation κ_m



(d) Optimal CBDC rate with variation κ_m



(e) Calibrated τ with variation κ_c



(f) Optimal CBDC rate with variation κ_c

Figure 9: Sensitivity Test

5.3 Card nonmembership fee adoption cost (κ)

Next, the assumption of the parameters about the adoption costs of holding a card that is not paid to the card network is relaxed. You can see the results of the variation of κ_m in the middle panel of Figure 9. At a low level of κ_m, the matched τ_m and τ_c look mesh slightly, but as κ_m gets higher, it is clear that τ_c converges to 50bp and τ_m increases consistently. Since κ_m is the adoption cost of the card for merchants, it can be seen that the optimal CBDC rate is relatively less sensitive to an increase in κ_m.

In the case of κ_c, the results have appeared in the lower panel of Figure 9. As you can recognize, the relationship of matched τ_m and τ_c appeared similar to that of d. Given the share of card holders, as the opportunity cost of a consumer's card issuance decreases, then the difference between the transaction cost of the card and cash for consumers diverges. At the same time, the calibrated transaction cost of cash for merchants increases. The socially optimal CBDC rate generally decreases when κ_c rises.

Parameter			optimal CBDC rate	
notation	meaning	domain	direction	range
$\theta + (\eta_2 - \eta_1)$	the non-pecuniary CBDC remuneration	[0.00, 0.06]	(-)/(+)	[0.27, 0.59]
d	the card service marginal cost	[0.00, 1.50]	(-)	[0.12, 1.08]
κ_m	the adoption cost of card for merchants	[1.00, 3.00]	unclear	[0.49, 0.74]
κ_c	the adoption cost of card for consumers	[0.00, 0.30]	(-)	[0.04, 0.97]

Table 10: The summary for sensitivity checks

Note: 1. The unit of the figures in the table is a percentage(%). The meaning of percentage for κ_m and κ_c refers to the share of average income. 2. κ_m under 1.0% has a poor matching performance, so I exclude them.

Table 10 summarizes the sensitivity checks results. This table shows how the optimal CBDC rate changes when assumptions about the parameters are perturbed. It should be noted that the optimal CBDC rate range of digital and privacy benefit is corresponding to the range of the optimal CBDC remuneration rate, but the ranges of the optimal rate of the other parameters indicate the range of the upper limit of the optimal CBDC remuneration rate.

5.4 Using the Debit Card Fee

Regarding the criticism in previous studies that simple comparison is not reasonable due to the credit function included in credit cards, many previous studies presented the results when calibration is used only as a statistic for a debit card. Here, I simply present the results of using the debit card fee rate of 1.47%. The estimated parameter values were

$\tau_m = 2.73\%$ and $\tau_c = 0.28\%$, and the optimal CBDC remuneration rate was estimated to be 28bp, which was slightly lower than the previous benchmark.

6 Conclusion

In this report, I looked into the impact of the introduction of CBDC in Korea on the payment system such as card fees, profit of card network, and social welfare by extending the static model in Li et al.(2020). I reorganize the results of this paper as following three implications.

First, the economic policy authorities should consider the reaction of private agents in implementing the policy. It is consistent with the implications of many previous studies on the macroeconomic and game theory. The central bank needs to consider the expected behavior of the card network, consumers, and merchants in the payment market when they initiate the introduction of CBDC. Furthermore, it was shown that CBDC can improve social welfare through indirect effects according to the reactions of private agents. This is because the card network has monopoly profits, and the central bank has the effect of improving social welfare by eroding this monopoly power. This analysis is also consistent with previous studies.

Second, I propose two conflicting effects of CBDC on redistribution policies. If the card network lowers the condition for card issuance with the introduction of CBDC, low income consumers who used only cash before the CBDC introduction will be able to retain a card. In this regard, the CBDC can foster financial inclusion. However, as the benefits from the introduction of CBDCs are mainly concentrated on the high income class which has a large amount of trading volume, CBDC may play a kind of regressive subsidy and increase inequality. Therefore, if monetary authorities consider inequality rather than overall social welfare, the optimal CBDC rate may need to be set low.

Third, the optimal CBDC rate depends on the specific design of CBDC. According to the results of this paper, the optimal CBDC remuneration rate was the transaction cost of trading cash relative to the card. However, if the non-pecuniary CBDC remuneration benefits exist, then the optimal CBDC rate is not equal to the optimal CBDC remuneration rate. In other words, the optimal CBDC rate depends on how much the central bank pays attention to the digital accessibility and protection of private information of the CBDC. Also, it suggests that even if the CBDC is a kind of noninterest-bearing, it can sufficiently achieve the improvement or maximization of social welfare.

However, there are still many limitations in this paper. The CBDC rate used in this model means the percentage as a unit of fee for payment volume. Since interest is paid on an outstanding account, the figures in this paper are not interchangable with the CBDC interest rate. In addition, this paper is silent on the way to estimate the parameters needed to determine the specific optimal remuneration rate. Also, the payment cap of CBDC and the short-term loan function of credit cards were not taken into account in this paper.

The payment market are exposed to the unignorable impact of introduction of CBDC. Although monetary policy and financial stability related to bank lending are also significant issues for central banks, at least in the CBDC analysis, it is most significant to examine how it changes the payment market and how it affects social welfare and financial inclusion. In this regard, I think this paper contributes to CBDC studies for analyzing the effect of the introduction of CBDC on the Korean payment market based on a specific quantitative model for the first time.

References

Andolfatto, David Andolfatto(2018), “Assessing the Impact of Central Bank Digital Currency on Private Banks”, Federal Reserve Bank of St. Louis Working Paper 2018-026

Auer, Raphael, Holti Banka, Nana Yaa Boakye-Adjei, Ahmed Faragallah, Jon Frost, Harish Natarajan and Jermy Prenio(2019), “Central Bank Digital Currencies: A New Tool in the Financial Inclusion Toolkit?”, BIS FSI Insights on policy implementation No 41

BOK(2022), “Survey Result on the Payment and Mobile Financial Service in 2021”, Payment and settlement research No. 2022-1

Barontini, Christian and Henry Holden(2019), “Proceeding with Caution – A Survey on Central Bank Digital Currency”, BIS Papers No 101 2019

Bergman, Mats(2020), “Competitive Aspects of an E-krona”, Sveriges RIKS BANK Economic review 2020:2

Broadbent, Ben(2016), “Central Banks and Digital Currencies”, Speech by Mr Ben Broadbent, Deputy Governor for Monetary Policy of the Bank of England, at the London School of Economics, London, 2 March 2016.

Chiu, Jonathan, S. Mohammad R. Davoodalhosseini, Janet Hua Jiang, and Yu Zhu(2019), “Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment”, Bank of Canada Staff Working Paper 2019-20

Choi, Syngjoo, Bongseop Kim, Young Sik Kim, and Ohik Kwon(2022), “Central Bank Digital Currency and Privacy: A Randomized Survey Experiment”, 2022

Dyson, B. and Hodgson, G. (2016), “Digital cash: Why Central Banks Should Start Issuing Electronic Money”,Positive Money.

ECB(2020), “Study on the Payment Attitudes of Consumers in the Euro Area (SPACE)”, December 2020

Hayashi, Fumiko(2021), “Cash or Debit Cards? Payment Acceptance Costs for Merchants”, Federal Reserve Bank of Kansas City, 2021

Khiaonarong, Tanai and David Humphrey(2022), "Falling Use of Cash and Demand for Retail Central Bank Digital Currency", IMF Working Paper 2022-27

Lagos, Ricardo and Randall Wright(2005), "A Unified Framework for Monetary Theory and Policy Analysis" *Journal of Political Economy*, 113, pp.463-484.

Li, Bin Grace, James McAndrews, and Zhu Wang(2020), "Two-Sided market, R&D, and Payments System Evolution", *Journal of Monetary Economics*, Volume 115, 2020, pp.180-199

Rochet, Jean-Charles and Jean Tirole (2002), "Cooperation among Competitors: Some Economics of Payment Card Associations", *RAND Journal of Economics*, 33(4), pp.549-570.

Rochet, Jean-Charles and Jean Tirole (2003), "Platform competition in two-sided markets" *Journal of European Economics Association*, Volume 1, Issue 4, pp.992-1029.

Rochet, Jean-Charles and Julian Wright(2010) "Credit Card Interchange Fees", *Journal of Banking & Finance* Volume 34, pp.1788–1797

Williamson, Stephen(2022), "Central Bank Digital Currency: Welfare and Policy Implications"

Appendix

A.1. Deriving the Consumer Surplus

In this appendix, I assume $\mu = 1$. First, we look into the case of the cash only economy. Since $P_{cash} = 1/(1-\tau_m)$ and $E[\alpha] = 0.5$, then

$$\begin{aligned}
\ln V_I^{cash} &= \int_0^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha I}{E(\alpha)(1+\tau_c)P_{cash}}\right) dG(\alpha) \\
&= \int_0^1 2\alpha \ln \alpha dG(\alpha) + \int_0^1 2\alpha \ln\left(\frac{2I(1-\tau_m)}{(1+\tau_c)}\right) dG(\alpha) \\
&= \left[2\alpha^2\left(\frac{\ln\alpha}{2} - \frac{1}{4}\right)\right]_0^1 + \ln\left(\frac{2I(1-\tau_m)}{(1+\tau_c)}\right) \\
&= -\frac{1}{2} + \ln\left(\frac{2I(1-\tau_m)}{(1+\tau_c)}\right).
\end{aligned}$$

By taking the exponential function on both sides, the level value of the indirect utility function is derived.

$$V_I^{cash} = I\frac{2(1-\tau_m)}{(1+\tau_c)}e^{-0.5}$$

Since the log utility function is assumed, the income elasticity of indirect utility is equal to 1, and the level value of indirect utility is proportional to the income level. Take the integral over all consumers, and get the total consumer surplus.

$$(CS) = \int_0^\infty V_I^{cash} dF(I) = \left(\frac{1}{\lambda}\right)\left(\frac{2(1-\tau_m)}{(1+\tau_c)}\right)e^{-0.5}$$

Next, we look into the case of card and cash economy. Here, I substitues τ as y because we want to see the case of introduction of CBDC. For cash-only consumers, the log indirect utility function is

$$\begin{aligned}
\ln V_I^{Notcard} &= \int_0^{min\{\alpha_1,1\}} \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha I}{E(\alpha)(1+y_c)P_{CBDC}}\right) dG(\alpha) \\
&\quad + \int_{min\{\alpha_1,1\}}^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha I}{E(\alpha)(1+y_c)P_{card}}\right) dG(\alpha) \\
&= \int_0^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha I(1-y_m)}{E(\alpha)(1+y_c)}\right) dG(\alpha) + \int_{min\{\alpha_1,1\}}^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{P_{CBDC}}{P_{card}}\right) dG(\alpha) \\
&= \ln\left[I\left(\frac{1-y_m}{1+y_c}\right)2e^{\Phi_1 - 0.5}\right]
\end{aligned}$$

where $\Phi_1 = \int_{min\{\alpha_1,1\}}^1 \frac{\alpha}{E(\alpha)} \ln\left(\frac{P_{CBDC}}{P_{card}}\right) dG(\alpha)$. Note that Φ_1 does not depend on the income level, I. For card holders, the log indirect utility function is

$$\begin{aligned}
\ln V_I^{card} &= \int_0^{\alpha_0} \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha(I-K_c)}{E(\alpha)(1+y_c)P_{CBDC}}\right) dG(\alpha) \\
&+ \int_{\alpha_0}^{1} \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha(I-K_c)}{E(\alpha)(1+f_c)P_{card}}\right) dG(\alpha) \\
&= \int_0^{1} \frac{\alpha}{E(\alpha)} \ln\left(\frac{\alpha(I-K_c)(1-y_m)}{E(\alpha)(1+y_c)}\right) dG(\alpha) + \int_{\alpha_0}^{1} \frac{\alpha}{E(\alpha)} \ln\left(\frac{(1+y_c)P_{CBDC}}{(1+f_c)P_{card}}\right) dG(\alpha) \\
&= \ln\left[(I-K_c)\left(\frac{1-y_m}{1+y_c}\right)2e^{-0.5}\right] + (1-\alpha_0^2)\ln\left(\frac{(1+y_c)}{(1+f_c)}\right) + \int_{\alpha_0}^{1} \frac{\alpha}{E(\alpha)} \ln\left(\frac{P_{CBDC}}{P_{card}}\right) dG(\alpha).
\end{aligned}$$

The last term is equal to

$$\int_{\alpha_0}^{1} \frac{\alpha}{E(\alpha)} \ln\left(\frac{P_{CBDC}}{P_{card}}\right) dG(\alpha) = \Phi_1 + \int_{\alpha_0}^{min\{\alpha_1,1\}} \frac{\alpha}{E(\alpha)} \ln\left(\frac{P_{CBDC}}{P_{card}}\right) dG(\alpha) \equiv \Phi_2.$$

Hence,

$$\ln V_I^{card} = \ln\left[(I-K_c)\left(\frac{1-y_m}{1+y_c}\right)2e^{\Phi_2-0.5}\left(\frac{(1+y_c)}{(1+f_c)}\right)^{(1-\alpha_0^2)}\right].$$

Then, we can aggregate the consumer surplus as follows.

$$\begin{aligned}
(CS) &= \int_0^{I_0} V_I^{Notcard} dF(I) + \int_{I_0}^{\infty} V_I^{card} dF(I) \\
&= \int_0^{I_0} \left[I\left(\frac{1-y_m}{1+y_c}\right)2e^{\Phi_1-0.5}\right]\lambda e^{-\lambda I} dI \\
&+ \int_{I_0}^{\infty} \left[(I-K_c)\left(\frac{1-y_m}{1+y_c}\right)2e^{\Phi_2-0.5}\left(\frac{(1+y_c)}{(1+f_c)}\right)^{(1-\alpha_0^2)}\right]\lambda e^{-\lambda I} dI \\
&= \frac{2(1-y_m)}{(1+y_c)}\frac{e^{\Phi_1-0.5}}{\lambda}(1-e^{-\lambda I_0}-\lambda I_0 e^{-\lambda I_0}) \\
&+ \frac{2(1-y_m)}{(1+y_c)}\left(\frac{1+y_c}{1+f_c}\right)^{(1-\alpha_0^2)}\frac{e^{\Phi_2-0.5-\lambda I_0}}{\lambda}(1+\lambda I_0-k_c)
\end{aligned}$$

In case of $y_c < \tau_c$, all consumer use the card. So, the calculation of the consumer surplus is same with the above cardholders' utility. But we need to change the α_0 to α_1.

$$\begin{aligned}
(CS) &= \frac{2(1-y_m)}{(1+y_c)}\left(\frac{1+y_c}{1+f_c}\right)^{(1-\alpha_1^2)}\frac{e^{\Phi_3-0.5}}{\lambda} \\
&\text{where } \Phi_3 = \int_{\alpha_1}^{1} \frac{\alpha}{E(\alpha)} \ln\left(\frac{P_{CBDC}}{P_{card}}\right) dG(\alpha)
\end{aligned}$$

A.2. Proof for Proposition 2

Here, I will show that how the equation (4) is derived. Since $f_c = 0$, $k'_m = 0$, $k'_c = -\kappa_c (k_c = 0)$, and $I_0 = 0$, then α_1 is calculated easily.

$$\begin{aligned}\alpha_1 &= \frac{k_m}{2e^{-\lambda I_0}(\lambda I_0 + 1 - k_c)\left(\frac{y_m - f_m}{1+f_c}\right)} \\ &= \frac{\kappa_m}{2(y_m - f_m)}\end{aligned}$$

By using above facts, the profit maximization problem of the card network can be reorganized.

$$\begin{aligned}\Pi(y_c, y_m, \kappa_m, \kappa_c, d) &= \max_{\{f_m\}} \int_{\alpha_1}^{1} (f_m - d)\frac{\alpha(1-k_c)}{E(\alpha)\lambda} dG(\alpha) - \frac{\kappa_c}{\lambda} \\ &= \max_{\{f_m\}} \frac{(1-\alpha_1^2)(f_m - d)}{\lambda} - \frac{\kappa_c}{\lambda} \\ &= \max_{\{f_m\}} \left(1 - \frac{\kappa_m^2}{4(y_m - f_m)^2}\right)\left(\frac{(f_m - d)}{\lambda}\right) - \frac{\kappa_c}{\lambda}\end{aligned}$$

The first order condition can be derived as follow.

$$\begin{aligned}[f_m] \quad & -2\frac{\kappa^2}{4(y_m - f_m)^3}\left(\frac{f_m - d}{\lambda}\right) + \left(1 - \frac{\kappa_m^2}{4(y_m - f_m)^2}\right)\frac{1}{\lambda} = 0 \\ & \Rightarrow -2\kappa_m^2(f_m - d) + 4(y_m - f_m)^3 - \kappa_m^2(y_m - f_m) = 0\end{aligned}$$

Note that the objective function is definitely concave on $[d, y_m]$ in the reasonable range of κ_m as shown in the following Figure.

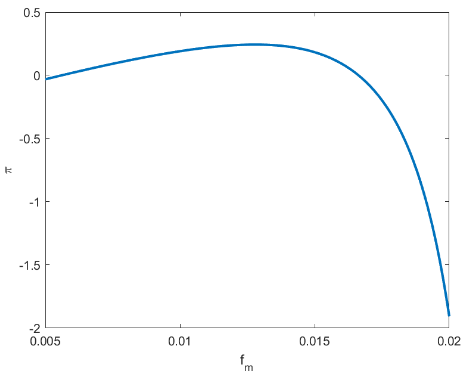


Figure 10: The profit function in dominant CBDC case

Note: All parameters is set to the benchmark model and the $y_m = 0.0243$ which is corresponding to $\rho = \tau_c$

A.3. The matched τs with different card trading shares.

There are two surveys in the Bank of Korea that announce the share of card transactions. One is from the payment and mobile financial services survey announced by Payment & Settlement Systems department. (survey A) The other is from the cash use status survey announced by Currency department. (survey B) The matching error means that sum of squared error of targets(the share of card tradings and cardholders)

share	calculating methods	τ_m	τ_c	matching error
83.6	cash/(cash+card) in survey A	3.62	0.39	0.0006
74.4	**card/(total) in survey A**	**3.02**	**0.59**	**0.0004**
73.0	cash/(cash+card) in survey B	2.87	0.70	0.0001
58.3	card/(total) in survey B	2.63	0.66	0.0689