GRADE 10 MATHEMATICS

Lessons:

1. **Sequences**
2. **Polynomial Functions**
3. **Permutations and Combinations**
4. **Measures of Position**

❖ SEQUENCES

A sequence is an arrangement of any objects or a set of numbers in a particular order followed by some rule. If a_1, a_2, a_3, a_4, ……… etc. denote the terms of a sequence, then 1,2,3,4,…..denotes the position of the term.

A sequence can be defined based on the number of terms i.e. either finite sequence or infinite sequence.

If a_1, a_2, a_3, a_4, ……. is a sequence, then the corresponding series is given by

$SN = a_1+a_2+a_3 + .. + a_N$

Types of Sequence

Some of the most common examples of sequence are the following:

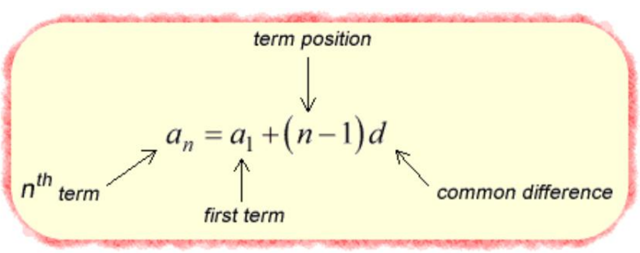
1. Arithmetic Sequence
2. Geometric Sequence
3. Harmonic Sequence
4. Fibonacci Sequence

Arithmetic Sequence

An arithmetic sequence is a sequence where every term after the first is obtained by adding a constant called the common difference.

The sequences 1, 4, 7, 10, … and 15, 11, 7, 3, ... are examples of arithmetic sequences since each one has a common difference of 3 and -4, respectively.

Formula: **$a_n = a_1 + (n-1)\,d$**



Geometric Sequences

A geometric sequence is a sequence where each term after the first is obtained by multiplying the preceding term by a nonzero constant called the common ratio.

The common ratio, r, can be determined by dividing any term in the sequence by the term that precedes it. Thus, in the geometric sequence 32, 16, 8, 4, 2, ... , the common ratio is 1/2 since 16/32 = 1/2.

Inserting a certain number of terms between two given terms of a geometric sequence is an interesting activity in studying geometric sequences. We call the terms between any two given terms of a geometric sequence the geometric means.

Formula : **$a + ar + ar^2 + ar^3 + \ldots$**

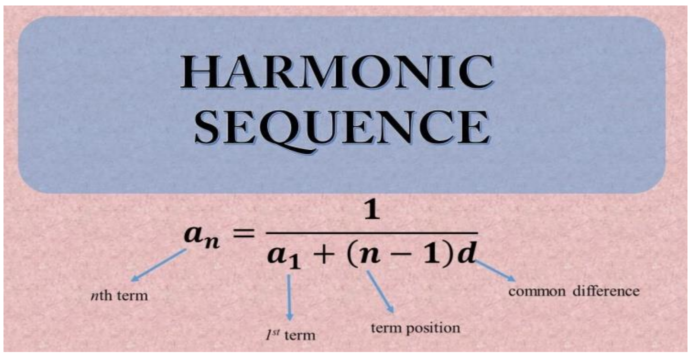
Where a = start term

r = common ratio

Harmonic Sequence

A series of numbers is said to be in harmonic sequence if the reciprocals of all the elements of the sequence form an arithmetic sequence.

Formula :



Fibonacci Sequence

A Fibonacci sequence is a sequence where its first two terms are either both 1, or 0 and 1; and each term, thereafter, is obtained by adding the two preceding terms. Sequence is defined as, $F_0 = 0$ and $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$

Formula: **$F_n = F_{n-1} + F_{n-2}$, where n is the nth term**

❖ Polynomial Functions

A polynomial function is a function that can be expressed in the form of a polynomial. The definition can be derived from the definition of a polynomial equation. A polynomial is generally represented as P(x). The highest power of the variable of P(x) is known as its degree. Degree of a polynomial function is very important as it tells us about the behaviour of the function P(x) when x becomes very large. The domain of a polynomial function is entire real numbers (R).

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots\ldots + a_2 x^2 + a_1 x + a_0$, then for $x \gg 0$ or $x \ll 0$, $P(x) \approx$ an xn. Thus, polynomial functions approach power functions for very large values of their variables.

Examples:

A polynomial function has only positive integers as exponents. We can even perform different types of arithmetic operations for such functions like addition, subtraction, multiplication and division.

1) x^2+2x+1
2) $3x-7$
3) $7x^3$+x2-2

All three expressions above are polynomial since all of the variables have positive integer exponents. But expressions like;

1) $5x^{-1}+1$
2) $4x^{1/2}+3x+1$
3) $(9x+1) \div (x)$

are not polynomials, we cannot consider negative integer exponents or fraction exponent or division here.

Types of Polynomial Functions

There are various types of polynomial functions based on the degree of the polynomial. The most common types are:

1. Constant Polynomial Function: $P(x) = a = ax^0$
2. Zero Polynomial Function: $P(x) = 0$; where all a_i's are zero, $i = 0, 1, 2, 3, \ldots, n$.
3. Linear Polynomial Function: $P(x) = ax + b$
4. Quadratic Polynomial Function: $P(x) = ax^2+bx+c$
5. Cubic Polynomial Function: ax^3+bx^2+cx+d
6. Quartic Polynomial Function: $ax^4+bx^3+cx^2+dx+e$

Constant Polynomial Function

Degree 0 (Constant Functions)

- Standard form: P(x) = a = a.x0, where a is a constant.
- Graph: A horizontal line indicates that the output of the function is constant. It doesn't depend on the input.

Zero Polynomial Function

A constant polynomial function whose value is zero. In other words, zero polynomial function maps every real number to zero, $f: R \to \{0\}$ defined by $f(x) = 0 \ \forall \ x \in R$. For example, let f be an additive inverse function, that is, $f(x) = x + (-x)$ is zero polynomial function.

Linear Polynomial Functions

Degree 1, Linear Functions

- Standard form: $P(x) = ax + b$, where a and b are constants. It forms a straight line.
- Graph: Linear functions have one dependent variable and one independent which are x and y, respectively.

In the standard formula for degree 1, a represents the slope of a line, the constant b represents the y-intercept of a line.

Quadratic Polynomial Functions

Degree 2, Quadratic Functions

- Standard form: $P(x) = ax^2+bx+c$, where a, b and c are constant.
- Graph: A parabola is a curve with one extreme point called the vertex. A parabola is a mirror-symmetric curve where any point is at an equal distance from a fixed point known as Focus.

In the standard form, the constant 'a' represents the wideness of the parabola. As 'a' decreases, the wideness of the parabola increases. This can be visualized by considering the boundary case when a=0, the parabola becomes a straight line. The constant c represents the y-intercept of the parabola.

The vertex of the parabola is given by $(h,k) = (-b/2a, -D/4a)$, where D is the discriminant and is equal to (b^2-4ac).

Graphs of Higher Degree Polynomial Functions

- Standard form – $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots + a_0$, where $a_0, a_1, \ldots\ldots, a_n$ are all constants.
- Graph: Depends on the degree, if P(x) has degree n, then any straight line can intersect it at a maximum of n points. The constant term in the polynomial expression, i.e. a0 here represents the y-intercept.

❖ PERMUTATIONS AND COMBINATIONS

Permutations

In mathematics, permutation relates to the act of arranging all the members of a set into some sequence or order. In other words, if the set is already ordered, then the rearranging of its elements is called the process of permuting. Permutations occur, in more or less prominent ways, in almost every area of mathematics. They often arise when different orderings on certain finite sets are considered.

Formula: **$_nP_r = (n!)/(n-r)!$**

$_nP_r$ = permutation

n = total number of objects

r = number of objects selected

Combinations

The combination is a way of selecting items from a collection, such that (unlike permutations) the order of selection does not matter. In smaller cases, it is possible to count the number of combinations. Combination refers to the combination of n things taken k at a time without repetition. To refer to combinations in which repetition is allowed, the terms k-selection or k-combination with repetition are often used.

Formula: **$_nC_r = (n!)/r!(n-r)!$**

$_nC_r$ = number of combinations

n = total number of objects in the set

r = number of choosing objects from the set

Difference Between Permutation and Combination

Permutation	Combination
Arranging people, digits, numbers, alphabets, letters, and colours	Selection of menu, food, clothes, subjects, team.
Picking a team captain, pitcher and shortstop from a group.	Picking three team members from a group.
Picking two favourite colours, in order, from a colour brochure.	Picking two colours from a colour brochure.
Picking first, second and third place winners.	Picking three winners.

Uses of Permutation and Combination

A permutation is used for the list of data (where the order of the data matters) and the combination is used for a group of data (where the order of data doesn't matter).

❖ MEASURES OF POSITION

Measures of position give us a way to see where a certain data point or value falls in a sample or distribution. A measure can tell us whether a value is about the average, or whether it's unusually high or low. Measures of position are used for quantitative data that falls on some numerical scale. Sometimes, measures can be applied to ordinal variables— those variables that have an order, like first, second…fiftieth.

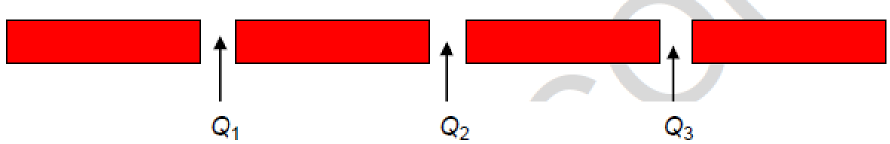
Measures of position can also show how to values from different distributions or measurement scales compare. For example, a person's height (measured in feet) and weight (measured in pounds) can be compared by converting the measurements to z-scores.

Measures of Position for Ungrouped Data

1. The Quartile for Ungrouped Data

The quartiles are the score points which divide a distribution into four equal parts. Twenty-five percent (25%) of the distribution are below the first quartile, fifty percent (50%) are below the second quartile, and seventy-five percent (75%) are below the third quartile. 1 Q is called the lower quartile and 3 Q is the upper quartile. 1 Q < 2 Q < 3 Q , where 2 Q is nothing but the median. The difference between 3 Q and 1 Q is the interquartile range.

Since the second quartile is equal to the median, the steps in the computation of median by identifying the median class is the same as the steps in identifying the Q1 class and the Q3 class.



a) 25% of the data has a value ≤ Q1
b) 50% of the data has a value ≤ X or Q2
c) 75% of the data has a value ≤ Q3

Example:

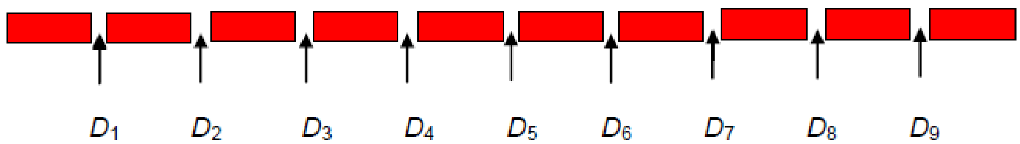
The owner of a coffee shop recorded the number of customers who came into his café each hour in a day. The results were 14, 10, 12, 9, 17, 5, 8, 9, 14, 10, and 11. Find the lower quartile and upper quartile of the data.

Solution:

- In ascending order, the data are 5, 8, 9, 9, 10, 10, 11, 12, 14, 14, 17
- The least value in the data is 5 and the greatest value in the data is 17.
- The middle value in the data is 10.
- The lower quartile is the value that is between the middle value and the least value in the data set.
- So, the lower quartile is 9.
- The upper quartile is the value that is between the middle value and the greatest value in the data set.
- So, the upper quartile is 14.

2. The Deciles for Ungrouped Data

The deciles are the nine score points which divide a distribution into ten equal parts. They are deciles and are denoted as D1, D2, D3,…, D9. They are computed in the same way that the quartiles are calculated.



Example:

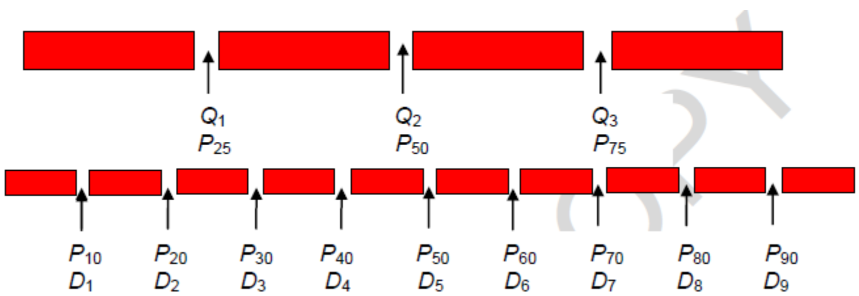
Anthony is a secretary in one big company in Metro Manila. His salary is in the 7th decile. Should Anthony be glad about his salary or not? Explain your answer.

Solution:

70% of employees receive a salary that is less than or equal to his salary and 30% of the employees receive a salary that is greater than his salary. Anthony should be pleased with his salary.

3. **The Percentile for Ungrouped Data**

The percentiles are the ninety-nine score points which divide a distribution into one hundred equal parts, so that each part represents the data set. It is used to characterize values according to the percentage below them. For example, the first percentile (P1) separates the lowest 1% from the other 99%, the second percentile (P2) separates the lowest 2% from the other 98%, and so on.



The percentiles determine the value for 1%, 2%,..., and 99% of the data. P_{30} or 30th percentile of the data means 30% of the data have values less than or equal to P_{30}.

The 1st decile is the 10th percentile (P_{10}). It means 10% of the data is less than or equal to the value of P_{10} or D_1, and so on.

Example:

Find the 30th percentile or P30 of the following test scores of a random sample of ten students: 35, 42, 40, 28, 15, 23, 33, 20, 18, and 28.

Solution:

First, arrange the scores from the lowest to the highest.

15 18 20 23 28 28 33 35 40 42

Steps to find percentile value on a data with n elements:

To find its P_{30} position, use the formula $\frac{k(n+1)}{100}$ and round off to the nearest integer.

P_{30} = 30(10 + 1)/100

= 30(11)/100

= 300/100

$= 3.3$

$= 3.3 \approx 3$, Therefore, P_{30} is the third element which is 20. Thus, $P_{30} = 20$.

Measures of Position for Ungrouped Data

1. **The Quartile for Grouped Data**

Recall that quartiles divide the distribution into four equal parts.

The steps in computing the median are similar to that of Q1 and Q3. In finding the median, we first need to determine the median class. In the same manner, the Q1 and the Q3 class must be determined first before computing for the value of Q1 and Q3. The Q1 class is the class interval where the (N/4)th score is contained, while the class interval that contains the (3N/4)th score is the Q_3 class.

In computing the quartiles of grouped data, the following formula is used:

$$Q_k = LB + \left(\frac{\frac{kN}{4} - cf_b}{f_{Qk}} \right) i$$

where:

LB = lower boundary of the Q_k class

N = total frequency

cf_b = cumulative frequency of the class before the Q_k class

f_{Q_1} = frequency of the Q_k class

i = size of class interval

k = nth quartile, where n = 1, 2, and 3

2. **The Deciles for Grouped Data**

Deciles are those values that divide the total frequency into 10 equal parts. The kth decile denoted by D_k is computed as follows:

$$D_k = LB + \left(\frac{\frac{kN}{10} - cf_b}{f_{D_k}} \right) i$$

where:

LB = lower boundary of the D_k class

N = total frequency

cf_b = cumulative frequency before the D_k class

f_{D_k} = frequency of the D_k class

i = size of class interval

k = nth decile where n = 1, 2, 3, 4, 5, 6, 7, 8, and 9

3. **The Percentile for Grouped Data**

The percentile of grouped data is used to characterize values according to the percentage below them.

Early on, you have already learned that kth quartile denoted by Q_k and the kth deciles denoted by D_k are computed, respectively, as follows:

$$Q_k = LB + \left(\frac{\frac{kN}{4} - cf_b}{f_{Q_k}} \right) i \quad \text{and} \quad D_k = LB + \left(\frac{\frac{kN}{10} - cf_b}{f_{D_k}} \right) i$$

Finding percentiles of a grouped data is similar to that of finding quartiles and deciles of a grouped data. The kth percentile, denoted by P_k is computed as follows:

$$P_k = LB + \left(\frac{\frac{kN}{100} - cf_b}{f_{P_k}} \right) i$$

where:

LB = lower boundary of the kth percentile class
N = total frequency
cf_b = cumulative frequency before the percentile class
f_{P_k} = frequency of the percentile class
i = size of class interval
k = nth percentile where n = 1, 2, 3,…, 97, 98, and 99

Percentile Rank

Percentile ranks are particularly useful in relating individual scores to their positions in the entire group. A percentile rank is typically defined as the proportion of scores in a distribution that a specific score is greater than or equal to. For instance, if you received a score of 95 on a mathematics test and this score was greater than or equal to the scores of 88% of the students taking the test, then your percentile rank would be 88.

An example is the National Career Assessment Examination (NCAE) given to Grade 9 students. The scores of students are represented by their percentile ranks.

Formula:

$$P_{PR} = \frac{100}{N} + \left[\frac{(P - LB) f_P}{i} + cf_P \right]$$

where:

PR = percentile rank, the answer will be a percentage
cf_P = cumulative frequency of all the values below the critical value
P = raw score or value for which one wants to find a percentile rank
LB = lower boundary of the kth percentile class
N = total frequency
i = size of the class interval

NOTES:

- Deciles - the nine score points which divide a distribution into ten equal parts. These deciles are denoted as D_1, D_2, D_3,… D_9.
- Percentiles - the ninety-nine score points which divide a distribution into one hundred equal parts so that each part represents 1/100 of the data set. They are used to characterize values according to the percentage below them.
- Quantiles - measures of positions that divide a distribution into four, ten, and hundred equal parts. Such measures of positions are quartiles, deciles, and percentiles.
- Quartiles - the score points which divide a distribution into four equal parts. Twenty-five percent (25%) of the distribution fall below the first quartile, fifty percent (50%) fall below the second quartile, and seventy-five percent (75%) fall below the third quartile.

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