

AN ABSTRACT OF THE DISSERTATION OF

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Over the last two decades, satisfiability and satisfiability-modulo theory (SAT/SMT) solvers have grown powerful enough to be general purpose reasoning engines throughout software engineering and computer science. However, most practical use cases of SAT/SMT solvers require not just solving a single SAT/SMT problem, but solving sets of related SAT/SMT problems. This discrepancy was directly addressed by the SAT/SMT community with the invention of incremental SAT/SMT solving. However, incremental SAT/SMT solvers require end-users to hand write a program which dictates the terms that are shared between problems and terms which are unique. By placing the onus on end-users, incremental solvers couple the end-users' solution to the end-users' *exact* sequence of SAT/SMT problems—making the solution overly specific—and require the end-user to write extra infrastructure to coordinate or handle the results.

This dissertation argues that the aforementioned problems are caused from the lack of variation as a computation concept, similar to that of a `while` loop. To demonstrate the argument, this thesis applies theory from *variational* programming to the domain of SAT/SMT solvers to create the first variational SAT solver. The thesis formalizes a variational propositional logic and specifies variational SAT solving as a transpiler, which transpiles variational SAT problems to non-variational SAT that are then processed by an industrial SAT solver. It shows that the transpiler is an instance of a variational fold and uses that fact to extend the variational SAT solver to an asynchronous variational SMT solver. Finally, it defines a general algorithm to construct a single variational string from a set of non-variational strings.

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Variational Satisfiability Solving

by

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APPROVED:

Major Professor, representing Computer Science

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I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

Jeffrey M. Young, Author

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Chapter 1: Introduction

One of the most important aspects of any programming language is the ability to control complexity, especially as software written in that language grows. The burgeoning field of *variation theory* and *variational programming* [7, 12, 13, 17, 30] attempt to control complexity which is induced into a software artifact when many *similar yet distinct* kinds of the same software artifact must coexist. For example, software is often *ported* to other platforms, creating similar, yet distinct instances of that software which must be maintained. Such instances of variation are ubiquitous: Web applications are tested on multiple servers; programming languages maintain backwards compatibility and so do software libraries; databases evolve over time, locale and data; and device drivers must work with varying processors and architectures. Variation theory and variational programming have been successful in small systems, yet it has not been tested in a performance demanding practical domain. In the words of Joe Armstrong [2], “No theory is complete without proof that the ideas work in practice”; this is the central project of this thesis, to put the ideas of *variation* and *variational programming* to the test in the practical domain of satisfiability solving (SAT).

cite variational data structures and images

The major contribution of this thesis is the formalization of a *variational propositional logic* (VPL), *variational satisfiability solving*, and the construction of a *variational SAT solver*. In the next section I motivate the use of variation theory and variational techniques in satisfiability solving. In addition to work on variational SAT several other contributions are made. The thesis extends variational satisfiability solving to variational satisfiability-modulo theories (SMT). It demonstrates reusable techniques and architecture for constructing *variational or variation-aware* systems using the non-variational counterparts of these systems for other domains. It shows that, with the concept of variation, the variational SMT and SAT solvers can be trivially parallelized. Lastly, the thesis provides a general algorithm to construct variational strings from a set of non-variational strings and argues for the proliferation of variation theory to other domains in computer science.

1.1 Motivation and Impact

Classic SAT, which solves the boolean satisfiability problem [5] has been one of the largest success stories in computer science over the last two decades. Although SAT solving is known to be NP-complete [8], SAT solvers based on conflict-driven clause learning (CDCL) [3, 20, 24] have been able to solve boolean formulae with millions variables quickly enough for use in real-world applications [28]. Leading to their proliferation into several fields of scientific inquiry ranging from software engineering to Bioinformatics [16, 19].

The majority of research in the SAT community focuses on solving a single SAT problem as fast as possible, yet many practical applications of SAT solvers [6, 9, 10, 14, 23, 25, 31] require solving a set of related SAT problems [10, 23, 25]. To take just one example, software product-lines (SPL) utilizes SAT solvers for a diverse range of analyses including: automated feature model analysis [4, 15, 27], feature model sampling [22, 29], anomaly detection [1, 18, 21], and dead code analysis [26].

This misalignment between the SAT research community and the practical use cases of SAT solvers is well known. To address the misalignment, modern solvers attempt to propagate information from one solving instance, on one problem, to future instances in the problem set. Initial attempts focused on clause sharing (CS) [23, 31] where learned clauses from one problem in the problem set are propagated forward to future problems. Although, modern solvers are based on a major breakthrough that occurred with *incremental SAT under assumptions*, introduced in Minisat [11].

Incremental SAT under assumptions, made two major contributions: a performance contribution, where information including learned clauses, restart and clause-detection heuristics are carried forward. A usability contribution; Minisat exposed an interface that allowed the end-user to directly program the solver. Through the interface the user can add or remove clauses and dictate which clauses or variables are shared and which are unique to the problem set, thus directly addressing the practical use case of SAT solvers.

Despite its success, the incremental interface introduced a programming language that required an extra input, the set of SAT problems, *and* a program to direct the solver with side-effectual statements. This places further burden on the end-user: the system is less-declarative as the user must be concerned with the internals of the solver. A new class of errors is possible as the input program could misuse the introduced side-effectual statements. By requiring the

user to direct the solver, the users' solution is specific to the exact set of satisfiability problems at hand, thus the programmed solution is specific to the problem set and therefore to the solver input. Should the user be interested in the assignment of variables under which the problem at hand was found to be satisfiable, then the user must create additional infrastructure to track results; which again couples to the input and is therefore difficult to reuse.

I argue that solving a set of related SAT problems *is a variational programming problem* and that by directly addressing the problem's variational nature the incremental SAT interface and performance can be improved. The essence of variational programming is a formal language called the *choice calculus*. With the choice calculus, sets of problems in the SAT domain can be expressed syntactically as a single *variational artifact*. The benefits are numerous:

1. The side-effectual statements are hidden from the user, recovering the declarative nature of non-incremental SAT solving.
2. Malformed programs built around the control flow operators become syntactically impossible.
3. The end-user's programmed solution is decoupled from the specific problem set, increasing software reuse.
4. The solver has enough syntactic information to produce results which previously required extra infrastructure constructed by the end-user.
5. Previously difficult optimizations can be syntactically detected and applied before the runtime of the solver.

This work is applied programming language theory in the domain of satisfiability solvers. Due to the ubiquity of satisfiability solvers estimating the impact is difficult although the surface area of possible applications is large. For example, many analyses in the software product-lines community use incremental SAT solvers. By creating a variational SAT solver such analyses directly benefit from this work, and thus advance the state of the art. For researchers in the incremental satisfiability solving community, this work serves as an avenue to construct new incremental SAT solvers which efficiently solve classes of problems that deal with variation.

For researchers studying variation the significance and impact is several fold. By utilizing results in variational research, this work adds validity to variational theory and serves as an empirical case study. At the time of this writing, and to my knowledge, this work is the first

to directly use results in the variational research community to parallelize a variation unaware tool. Thus by directly handling variation, this work demonstrates direct benefits to be gained for researchers in other domains and magnifies the impact of any results produced by the variational research community. Lastly, the result of my thesis, a variational SAT solver, provides a new logic and tool to reason about variation itself.

1.2 Contributions and Outline of this Thesis

The high-level goal of this thesis is to use variation theory to formalize and construct a variational satisfiability solver that understands and can solve SAT problems that contain *variational values* in addition to boolean values. It is our desire that the work not only be of theoretical interest but of practical use. Thus, the thesis provides numerous examples of variational SAT and variational SMT problems to motivate and demonstrate the solver. The rest of this section outlines the thesis and expands on the contributions of each chapter:

1. [Chapter 2](#)

Chapter 2: Background

Chapter 3: Variational Propositional Logic

Chapter 4: Variational Satisfiability Solving

Chapter 5: Variational Satisfiability-Modulo Theory Solving

Chapter 6: Case Studies

Chapter 7: Related Work

Chapter 8: Conclusion

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APPENDICES

Appendix A: Redundancy

This appendix is inoperable.

