GRADE 9 MATHEMATICS

Lessons:

1. **Quadratic Functions**
2. **Variations**
3. **Quadrilaterals**
4. **Trigonometry**

❖ QUADRATIC FUNCTIONS

Quadratic Function - A quadratic function is a polynomial function with one or more variables in which the highest exponent of the variable is two. Since the highest degree term in a quadratic function is of the second degree, therefore it is also called the polynomial of degree 2. A quadratic function has a minimum of one term which is of the second degree. It is an algebraic function.

Standard Form of Quadratic Equation

The parent quadratic function is of the form $f(x) = x^2$ and it connects the points whose coordinates are of the form (number, $number^2$). Transformations can be applied on this function on which it typically looks of the form $f(x) = a(x - h)^2 + k$ and further it can be converted into the form $f(x) = ax^2 + bx + c$.

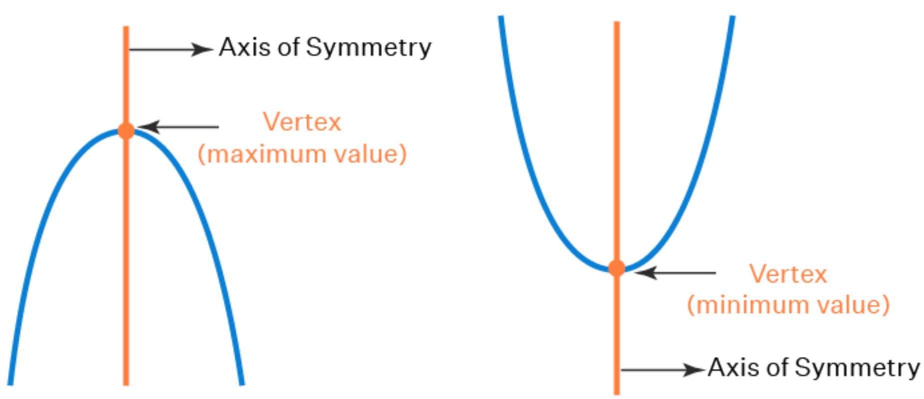
The standard form of a quadratic function is of the form **f(x) = ax2 + bx + c**, where a, b, and c are real numbers with $a \neq 0$.

Examples of Quadratic Function

1) $f(x) = 2x^2 + 4x - 5$; Here a = 2, b = 4, c = -5
2) $f(x) = 3x^2 - 9$; Here a = 3, b = 0, c = -9
3) $f(x) = x^2 - x$; Here a = 1, b = -1, c = 0

Vertex of Quadratic Function

The vertex of a quadratic function (which is in U shape) is where the function has a maximum value or a minimum value. The axis of symmetry of the quadratic function intersects the function (parabola) at the vertex.



Quadratic Functions Formula

A quadratic function can always be factorized, but the factorization process may be difficult if the zeroes of the expression are non-integer real numbers or non-real numbers. In such cases, we can use the quadratic formula to determine the zeroes of the expression. The general form of a quadratic function is given as: $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers with $a \neq 0$. The roots of the quadratic function f(x) can be calculated using the formula of the quadratic function which is:

- $\mathbf{x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a}$

Different Forms of Quadratic Function

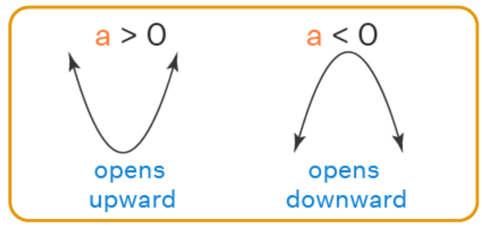
A quadratic function can be in different forms: standard form, vertex form, and intercept form. Here are the general forms of each of them:

1. **Standard Form**: $f(x) = ax^2 + bx + c$, where $a \neq 0$.
2. **Vertex Form**: $f(x) = a(x - h)^2 + k$, where $a \neq 0$ and (h, k) is the vertex of the parabola representing the quadratic function.
3. **Intercept Form**: $f(x) = a(x - p)(x - q)$, where $a \neq 0$ and $(p, 0)$ and $(q, 0)$ are the x-intercepts of the parabola representing the quadratic function.

PARABOLA

The graph of a quadratic function is a curve which is called a parabola. Parabolas may open upward or downward and vary in "width" or "steepness", but they all have the same basic "U" shape.

a) If $a > 0$, then the parabola opens upward.
b) If $a < 0$, then the parabola opens downward.



Converting Standard Form of Quadratic Function into Vertex Form

A quadratic function $f(x) = ax^2 + bx + c$ can be easily converted into the vertex form $f(x) = a(x - h)^2 + k$ by using the values $h = -b/2a$ and $k = f(-b/2a)$.

Example: Convert the quadratic function $f(x) = 2x^2 - 8x + 3$ into the vertex form.

Step 1: By comparing the given function with $f(x) = ax^2 + bx + c$, we get $a = 2$, $b = -8$, and $c = 3$.

Step 2: Find 'h' using the formula: $h = -b/2a = -(-8)/2(2) = 2$.

Step 3: Find 'k' using the formula: $k = f(-b/2a) = f(2) = 2(2)^2 - 8(2) + 3 = 8 - 16 + 3 = -5$.

Step 4: Substitute the values into the vertex form: $f(x) = 2(x - 2)^2 - 5$.

Converting Standard Form of Quadratic Function into Intercept Form

A quadratic function $f(x) = ax^2 + bx + c$ can be easily converted into the vertex form $f(x) = a(x - p)(x - q)$ by using the values of p and q (x-intercepts) by solving the quadratic equation $ax^2 + bx + c = 0$.

Example: Convert the quadratic function $f(x) = x^2 - 5x + 6$ into the intercept form.

Step 1: By comparing the given function with $f(x) = ax^2 + bx + c$, we get $a = 1$.

Step 2: Solve the quadratic equation: $x^2 - 5x + 6 = 0$. By factoring the left side part, we get

$$(x - 3)(x - 2) = 0$$
$$x = 3, x = 2$$

Step 3: Substitute the values into the intercept form: $f(x) = 1(x - 3)(x - 2)$.

Domain and Range of Quadratic Function

- **DOMAIN**
  A quadratic function is a polynomial function that is defined for all real values of x. So, the domain of a quadratic function is the set of real numbers, that is, R. In interval notation, the domain of any quadratic function is $(-\infty, \infty)$.
- **RANGE**
  The range of the quadratic function depends on the graph's opening side and vertex. So, look for the lowermost and uppermost f(x) values on the graph of the function to determine the range of the quadratic function. The range of any quadratic function with vertex (h, k) and the equation $f(x) = a(x - h)^2 + k$ is:
  a) $y \geq k$ (or) $[k, \infty)$ when $a > 0$ (as the parabola opens up when $a > 0$).
  b) $y \leq k$ (or) $(-\infty, k]$ when $a < 0$ (as the parabola opens down when $a < 0$).

Maxima and Minima of Quadratic Function

Maxima or minima of quadratic functions occur at its vertex. It can also be found by using differentiation.

a) If $a > 0$, the parabola will open upwards and hence it will have a minima.
   Minimum value – the minimum value of $f(x) = ax2 + bx + c$ where $a > 0$, is the y-coordinate of the vertex.
b) If $a < 0$, the parabola will open downwards and hence it will have a maxima.
   Maximum value – the maximum value of $f(x) = ax2 + bx + c$ where $a < 0$, is the y-coordinate of the vertex.

Graphing Quadratic Function

The graph of a quadratic function is a parabola. i.e., it opens up or down in the U-shape. Here are the steps for graphing a quadratic function.

Step 1: Find the vertex.

Step 2: Compute a quadratic function table with two columns x and y with 5 rows (we can take more rows as well) with vertex to be one of the points and take two random values on either side of it.

Step 3: Find the corresponding values of y by substituting each x value in the given quadratic function.

Step 4: Now, we have two points on either side of the vertex so that by plotting them on the coordinate plane and joining them by a curve, we can get the perfect shape. Also, extend the graph on both sides. Here is the quadratic function graph.

❖ <u>VARIATIONS</u>

A variation is a relation between a set of values of one variable and a set of values of other variables.

Four Types of Variation

1. **Direct Variation**
2. **Inverse Variation**
3. **Joint Variation**
4. **Combined Variation**

Direct Variation

There is direct variation whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:

"y varies directly as x"
"y is directly proportional to x" and
"y is proportional to x"

may be translated mathematically as $y = kx$, where k is the constant of variation.

For two quantities, x and y, an increase in x causes an increase in y as well. Similarly, a decrease in x causes a decrease in y.

Inverse Variation

Inverse variation occurs whenever a situation produces pairs of numbers whose product is constant. It is when one of the variables increases, the other one decreases (their product is constant). For example, the temperature in my house varies indirectly (same or inversely) with the amount of time the air conditioning is running. Or, the number of people I invite to my bowling party varies inversely with the number of games they might get to play (or you can say is proportional to the inverse of).

For two quantities x and y, an increase in x causes a decrease in y or vice versa. We can say that y varies inversely as x or y = $\frac{k}{x}$.

The statement, "y varies inversely to x," translates to y = $\frac{k}{x}$, where k is the constant of variation.

Joint Variation

Joint variation is just like direct variation, but involves more than one other variable. All the variables are directly proportional, taken one at a time. It is when at least two variables are related directly. For example, the area of a triangle is jointly related to both its height and base.

The statement a varies jointly as b and c means a = kbc, or k = $\frac{a}{bc}$, where k is the constant of variation.

Combined Variation

Combined Variation involves a combination of direct or joint variation, and indirect variation. For example, the average number of phone calls per day between two cities has found to be jointly proportional to the populations of the cities, and inversely proportional to the square of the distance between the two cities.

The statement t varies directly as x and inversely as y means t = $\frac{kx}{y}$, or k = $\frac{ty}{x}$, where k is the constant of variation.

❖ QUADRILATERALS

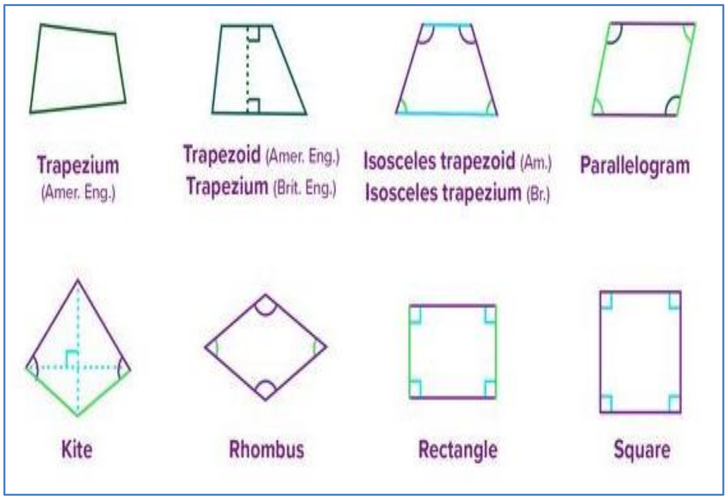
A **quadrilateral** is a closed shape and a type of polygon that has four sides, four vertices and four angles. The angles are present at the four vertices or corners of the quadrilateral. If ABCD is a quadrilateral then angles at the vertices are ∠A, ∠B, ∠C and ∠D. The sides of a quadrilateral are AB, BC, CD and DA. It is formed by joining four non-collinear points. The sum of interior angles of quadrilaterals is always equal to 360 degrees.

The word quadrilateral is derived from the Latin words 'Quadra' which means four and 'Latus' means 'sides'. It is not necessary that all the four sides of a quadrilateral are equal in length. Hence, we can have different types of quadrilaterals based on sides and angles.

Types of Quadrilaterals

The types of quadrilaterals are defined based on the measure of the angles and lengths of their sides. As the word 'Quad' means four, all these types of a quadrilateral have four sides, and the sum of angles of these shapes is 360 degrees. The lists of types of quadrilaterals are:

1. Trapezium
2. Parallelogram
3. Squares
4. Rectangle
5. Rhombus
6. Kite



Convex, Concave and Intersecting Quadrilaterals

Another way to classify the types of quadrilaterals are:

a) **Convex Quadrilaterals**: Both the diagonals of a quadrilateral are completely contained within a figure.

b) **Concave Quadrilaterals**: At least one of the diagonals lies partly or entirely outside of the figure.

c) **Intersecting Quadrilaterals**: Intersecting quadrilaterals are not simple quadrilaterals in which the pair of non-adjacent sides intersects. These kinds of quadrilaterals are known as self-intersecting or crossed quadrilaterals

Properties of Quadrilateral

- Every quadrilateral has 4 vertices, 4 angles, and 4 sides
- The total of its interior angles = 360 degrees
- It has four sides: AB, BC, CD, and DA
- It has four vertices: Points A, B, C, and D
- It has four angles: ∠ABC, ∠BCD, ∠CDA, and ∠DAB
- ∠A and ∠B are adjacent angles
- ∠A and ∠C are the opposite angles
- AB and CD are the opposite sides
- AB and BC are the adjacent sides

Properties of Trapezium

- Only one pair of the opposite side of a trapezium is parallel to each other
- The two adjacent sides of a trapezium are supplementary (180 degrees)
- The diagonals of a trapezium bisect each other in the same ratio

Properties of Parallelogram

- The opposite side of the parallelogram are of the same length
- The opposite sides are parallel to each other
- The diagonals of a parallelogram bisect each other
- The opposite angles are of equal measure
- The sum of two adjacent angles of a parallelogram is equal to 180 degrees

Properties of Square

- All the sides of the square are of equal measure
- The sides are parallel to each other
- All the interior angles of a square are at 90 degrees (i.e., right angle)
- The diagonals of a square perpendicular bisect each other

Properties of Rectangle

- The opposite sides of a rectangle are of equal length
- The opposite sides are parallel to each other
- All the interior angles of a rectangle are 90 degrees.
- The diagonals of a rectangle bisect each other.

Properties of Rhombus

- All the four sides of a rhombus are of the same measure
- The opposite sides of the rhombus are parallel to each other
- The opposite angles are of the same measure
- The sum of any two adjacent angles of a rhombus is equal to 180 degrees
- The diagonals perpendicularly bisect each other

Properties of Kite

- The pair of adjacent sides of a kite are of the same length
- The largest diagonal of a kite bisect the smallest diagonal

- Only one pair of opposite angles are of the same measure.

QUADRILATERAL FORMULAS

There are two basic formulas for quadrilaterals, which are:

1. Area
   - Quadrilateral ABCD = (1/2) × d × h1 h 1 + (1/2) × d × h2 h 2 = (1/2) × d × (h1+h2 h 1 + h 2).
   - Parallelogram = Base x Height
   - Rectangle = Length x Width
   - Square = Side x Side
   - Rhombus = (1/2) x Diagonal 1 x Diagonal 2
   - Kite = 1/2 x Diagonal 1 x Diagonal 2
2. Perimeter
   - Quadrilateral = AB + BC + CD + AD
   - Parallelogram = 2(Base + Side)
   - Rectangle = 2(Length + Breadth)
   - Square = 4 x Side
   - Rhombus = 4 x Side
   - Kite = 2 (a + b), a and b are adjacent pairs

NOTES:

- **Adjacent Angles** – two angles sharing a common side and vertex but no interior points in common
- **Base Angles** – angles formed by a base and the legs
- **Complementary Angles** – two angles whose sum of the measures is 90°
- **Diagonal** – a line segment joining two non-consecutive vertices of a polygon
- **Vertical Angles** – two nonadjacent angles formed by two intersecting lines
- **Supplementary Angles** – two angles whose sum of the measures is 180°
- **Theorem** – a statement that needs to be proven before being accepted

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❖ TRIGONOMETRY

Trigonometry is one of the important branches in the history of mathematics that deals with the study of the relationship between the sides and angles of a right-angled triangle. This concept is given by the Greek mathematician Hipparchus. Trigonometry was derived from the Greek word ‘trigonon’ which means “triangle” and ‘metron’ which means “measure." Hence, from the derivations alone, we can conclude that trigonometry is all about triangles.

Six Trigonometric Ratios

1. **Sine (sin)** - Sine of an angle is defined as the ratio of the side opposite(perpendicular side) to that angle to the hypotenuse.
2. **Cosine (cos)** - Cosine of an angle is defined as the ratio of the side adjacent to that angle to the hypotenuse.
3. **Tangent (tan)** - Tangent of an angle is defined as the ratio of the side opposite to that angle to the side adjacent to that angle.
4. **Cotangent (cot)** - Cotangent is the multiplicative inverse of the tangent.
5. **Secant (sec)** - Secant is a multiplicative inverse of cosine.
6. **Cosecant (csc)** - Cosecant is a multiplicative inverse of sine.

Formula of Six Trigonometric Ratios

$$sin\ of\ \theta = sin\ \theta = \frac{opposite}{hypotenuse}$$

$$cosecant\ of\ \theta = csc\ \theta = \frac{hypotenuse}{opposite}$$

$$cosine\ of\ \theta = cos\ \theta = \frac{adjacent}{hypotenuse}$$

$$secant\ of\ \theta = sec\ \theta = \frac{hypotenuse}{adjacent}$$

$$tangent\ of\ \theta = tan\ \theta = \frac{opposite}{adjacent}$$

$$cotangent\ of\ \theta = cot\ \theta = \frac{adjacent}{opposite}$$

$$csc\ \theta = \frac{hypotenuse}{opposite} = \frac{1}{\frac{opposite}{hypotenuse}} = \frac{1}{sin\theta}$$

$$sec\ \theta = \frac{hypotenuse}{adjacent} = \frac{1}{\frac{adjacent}{hypotenuse}} = \frac{1}{cos\theta}$$

$$cot\ \theta = \frac{adjacent}{opposite} = \frac{1}{\frac{opposite}{adjacent}} = \frac{1}{tan\theta}$$

Trigonometric Ratios Table

The trigonometric ratios for some specific angles such as 0 °, 30 °, 45 °, 60 ° and 90° are given below, which are commonly used in mathematical calculations.

Angle	**0°**	**30°**	**45°**	**60°**	**90°**
Sin C	0	1/2	1/√2	√3/2	1
Cos C	1	√3/2	1/√2	1/2	0
Tan C	0	1/√3	1	√3	∞
Cot C	∞	√3	1	1/√3	0
Sec C	1	2/√3	√2	2	∞
Cosec C	∞	2	√2	2/√3	1

List of Trigonometry Formulas

1. **Pythagorean Identities**
    a) $\sin^2\theta + \cos^2\theta = 1$
    b) $\tan^2\theta + 1 = \sec^2\theta$
    c) $\cot^2\theta + 1 = \csc^2\theta$
    d) $\sin^2\theta = 2 \sin\theta \cos\theta$
    e) $\cos^2\theta = \cos^2\theta - \sin^2\theta$
    f) $\tan^2\theta = 2 \tan\theta / (1 - \tan^2\theta)$
    g) $\cot^2\theta = (\cot^2\theta - 1) / 2 \cot\theta$
2. **Sum and Difference Identities**

    For angles u and v, we have the following relationships:

    a) sin(u + v) = sin(u)cos(v) + cos(u)sin(v)

b) $\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$
c) $\tan(u+v) = \frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)}$
d) $\sin(u - v) = \sin(u)\cos(v) - \cos(u)\sin(v)$
e) $\cos(u - v) = \cos(u)\cos(v) + \sin(u)\sin(v)$
f) $\tan(u-v) = \frac{\tan(u)-\tan(v)}{1+\tan(u)\tan(v)}$

3. **If A, B and C are angles and a, b and c are the sides of a triangle, then,**
    3.1. **Sine Law**
        a) $a/\sin A = b/\sin B = c/\sin C$
    3.2. **Cosine Law**
        a) $c^2 = a^2 + b^2 - 2ab \cos C$
        b) $a^2 = b^2 + c^2 - 2bc \cos A$
        c) $b^2 = a^2 + c^2 - 2ac \cos B$

Applications of Trigonometry

- Its applications are in various fields like oceanography, seismology, meteorology, physical sciences, astronomy, acoustics, navigation, electronics, etc.
- It is also helpful to measure the height of the mountain, find the distance of long rivers, etc.

NOTES:

- **Acute Triangle** – a triangle whose angles are all less than 90°
- **Adjacent Side** – the side next to the reference angle in a right triangle
- **Angle** – the figure formed by two rays, called the sides of the angle, sharing a common endpoint called the vertex.
- **Angle Of Depression** – the angle from the horizontal line to the line of sight of the observer to the object below.
- **Angle Of Elevation** – is the angle from the horizontal to the line of sight of the observer to the object above it
- **Clinometer** – a device used to measure angles of elevation or depression
- **Line Of Sight** – an imaginary line that connects the eye of an observer to the object being observed
- **Oblique Triangle** – a triangle which does not contain any right angle
- **Obtuse Triangle** – a triangle in which one of the angles is more than 90°
- **Opposite Side** – the side across the reference angle in a right triangle
- **Ratio** – a relationship between two numbers of the same kind
- **Right Triangle** – a triangle in which one angle is a right angle (that is, a 90-degree angle). The relation between the sides and angles of a right triangle is the basis for trigonometry.
- **Special Angles** – angles with a reference angle of 30°, 45°, or 60°
- **Triangle** – one of the basic shapes in geometry: a polygon with three vertices and three sides, which are line segments

References:

https://myteachermommy.com/2022/06/10/grade-9-deped-modules-and-grade-9-modules-downloads/
https://drive.google.com/file/d/19vzmuGvzhFzam6U20RmDl-gYEURElQM-/view
https://www.cuemath.com/calculus/quadratic-functions/
http://dl.uncw.edu/digilib/Mathematics/Algebra/mat111hb/PandR/quadratic/quadratic.html#:~:text=A%20quadratic%20function%20is%20one,same%20basic%20%22U%22%20shape.
https://www.cliffsnotes.com/study-guides/algebra/algebra-i/functions-and-variations/variations#:~:text=A%20variation%20is%20a%20relation,is%20called%20a%20direct%20variation.
https://mathhints.com/direct-inverse-and-joint-variation/
https://byjus.com/maths/quadrilateral/
https://byjus.com/maths/trigonometry/#:~:text=Trigonometry%20is%20one%20of%20the,by%20the%20Greek%20mathematician%20Hipparchus.
https://www.mathsisfun.com/algebra/trigonometry.html
https://byjus.com/maths/trigonometry/#functions