Chapter 1 B: Picard's Method

(a) Use Picard's method with $\psi_0(x) = 1$ to obtain the next four successive approximations of the solution to

$$y'(x) = y(x), \qquad y(0) = 1$$

Show that these approximations are just the partial sums of the Maclaurin series for the actual solution e^x.

Solution

(b) Use Picard's method with $\psi + 0(x) = 0$ to obtain the next three successive approximations of the solution to the nonlinear problem

$$y'(x) = 3x - \left[y(x)^2\right], \qquad y(0) = 0$$

Graph these approximations for $0 \leq x \leq 1$.

Solution

(c) In Problem 29 in Exercises 1.2, we showed that the initial value problem

$$y'(x) = 3\left[y(x)^{2/3}, y(2) = 0\right]$$

does not have a unique solution. Show that Picard's method beginning with $\psi_0(x) = 0$ converges to the solution $y(x) = 0$, whereas Picard's method beginning with $\psi_0(x) = x - 2$ converges to the second solution $y(x) = (x - 2)^3$.

Solution