GRADE 7 MATHEMATICS

Lessons:

1. **Sets**
2. **Fundamental Operations on Integers**
3. **Rational and Irrational Numbers**
4. **Polynomials**

❖ SETS

Set - A set is a well-defined group of objects, called elements that share a common characteristic. It can be represented in set builder form or roster form. Usually, sets are represented in curly braces {}.

Example:

1. A = {1, 2, 3, 4, 5} is a set.
2. B = {January, February, March, April, May … December}

Set Builder Form - The set-builder notation or rule method is also used to express sets with an interval or an equation. This is used to write and represent the elements of sets, often for sets with an infinite number of elements. In this, one (or more) variable(s) is used that belongs to common types of numbers, such as integers, real numbers, and natural numbers.

Example:

1. A = {x | x < 10 and is an even natural number}
2. B = {x | x is prime number and 10 < x < 20}

Roster Form - In roster notation, the elements of a set are represented in a row surrounded by curly brackets and if the set contains more than one element then every two elements are separated by commas.

Example:

1. If A is the set of even natural numbers that is less than 10, the roster form can be represented by: A = {2, 4, 6, 8}
2. If B is the set of prime numbers between 10 and 20, the roster form can be represented by: B = {11, 13, 15, 17, 19}

Element – The objects in a set are called its elements. The mathematical notation for "is an element of" is ∈. For example, to denote that 2 is an element of the set E of positive even integers, one writes 2∈E. However, if 2 is not an element of E, write3 ∉E.

Example:

1. If set A = {2, 4, 6, 8}, then the elements of A are 2, 4, 6, 8.
2. If set B = {11, 13, 15, 17, 19}, then the elements of B are 11, 13, 15, 17, 19.

Subset – Set A is a subset of another set B if all elements of the set A are elements of the set B. In other words, the set A is contained inside the set B. The subset relationship is denoted as A ⊆ B.

Example:

If Set A = {1, 2, 3) and set B = {1, 2, 3}, we can say that set A is a subset of set B or A ⊆ B. See figure 1.

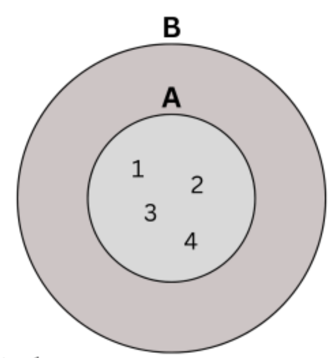

Figure 1.

Proper Subset – A proper subset of a set A is a subset of A that is not equal. In other words, if B is a proper subset of A, then all elements of B are in A but A contains at least one element that is not in set B. The proper subset relationship is denoted as B ⊂ A.

Example:

If set A = {1, 2, 3, 4, 5} and set B = {1, 2, 3}, we can say that set B is a proper subset of set A or B ⊂ A. See figure 1.1

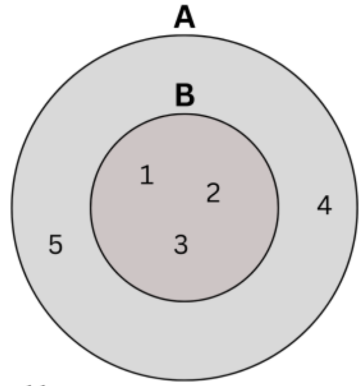

Figure 1.1

Universal Set – A universal set is a collection of elements of all related sets and subsets including itself. It is a set consisting of all the elements present in other given sets. It is usually represented by the symbol U.

Example:

1. Refer to figure 1 in subset, the universal set can be represented by: U = {1, 2, 3, 4}.
2. Refer to figure 2 in proper subset, the universal set can be represented by: U = {1, 2, 3, 4, 5}

Null Set – A null set or also known as empty set is a set that contains no elements. The symbol used to represent an empty set is {} or ϕ.

Example:

1. Let A = {x | 9 < x < 10, x is a natural number} will be a null set because there is NO natural number between numbers 9 and 10. Therefore, A = {} or A = ϕ.
2. Let B = {x | x > 8, x is the number of days in a week} will also be a void set because there are only 7 days in a week. Therefore, B = {} or B = ϕ.

Cardinality of the Set - The cardinality of a set is the total number of unique elements in a set. It can be finite or infinite. The cardinality of the set can be represented by n(A) or |A|.

Example:

1. If set A = {a, b, c, d, e}, then the cardinality of set A is: n(A) = 5 or |A| = 5.
2. If set B = {5, 10, 15, 20, 25, 30, 35}, then the cardinality of set B is: n(B) = 7 or |B| = 7.

Union – The union of two sets is a set containing all elements that are in A or in B (possibly both). To find the union of two given sets A and B is a set which consists of all the elements of A and all the elements of B such that no element is repeated. The symbol for denoting union of sets is 'U'.

Example:

1. If set A = {a, b c , d, e} and set B = {b, d, f, h, j}, the union of set A and set B or A ∪ $B = \{a, b, c, d, e, f, h, j\}$.
2. Refer to figure 1.2, A ∪ $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

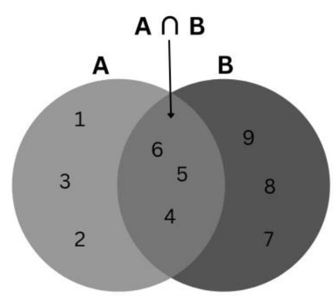

Figure 1.2

Intersection – The intersection of two sets A and B is the set of all those elements which are common or belongs to both A and B. Symbolically, we can represent the intersection of A and B as A ∩ B. An element x belongs to the intersection of the sets A and B if and only if x belongs to A and B. This can be expressed as: A ∩ B = {x : x ∈ A and x ∈ B}.

Example:

1. If set A = {a, b, c, d, e} and set B = {b, d, f, h, j}, the intersection of set A and set B or A ∩ B = {b, d}
2. Refer to figure 1.2, the intersection of set A and set B or A ∩ B = {6, 5, 4}.

Complement – The complement of a set is the set that includes all the elements of the universal set that are not present in the given set. In set theory, the complement of a set A which is often denoted by A^C or A' is the set of elements that are not in set A.

Example:

1. Let U = {1, 2, 3, 4, 5, 6, 7, 8} and A = {3, 5, 6}. Then A^C or A' = {1, 2, 4, 7, 8} and n(A') = 5.
2. Let U = {a, b, c , d, e, f, g, h} and B = {a, e}. Then A^c or A' = {b, c, d, f, g, h} and n(A') = 6.

❖ FUNDAMENTAL OPERATIONS ON INTEGERS

There are four fundamental operations on integers which are the basic arithmetic operation namely: Addition, Subtraction, Multiplication, and Division.

ADDITION OF INTEGERS

- Addition Of Integers Having The Same Sign
  1. The sum of two positive integers is the sum of their absolute values with a positive sign (+).
     Ex. (+ 6) + (+ 4) = +10
  2. The sum of two negative integers is the sum of their absolute values with negative sign (-).
     Ex. (-3) + (-4) = (-7)
- Addition Of Integers Having Opposite Signs
  The sum of two integers having opposite signs is equal to the difference of their absolute values with the sign of integer of greater absolute value. In other words, if the two integers have different sign, we will subtract the two integers instead of adding them and then get the sign of the integer with higher value.
  Example:
  a) (+6) + (-7) = -1. In this case, 7 is the higher value, that's why we took the sign of 7 which is negative(-).
  b) (-10) + (+17) = +7 or 7.

NOTE:

Positive + Positive = Positive

Negative + Negative = Negative

Negative + Positive or vise-versa = use the sign of the larger number and subtract

SUBTRACTION OF INTEGERS

- In subtraction, we change the sign of the integer which is to be subtracted and then add to the first integer. In other words, if a and b are two integers, then a – b = a + (-b).
  Example:
  a) Subtract 5 from 12. Solution: (12) – (5) = (12) + (-5) = 7
  b) Subtract -7 from -15. Solution: (-15) – (-7) = (-15) + (7)= -8
  c) Subtract 6 from -10. Solution: (-10) -(6) = (-10) + (- 6) = 16
  d) Subtract (-5) from 4. Solution: 4 – (-5) = 4 + (5) = 9

NOTE:

Negative - Positive = Negative

Positive - Negative = Positive

Negative - Negative or Negative + Positive= use the sign of the larger number and subtract

MULTIPLICATION OF INTEGERS

- Multiplication Of Integers Having The Same Sign
  When two integers have the same sign, their product is the product of their absolute values with positive sign.
  Example:
  a) (+6) × (+7) = + 42 or 42
  b) (-3) × (-5) = + 15 or 15
- Multiplication Of Integers Having Opposite Signs
  The product of two integers having opposite signs is the product of their absolute values with negative sign.
  Example:
  a) (-10) × (8) = (- 80)
  b) (12) × (-3) = (-36)

NOTE:

Positive x Positive = Positive

Negative x Negative = Positive

Negative x Positive = Negative

Positive x Negative = Negative

DIVISION OF INTEGERS

- Division Of Integers Having The Same Sign
  Division of two integers having the same sign is the division of their absolute value with a positive sign. If both integers have the same sign, then the quotient will be positive.
  Example:
  a) (+9) ÷ (+3) = (3)
  b) (-24) ÷ (-12) = (2)
- Division Of Integers Having Opposite Signs
  If both integers have different signs, the quotient will be negative.
  Example:
  a) 12 ÷ (-3) = (-4)
  b) (-18) ÷ (3) = (-6)

NOTE:

Positive ÷ Positive = Positive

Negative ÷ Negative = Positive

Negative ÷ Positive = Negative

Positive ÷ Negative = Negative

❖ RATIONAL AND IRRATIONAL NUMBERS

- **RATIONAL NUMBER -** A rational number is a number that can be made into a fraction. Decimals that repeat or terminate are rational because they can be changed into fractions.
  Examples: 1/3, 2/4, 1/5, 9/3, 0.3333

PROPERTIES OF RATIONAL NUMBERS

1. Closure Property - For two rational numbers say x and y the results of addition, subtraction and multiplication operations give a rational number. We can say that rational numbers are closed under addition, subtraction and multiplication.
   Example:
   a) (7/6)+(2/5) = 47/30
   b) (5/6) – (1/3) = 1/2
   a) $\frac{3}{4} + \frac{2}{4} = \left(\frac{3+2}{4}\right) = \frac{5}{4}$
   b) $\frac{3}{4}\ x\ \frac{2}{4} = \left(\frac{6}{16}\right)\ or\ \frac{3}{8}$
2. Commutative Property - For rational numbers, addition and multiplication are commutative.
   Commutative Law Of Addition: a+b = b+a
   Commutative Law Of Multiplication: a×b = b×a
3. Associative Property - Rational numbers follow the associative property for addition and multiplication. Suppose x, y and z are rational, then for addition: x+(y+z)=(x+y)+z. For multiplication: x(yz)=(xy)z.
4. Distributive Property - The distributive property states, if a, b and c are three rational numbers. Then, a x (b+c) = (a x b) + (a x c).
   Example: 1/2 x (1/2 + 1/4) = (1/2 x 1/2) + (1/2 x 1/4)
5. Identity Property – Zero (0) is an additive identity and 1 is a multiplicative identity for rational numbers.
   Example:
   a) 1/2 + 0 = 1/2 [Additive Identity]
   b) 1/2 x 1 = 1/2 [Multiplicative Identity]
6. Inverse Property - For a rational number x/y, the additive inverse is -x/y and y/x is the multiplicative inverse.
   Example:
   a) The additive inverse of 1/3 is -1/3. Hence, 1/3 + (-1/3) = 0
   b) The multiplicative inverse of 1/3 is 3. Hence, 1/3 x 3 = 1

- **IRRATIONAL NUMBER -** An irrational number is a number that cannot be made into a fraction. Decimals that do not repeat or end are irrational numbers. Pi is an irrational number.
  Examples: √2, √3, √5, √11, √21, π, 3.141141114

❖ POLYNOMIALS

Polynomial - A polynomial is a kind of algebraic expression where each term is a constant, a variable or a product of a constant and variable in which the variable has a whole number (non-negative number) exponent. A polynomial can be a monomial, binomial, trinomial or a multinomial.

An algebraic expression is NOT a polynomial if

a) The exponent of the variable is NOT a whole number {0, 1, 2, 3..}.
b) The variable is inside the radical sign.
c) The variable is in the denominator.

Kinds of Polynomial according to the number of terms

a) Monomial – is a polynomial with only one term
b) Binomial – is polynomial with two terms

c) Trinomial – is a polynomial with three terms
d) Polynomial – is a polynomial with four or more terms

Kinds of Polynomial according to its degree

a) Constant – a polynomial of degree zero
b) Linear – a polynomial of degree one
c) Quadratic – a polynomial of degree two
d) Cubic – a polynomial of degree three
e) Quartic – a polynomial of degree four
f) Quintic – a polynomial of degree five

* The next degrees that have no universal name yet, they are just called "polynomial of degree ____."

Definition of Terms

- **Term** - Term is a constant, a variable or a product of constant and variable. In the algebraic expression $3x^2 – x + 5$, $3x^2$, $-x$ and 5 are called the terms

- **Numerical Coefficient** - A numerical coefficient is defined as a fixed number that is multiplied to a variable. It is a constant/number.
  Example: In the term $3x^2$, 3 is called the numerical coefficient.

- **Literal Coefficient** - A literal coefficient is a variable used to represent a number. The number the variable represents can be either known or unknown. It can be our usual x or y, or it can be other letters, such as a, b, or c. In other words, literal coefficient is the variable including its exponent.
  Example: In the term $3x^2$, x^2 is called the literal coefficient.

  In literal coefficient x^2, 'x' is called the **base** and '2' is called the **exponent**.
  Degree is the highest exponent or the highest sum of exponents of the variables in a term.

- **Constant -** A constant term is a term that contains only a number. In other words, there is no variable in a constant term.
  Example: 4, 100, and -5.

- **Standard Form -** A polynomial is said to be in Standard Form if its terms are arranged from the term with the highest degree, up to the term with the lowest degree. The standard form of polynomial is given by, $f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1x + a_0$, where x is the variable and a_i are coefficients.
  Example: The standard form of $2x^2 – 5x^5 – 2x^3 + 3x – 10$ is $-5x^5 – 2x^3 + 2x^2 + 3x – 10$

  If the polynomial is in standard form the first term is called the **Leading Term**, the numerical coefficient of the leading term is called the **Leading Coefficient** and the exponent or the sum of the exponents of the variable in the leading term the **Degree of the polynomial**.

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