GRADE 8 MATHEMATICS

Lessons:

1. **Special Products and Factoring**
2. **Relations and Functions**
3. **Reasoning**

❖ SPECIAL PRODUCTS AND FACTORING

- **SPECIAL PRODUCTS**

Certain types of binomial multiplication sometimes produce results that are called **special products**. Special products have predictable terms. Although the distributive property can always be used to multiply any binomials, recognition of those that produce special products provides a problem-solving shortcut.

Special products are the result of binomials being multiplied or simplified further, and can be solved with ease using the FOIL method: first, outer, inner, last. One example of special product is the square of a binomial.

The square of a binomial consists of:

a) the square of the first term;
b) twice the product of the first and last terms; and
c) the square of the last term.

Remember that the square of a binomial is called a perfect square trinomial.

The square of a trinomial consists of:

a) the sum of the squares of the first, second and last terms;
b) twice the product of the first and the second terms;
c) twice the product of the first and the last terms; and
d) twice the product of the second and the last terms.

Special Products Involving Squares

Three Types of Special Products

1) Difference of Squares: $x^2 - y^2 = (x - y)(x + y)$
2) Square of Sum: $x^2 + 2xy + y^2 = (x + y)^2$
3) Square of Difference: $x^2 - 2xy + y^2 = (x - y)^2$

Special Products Involving Cubes

To find the cube of a binomial of the form $(x + y)^3$:

a) Find the cube of each term to get the first and the last terms.
   → $(x)^3$, $(y)^3$
b) The second term is three times the product of the square of the first term and the second term.
   → $3(x)^2(y)$
c) The third term is three times the product of the first term and the square of the second term.
   → $3(x)(y)^2$

Hence, $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

NOTE:

- The square of a binomial is expressed as $(x + y)^2$ or $(x + y)(x + y)$ and $(x – y)^2$ or $(x – y)(x – y)$.
- Product Rule: $(a^m)(a^n) = a^{m+n}$
- Raising a power to a power: $(a^m)^n = a^{mn}$
- Applying special products enables us to factorize polynomials more efficiently.

- **FACTORING POLYNOMIALS**

Factoring is the process of finding the factors of an expression. Factoring a polynomial is expressing the polynomial as a product of two or more factors; it is somewhat the reverse process of multiplying.

Ex: This polynomial $f(x) = x^2 + 5x + 6$ can be factored to become $f(x) = (x + 3)(x + 2)$.

Prime Numbers are numbers greater than 1. They only have two factors, 1 and the number itself. This means these numbers cannot be divided by any number other than 1 and the number itself without leaving a remainder.

Numbers that have more than 2 factors are known as **composite numbers.**

Types of Factoring

1. **Greatest Common Factor -** The greatest common factor (GCF) for a polynomial is the largest monomial that is a factor of (divides) each term of the polynomial. The GCF must be a factor of EVERY term in the polynomial.

   Example:

   To factor $12x^3y^5 – 20x^5y^2z$ follow the following steps:

   a) Find the greatest common factor of the numerical coefficients

      → The GCF of 12 and 20 is 4.

   b) Find the variable with the least exponent that appears in each term of the polynomial.

      → x and y are both common to all terms and 3 is the smallest exponent for x and 2 is the smallest exponent of y, thus, x^3y^2 is the GCF of the variables.

   c) The product of the greatest common factor in (a) and (b) is the GCF of the polynomial.

      → Hence, $4x^3y^2$ is the GCF of $12x^3y^5 – 20x^5y^2z$.

   d) To completely factor the given polynomial, divide the polynomial by its GCF, the resulting quotient is the other factor.

      → Thus, the factored form of $12x^3y^5 – 20x^5y^2z$ is $4x^3y^2(3y^3 – 5x^2z)$

2. **Difference of Two Squares -** The difference of two squares identity is a squared number subtracted from another squared number to get factorized in the form of $a^2 – b^2 = (a + b)(a – b)$.

   Example:

   a) Factor $x^2 – 25$.

      $x^2 – 25 = x^2 – 5^2$ → $(x + 5) (x – 5)$

   b) Factor $x^2 – 81$.

      $x^2 – 81 = x^2 - 9^2$ → $(x + 9) (x – 9)$

3. **Sum or Difference of Two Cubes -** A polynomial in the form $a^3 + b^3$ is called a **sum of cubes**. A polynomial in the form $a^3 – b^3$ is called a **difference of cubes**.

   In Sum of Cubes:

   $$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

   same sign

   always +

   opposite sign

   Example:

   a) Factor $x^3 + 125$

      $x^3 + 125 = (x)^3 + (5)^3$

      $= (x + 5) [x^2 - (x)(5) + (5)^2]$

      $= (x + 5) (x^2 – 5x + 25)$

In Difference of Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

same sign

always +

opposite sign

Example:

a) Factor $8x^3 - 27$.

$8x^3 - 27 = (2x)^3 - (3)^3$

$= (2x - 3)\,[(2x)^2 + (2x)(3) + (3)^2]$

$= (2x - 3)\,(4x^2 + 6x + 9)$

❖ RELATIONS AND FUNCTIONS

- **Rectangular Coordinate System**

The rectangular coordinate system consists of two real number lines that intersect at a right angle. The horizontal number line is called the x-axis, and the vertical number line is called the y-axis. These two number lines define a flat surface called a plane, and each point on this plane is associated with an ordered pair of real numbers (x, y). The first number is called the x-coordinate, and the second number is called the y-coordinate. The intersection of the two axes is known as the origin, which corresponds to the point (0, 0).

This system is often called the Cartesian coordinate system, named after the French mathematician René Descartes (1596– 1650). The x- and y-axis break the plane into four regions called quadrants, named using roman numerals I, II, III, and IV, as pictured in figure 2.1.

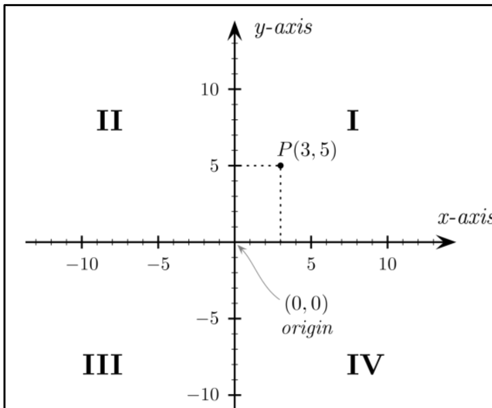
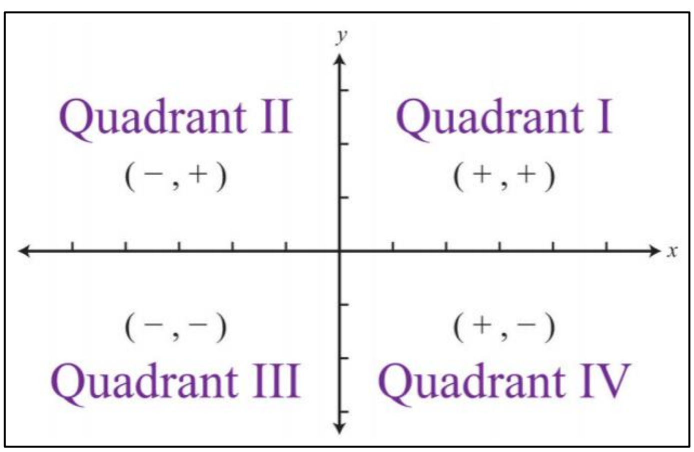


Figure 2.1



In quadrant I, both coordinates are positive.

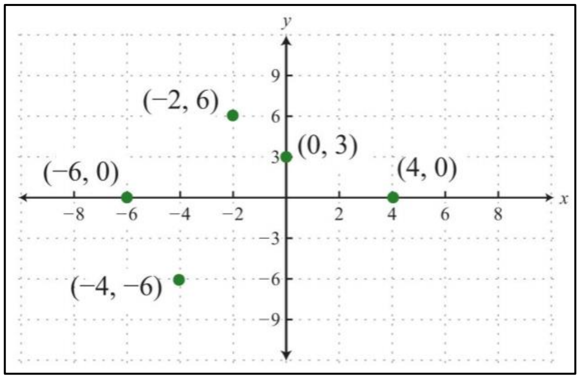
In quadrant II, the x-coordinate is negative and the y-coordinate is positive.

In quadrant III, both coordinates are negative. In quadrant IV, the x-coordinate is positive and the y-coordinate is negative.

Example:

Plot this set of ordered pairs: {(4,0),(−6,0),(0,3),(−2,6),(−4,−6)}.

Solution:



- **REPRESENTATIONS OF RELATIONS AND FUNCTIONS**

A **relation** is any set of ordered pairs. The set of all first coordinates is called the **domain** of the relation. The set of all second coordinates is called the **range** of the relation.

Consider the following set of ordered pairs. The first numbers in each pair are the first five natural numbers. The second number in each pair is twice that of the first. Eg. {(1,2),(2,4),(3,6),(4,8),(5,10)}

→ The domain is {1, 2, 3, 4, 5}.

→ The range is {2, 4, 6, 8, 10}.

Note that each value in the domain is also known as an input value, or independent variable, and is often labeled with the lowercase letter x. Each value in the range is also known as an output value, or dependent variable, and is often labeled lowercase letter y.

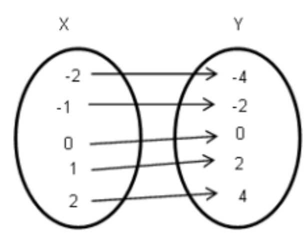
REPRESENTATION OF RELATIONS

Aside from ordered pairs, a relation may be represented in four other ways: (1) table, (2) mapping diagram, (3) graph, and (4) rule.

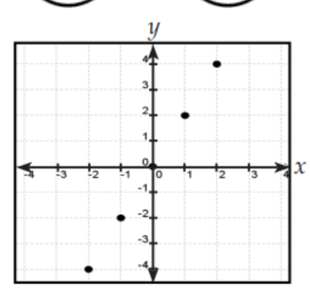
x	y
-2	-4
-1	-2
0	0
1	2
2	4

1. Table - The table describes clearly the behavior of the value of y as the value of x changes. Tables can be generated based on the graph. Below is an example of a table of values presented horizontally. At the right is also a table of values that is presented vertically.

x	-2	-1	0	1	2
y	-4	-2	0	2	4

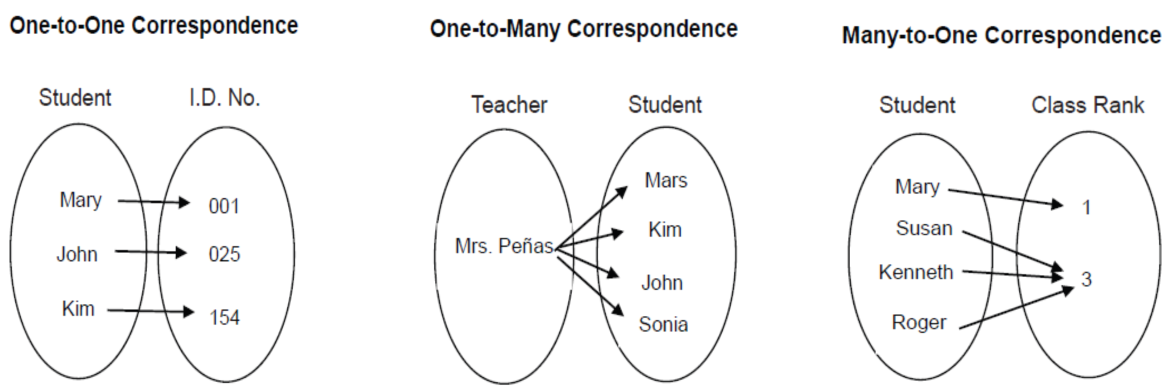


2. Mapping Diagram - Subsequently, a relation can be described by using a diagram as shown at the right. In this example, -2 is mapped to -4, -1 to -2, 0 to 0, 1 to 2, and 2 to 4.



3. Graph - At the right is an example of a graphical representation of a relation. It illustrates the relationship of the values of x and y.

4. Rule - Notice that the value of y is twice the value of x. In other words, this can be described by the equation y = 2x, where x is an integer from -2 to 2.



A correspondence may be classified as one-to-one, many-to-one, or one-to-many. It is one-to-one if every element in the domain is mapped to a unique element in the range, many-to-one if any two or more elements of the domain are mapped to the same element in the range; or one-to-many if each element in the domain is mapped to any two or more elements in the range.

A **function** is a special type of relation. It is a relation in which every element in the domain is mapped to exactly one element in the range. Thus, a set of ordered pairs is a function if no two distinct ordered pairs have equal abscissas.

Function Notation

The f(x) notation can also be used to define a function. If f is a function, the symbol f(x), read as "f of x," is used to denote the value of the function f at a given value of x. In simpler way, f(x) denotes the y-value (element of the range) that the function f associates with x-value (element of the domain). Thus, f(1) denotes the value of y at x = 1. Note that f(1) does not mean f times 1. The letters such as g, h and the like can also denote functions.

Furthermore, every element x in the domain of the function is called the pre-image. However, every element y or f(x) in the range is called the image. The figure at the right illustrates concretely the input (the value of x) and the output (the value of y or f(x)) in the rule or function. It shows that for every value of x there corresponds one and only one value of y.

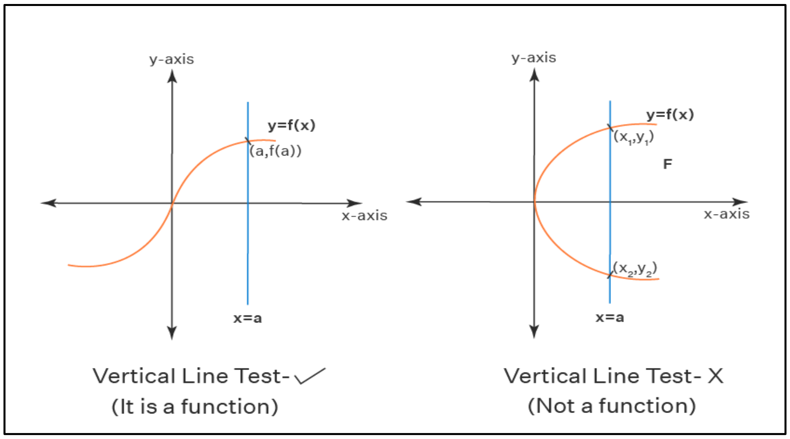
Horizontal and Vertical Lines

The horizontal line represents a function. It can be described by the equation y = c, where c is any constant. It is called a constant function. However, a vertical line which can be described by the equation x = c does not represent a function.

A relation may also be represented by an equation in two variables or the so-called rule. Consider the next example.

The Vertical Line Test

You may use the vertical line test to know if a given relation is a function or not. If every vertical line intersects the graph no more than once, the graph represents a function. Otherwise, if it intersects the graph more than once then it is not a function. See the figure below.



❖ REASONING

- **If Then Statements**

An **if-then statement** is composed of two clauses: the **if-clause** and the **then-clause**. We can denote a letter for each clause, p for the if-clause and q for the then-clause. The statement is in the form, "If p then q." Conditional statements are formed by joining two statements p and q using the words if and then. The p statement is called the **hypothesis** and the q statement is called the **conclusion**. A simple flow of reasoning from the if-clause to the then-clause is called **simple implication**.

Example:

1. All equiangular triangles are equilateral.
   If-then form: If a triangle is equiangular, then it is equilateral.
   Hypothesis (if p): If a triangle is equiangular
   Conclusion (then q): Then it is equilateral
2. Angles in a linear pair are supplementary.
   If-then form: If the angles are in a linear pair, then it is supplementary
   Hypothesis (if p): If the angles are in a linear pair
   Conclusion (then q): Then it is supplementary

CONVERSE, INVERSE, AND CONTRAPOSITIVE

Statement	If p, then q
Converse	If q, then p
Inverse	If not p, then not q
Contrapositive	If not q, then not p

Converse - To form the converse of the conditional statement, interchange the hypothesis and the conclusion.

Inverse - To form the inverse of the conditional statement, take the negation of both the hypothesis and the conclusion.

Contrapositive - To form the contrapositive of the conditional statement, interchange the hypothesis and the conclusion of the inverse statement.

Example:

1) Statement: If it rains, then they cancel school.
   Converse: If they cancel school, then it rains.
   Inverse: If it does not rain, then they do not cancel school.

Contrapositive: If they do not cancel school, then it does not rain.

2) Statement: If two angles are congruent, then they have the same measure.
   Converse: If two angles have the same measure, then they are congruent.
   Inverse: If two angles are not congruent, then they do not have the same measure.
   Contrapositive: If two angles do not have the same measure, then they are not congruent.

- **Inductive and Deductive Reasoning**

  Inductive Reasoning - Inductive reasoning uses specific examples to arrive at a general rule, generalizations, or conclusions.

  Example:

  1) Every time you eat peanuts, you start to cough. Therefore, you are allergic to peanuts.
  2) Every cat that you've observed purrs. Therefore, all cats must purr.

  Deductive Reasoning - Deductive reasoning uses basic and/or general statements to arrive at a conclusion.

  Example:

  1) An angle is acute if its measure is between 0^0 and 90^0. Angle B is acute. Therefore, angle B measures between 0^0 and 90^0.
  2) Filipinos are hospitable. Bonifacio is a Filipino. Therefore, Bonifacio is hospitable.

  The parts of a deductive reasoning are:

  • Hypothesis – the statement which is accepted or known at the beginning

  • Conclusion – the statement drawn from the hypothesis.

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