AVL TREES

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CS 146

AVL Tree is…

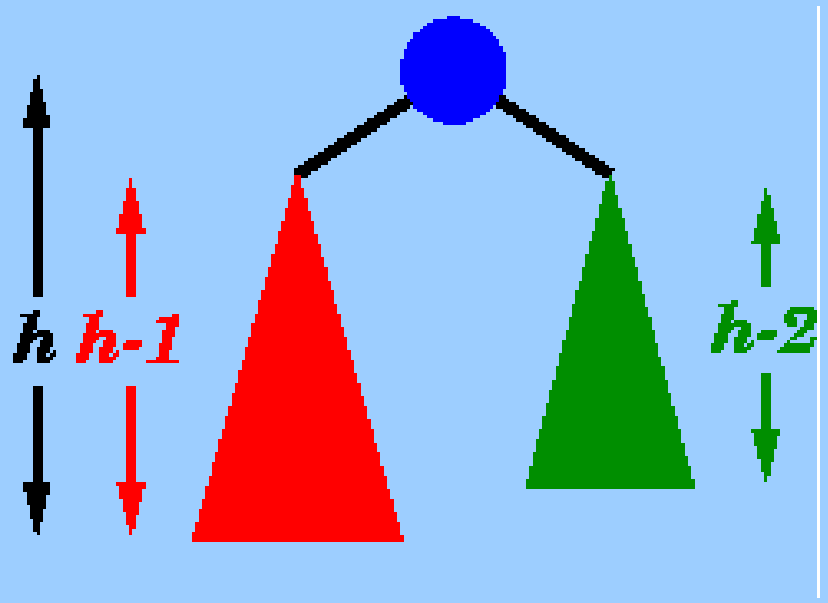
- named after **A**delson-**V**elskii and **L**andis
- the first dynamically balanced trees to be pr opose
- Binary search tree with **balance condition** i n which the sub-trees of each node can diffe r by at most 1 in their height

Definition of a balanced tree

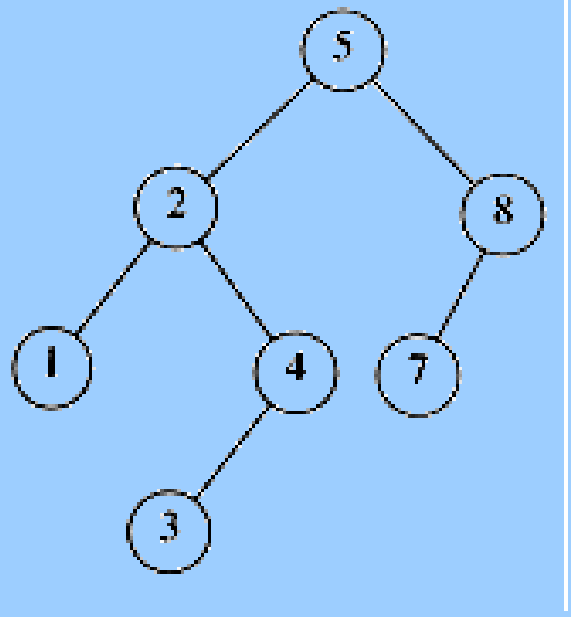
- Ensure the depth = O(log N)
- Take O(log N) time for searching, insertion, and deletion
- Every node must have left & right sub-trees of the same height

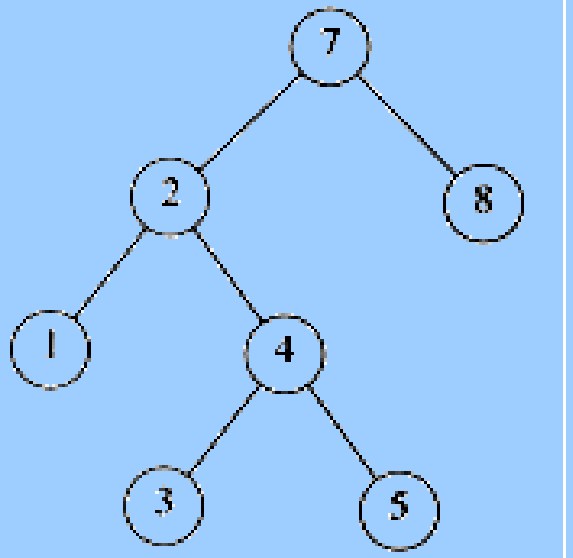
An AVL tree has the following properties:

1. Sub-trees of each node can differ by at most 1 in their height
2. Every sub-trees is an AVL tree



AVL tree?





YES

Each left sub-tree has height 1 greater than each right sub-tree

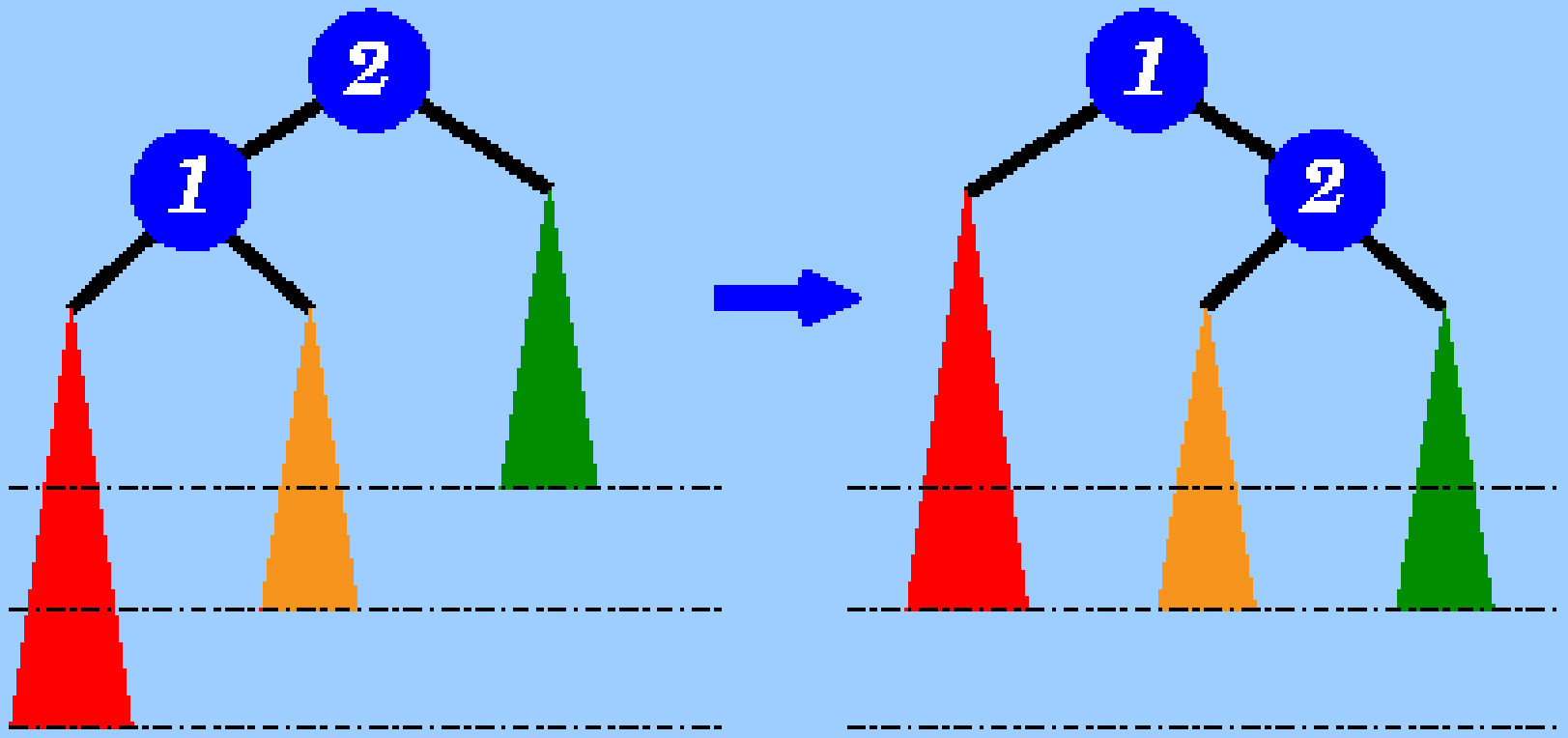
NO

Left sub-tree has height 3, but right sub-tree has height 1

Insertion and Deletions

- It is performed as in binary search trees
- If the balance is destroyed, rotation(s) is performed to correct balance
- For insertions, one rotation is sufficient
- For deletions, O(log n) rotations at most are needed

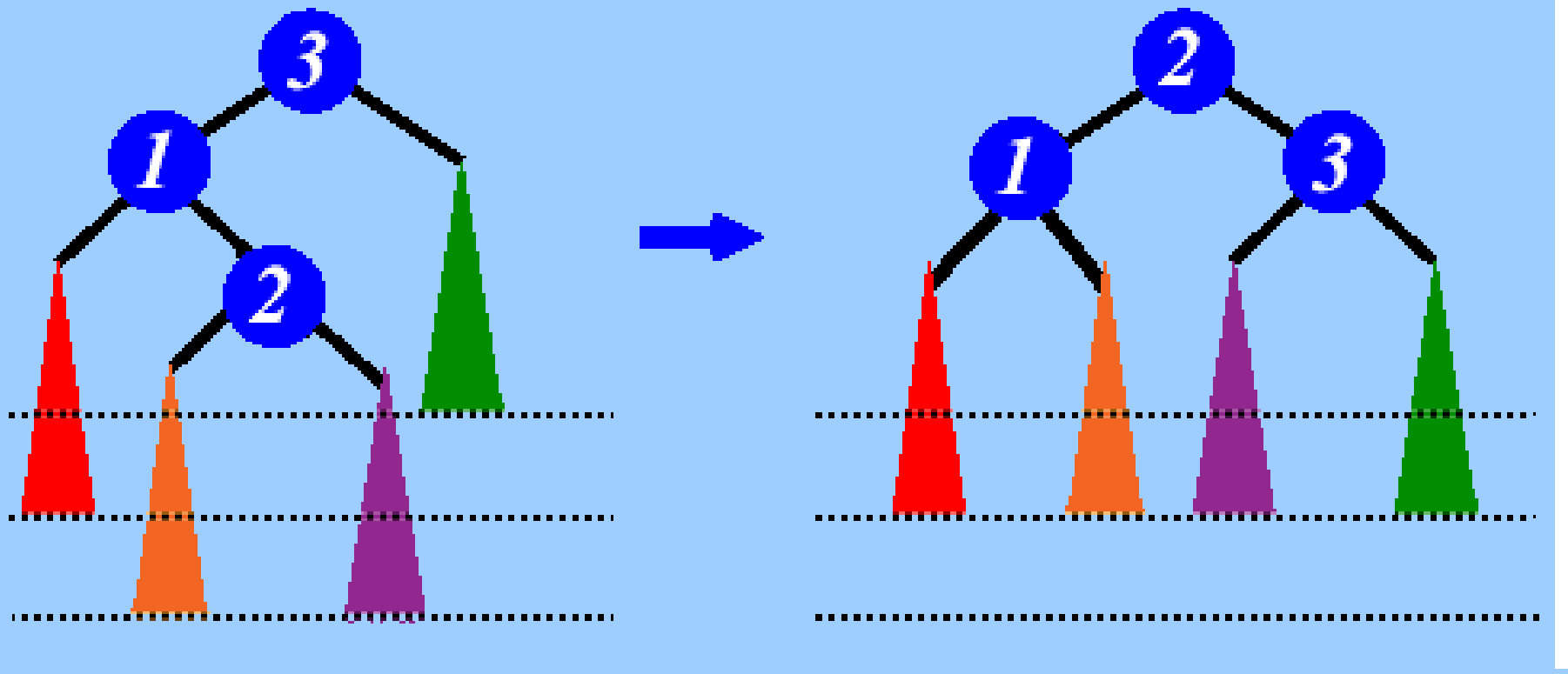
Single Rotation



left sub-tree is two level deeper than the right sub-tree

move ① up a level and ② down a level

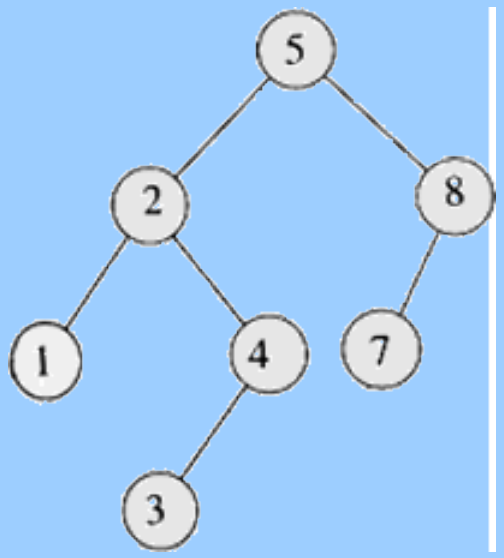
Double Rotation



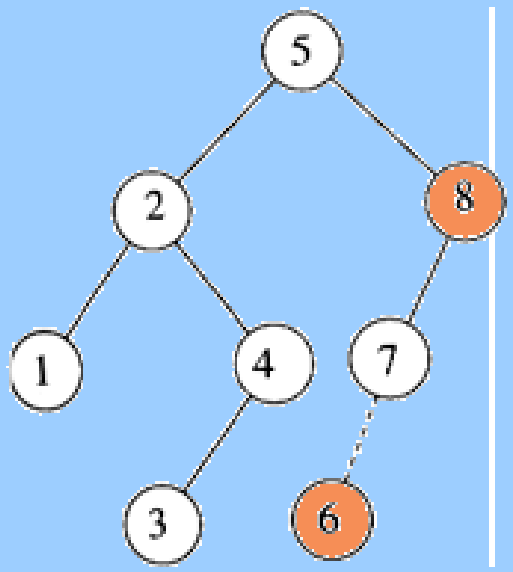
Left sub-tree is two level deeper than the right sub-tree

Move ② up two levels and ③ down a level

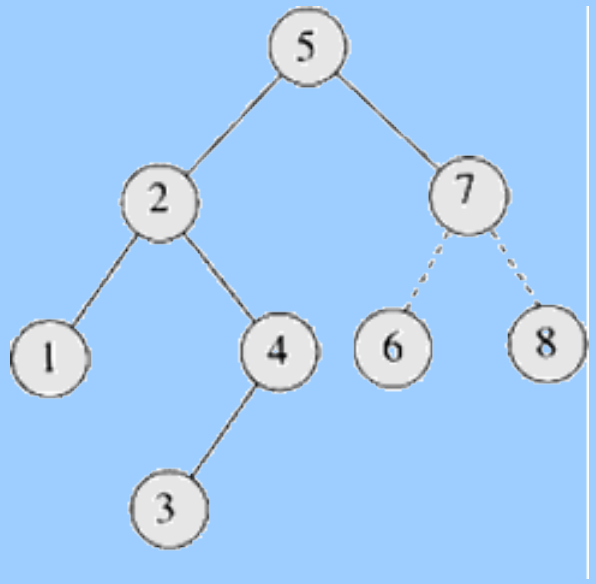
Insertion

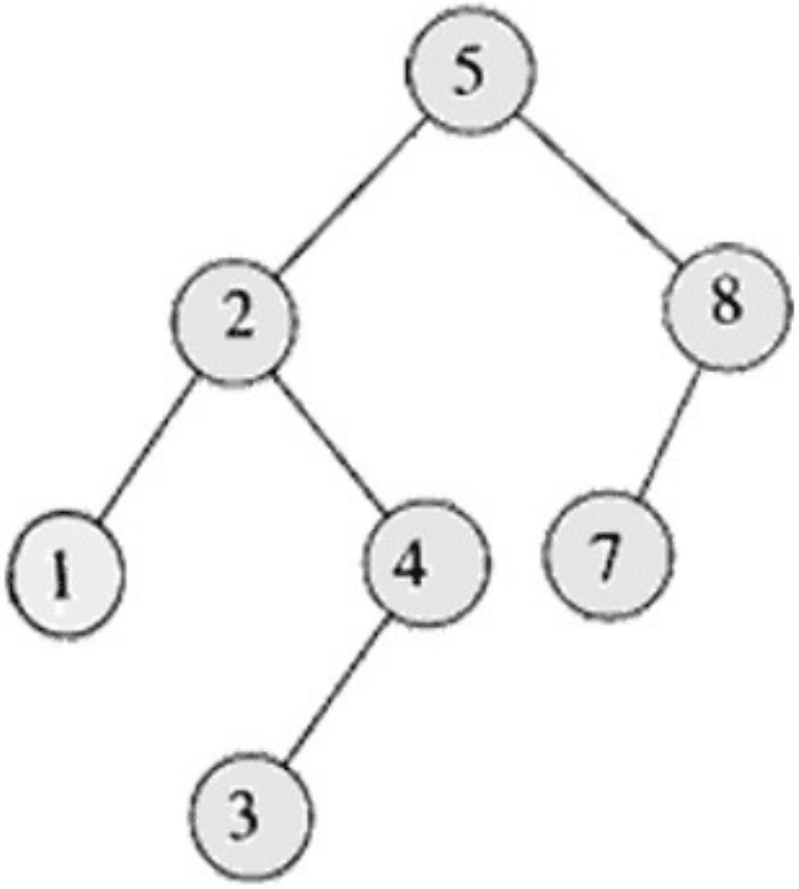


Insert 6

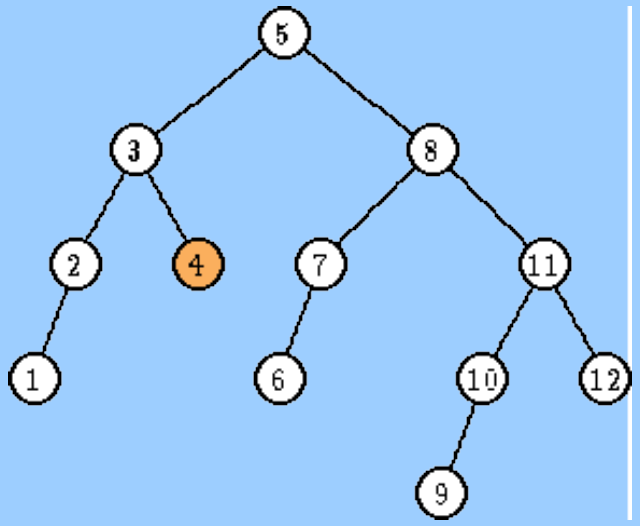


Imbalance at 8
Perform rotation with 7

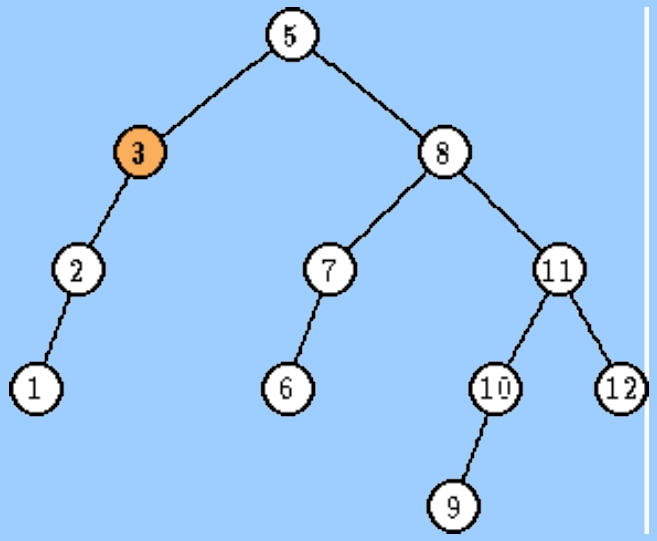



5
2
8
1
4
7
3

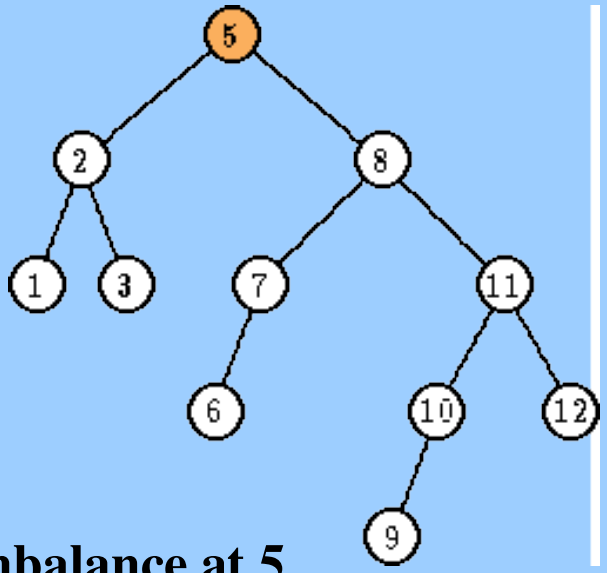
Deletion



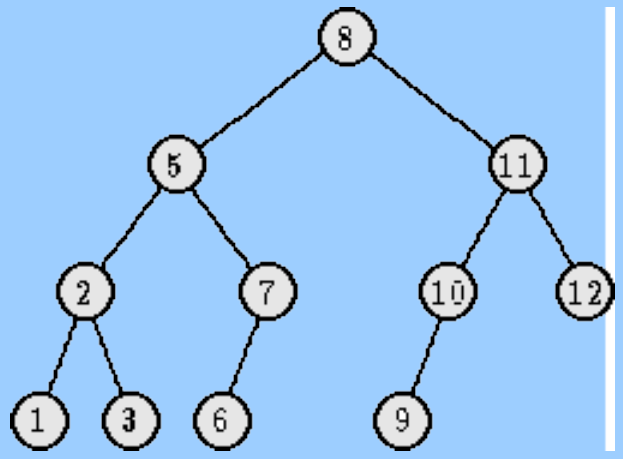
Delete 4

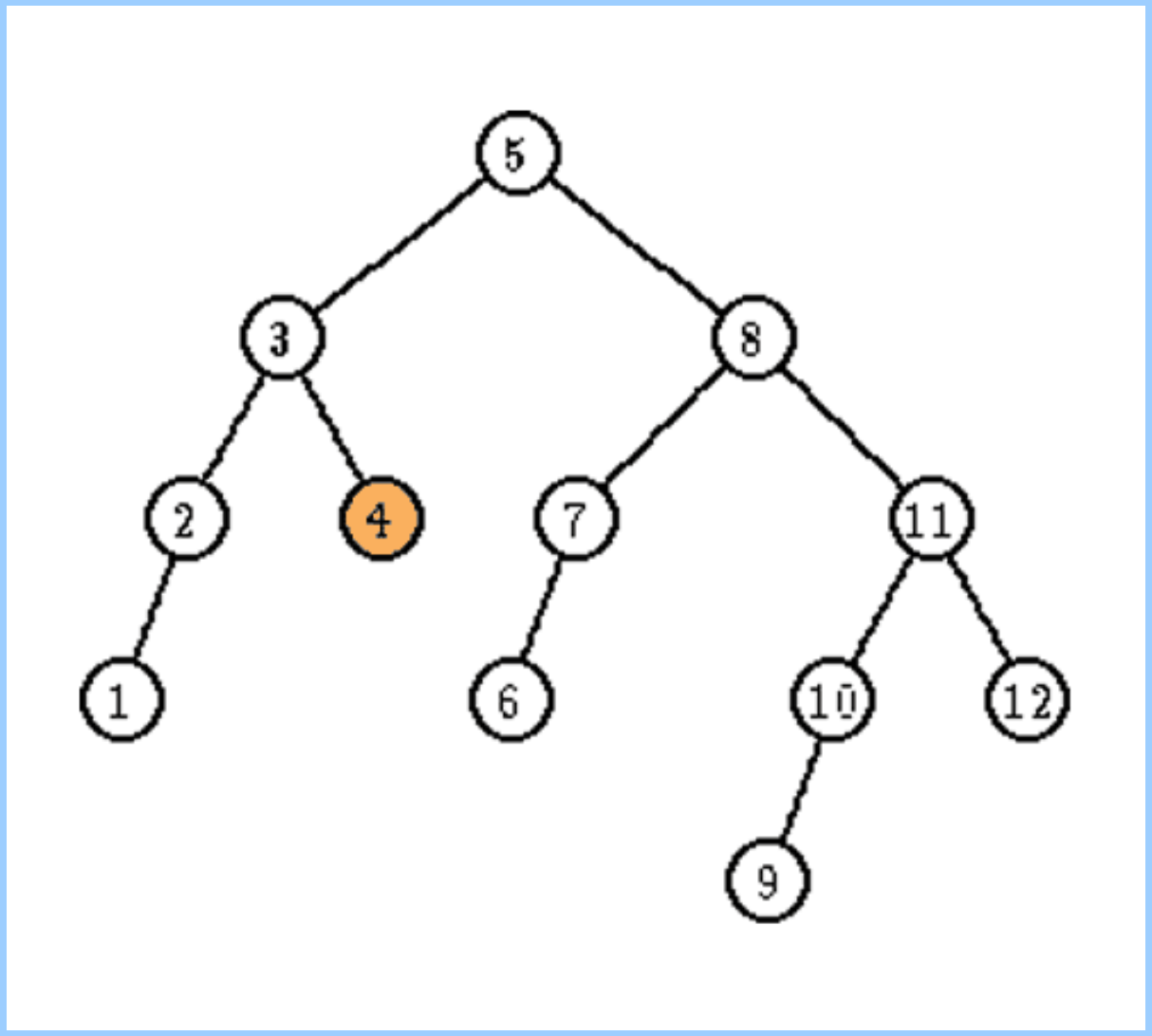


Imbalance at 3
Perform rotation with 2



Imbalance at 5
Perform rotation with 8




5
3
8
2
4
7
11
1
6
10
12
9

Key Points

- AVL tree remain balanced by applying rotations, therefore it guarantees O(log N) search time in a dynamic environment
- Tree can be re-balanced in at most O(log N) time