

# AVL TREES

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CS 146

# AVL Tree is...

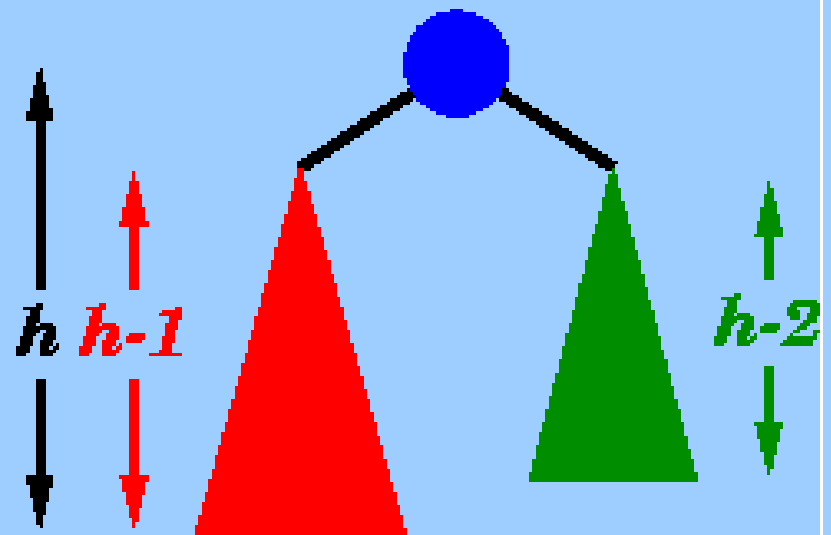
- named after **A**delson-**V**elskii and **L**andis
- the first dynamically balanced trees to be proposed
- Binary search tree with **balance condition** in which the sub-trees of each node can differ by at most 1 in their height

# Definition of a balanced tree

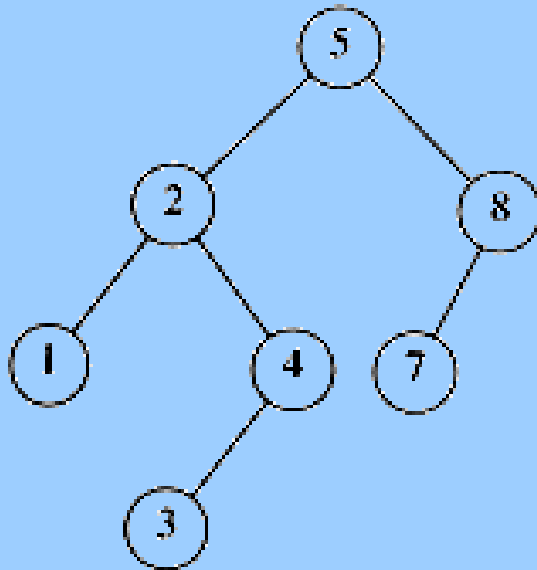
- Ensure the depth =  $O(\log N)$
- Take  $O(\log N)$  time for searching, insertion, and deletion
- Every node must have left & right sub-trees of the same height

# An AVL tree has the following properties:

1. Sub-trees of each node can differ by at most 1 in their height
2. Every sub-trees is an AVL tree

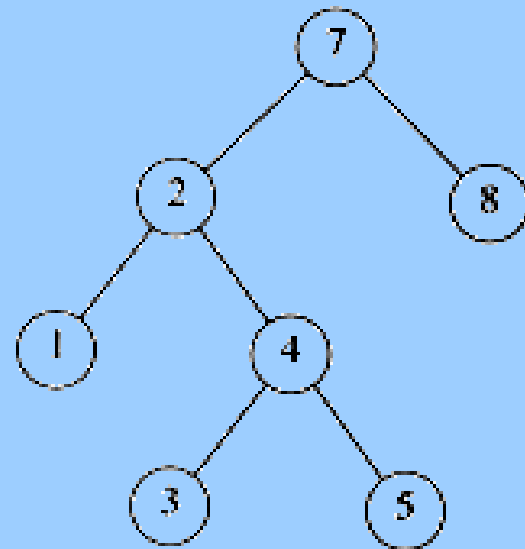


# AVL tree?



YES

Each left sub-tree has height 1 greater than each right sub-tree



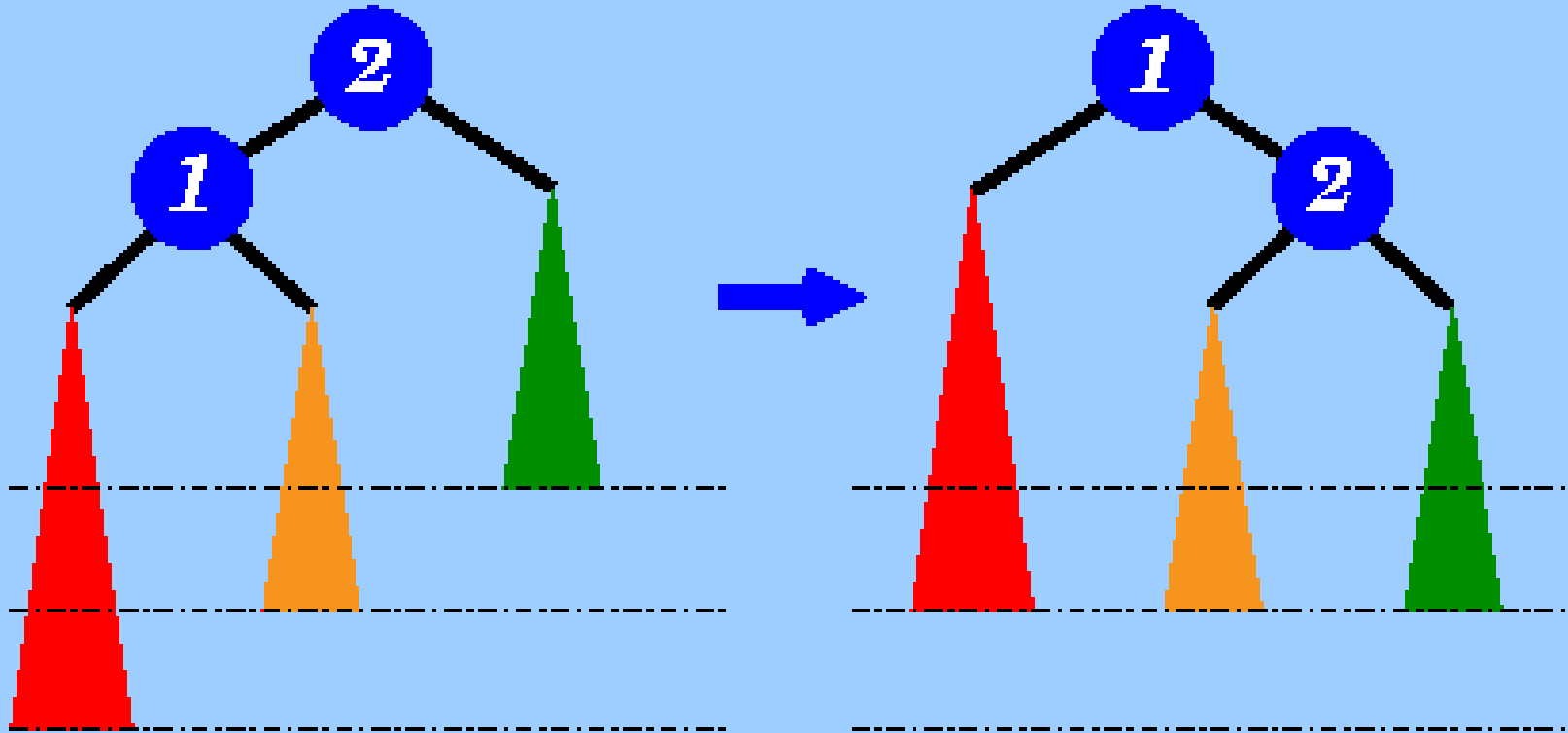
NO

Left sub-tree has height 3, but right sub-tree has height 1

# Insertion and Deletions

- It is performed as in binary search trees
- If the balance is destroyed, **rotation**(s) is performed to correct balance
- For insertions, one rotation is sufficient
- For deletions,  $O(\log n)$  rotations at most are needed

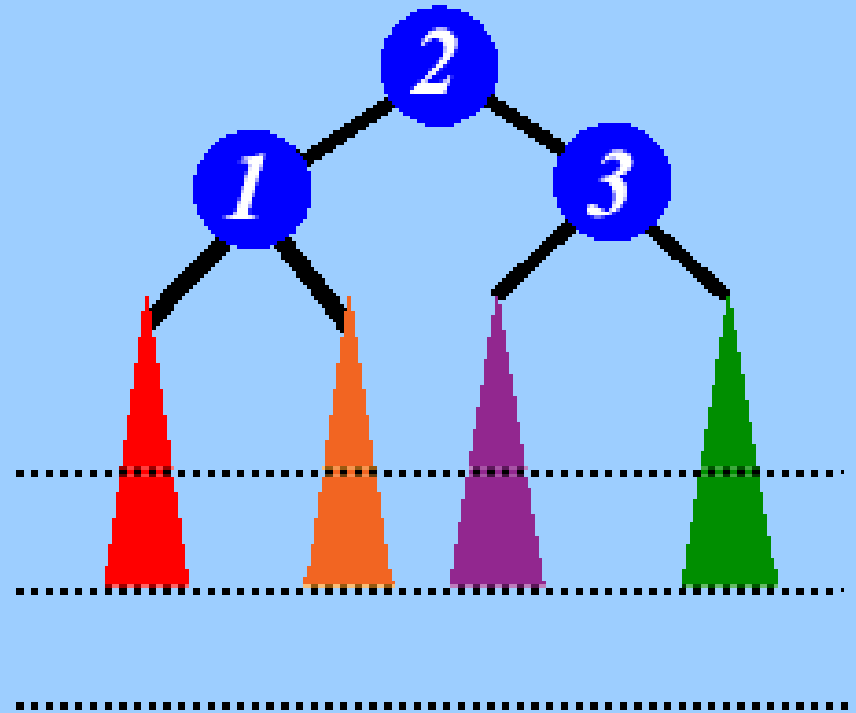
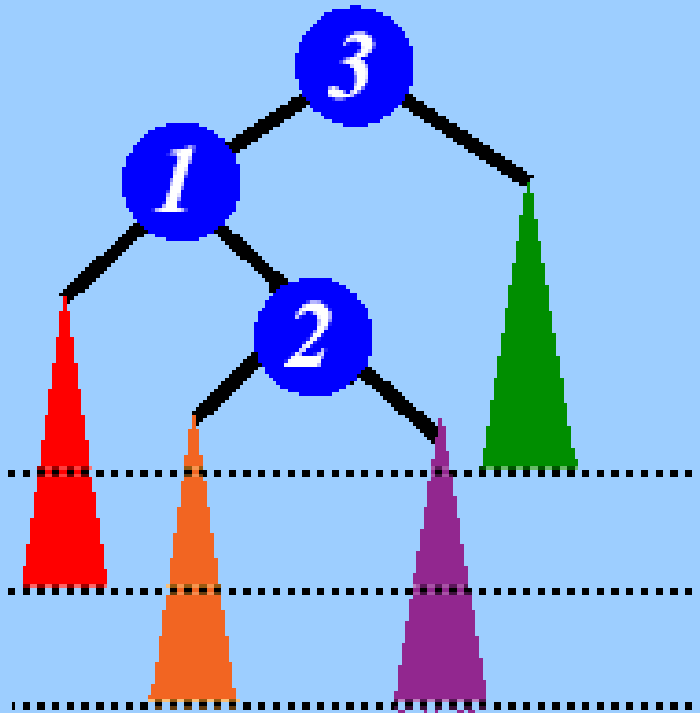
# Single Rotation



left sub-tree is two level  
deeper than the right sub-tree

move ① up a level and  
② down a level

# Double Rotation



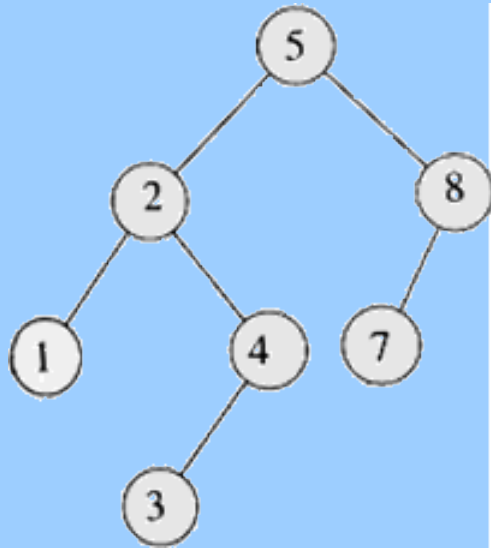
Left sub-tree is two level deeper than the right sub-tree

Move ② up two levels and ③ down a level

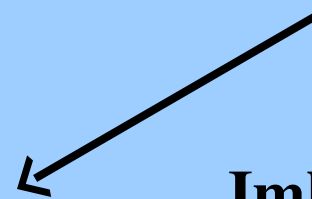
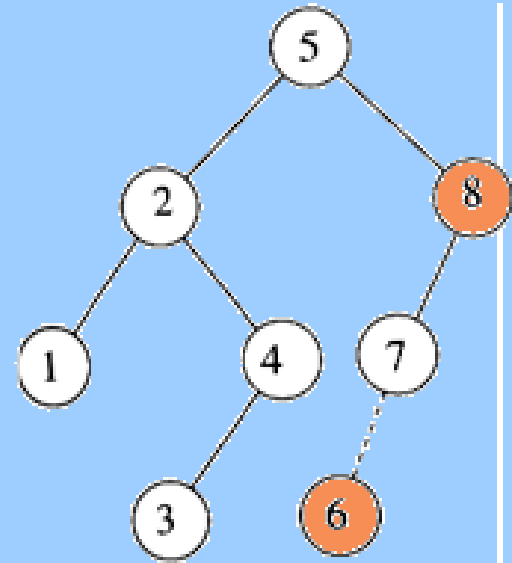


# Insertio

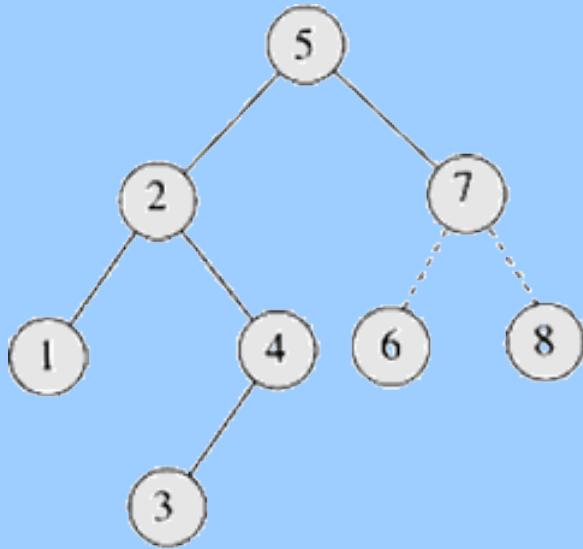
n

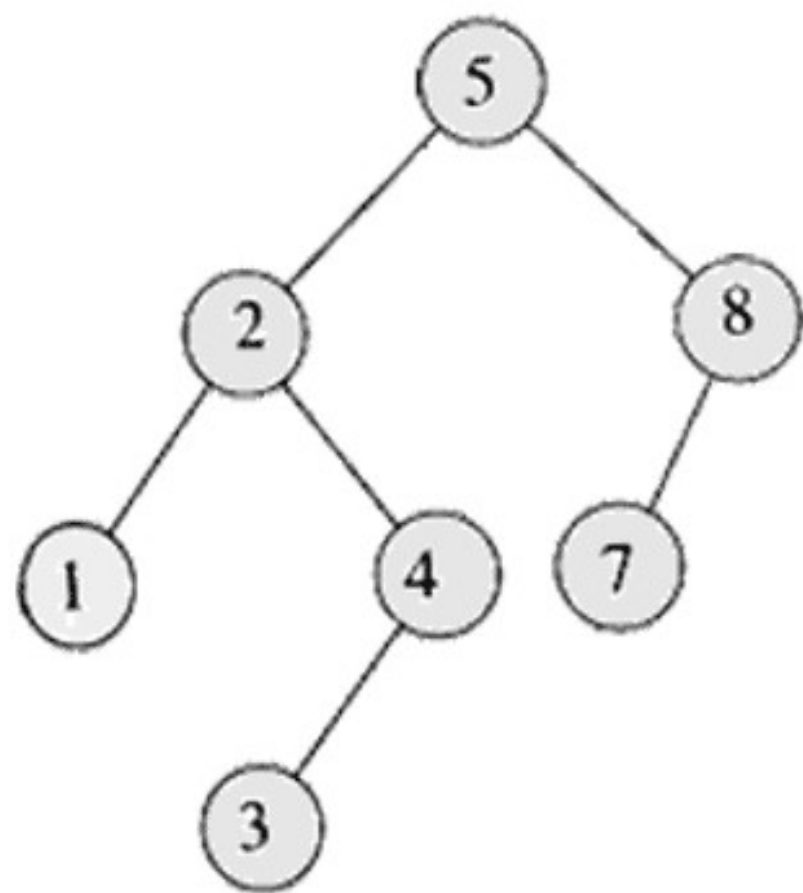


→  
**Insert 6**

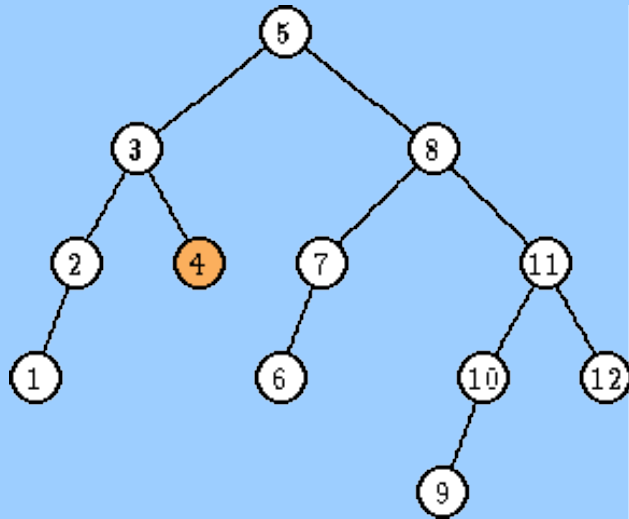


**Imbalance at 8**  
**Perform rotation with 7**

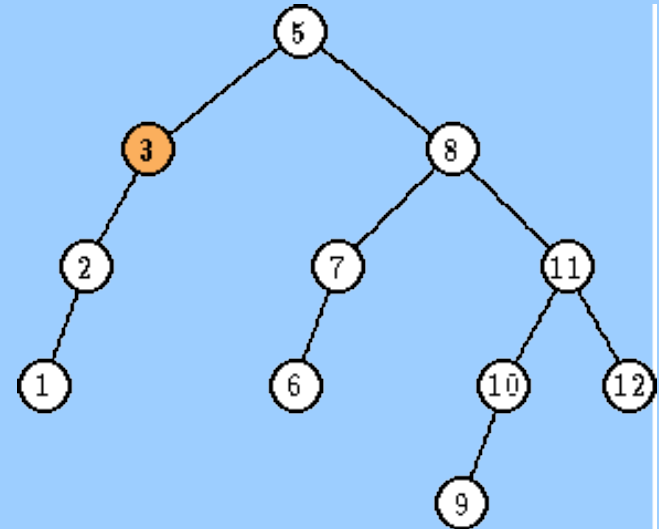




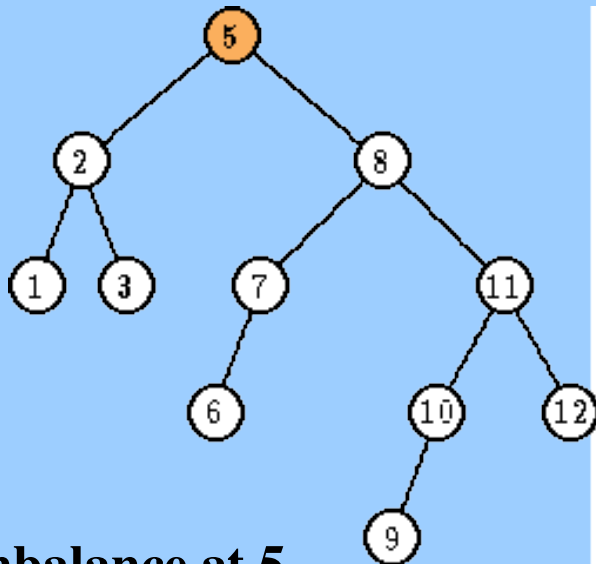
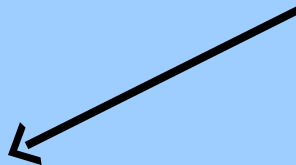
# Deletion



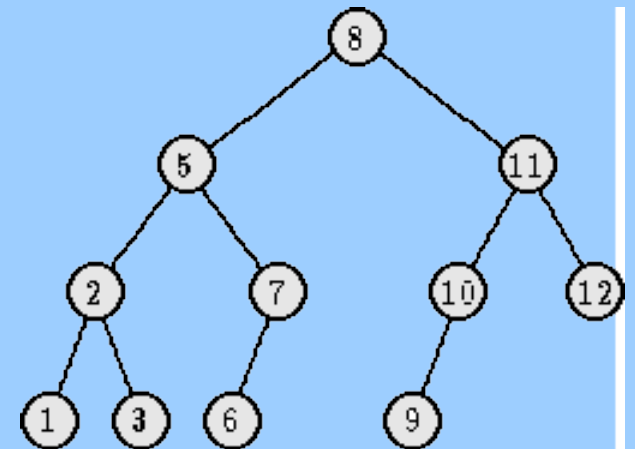
**Delete 4**

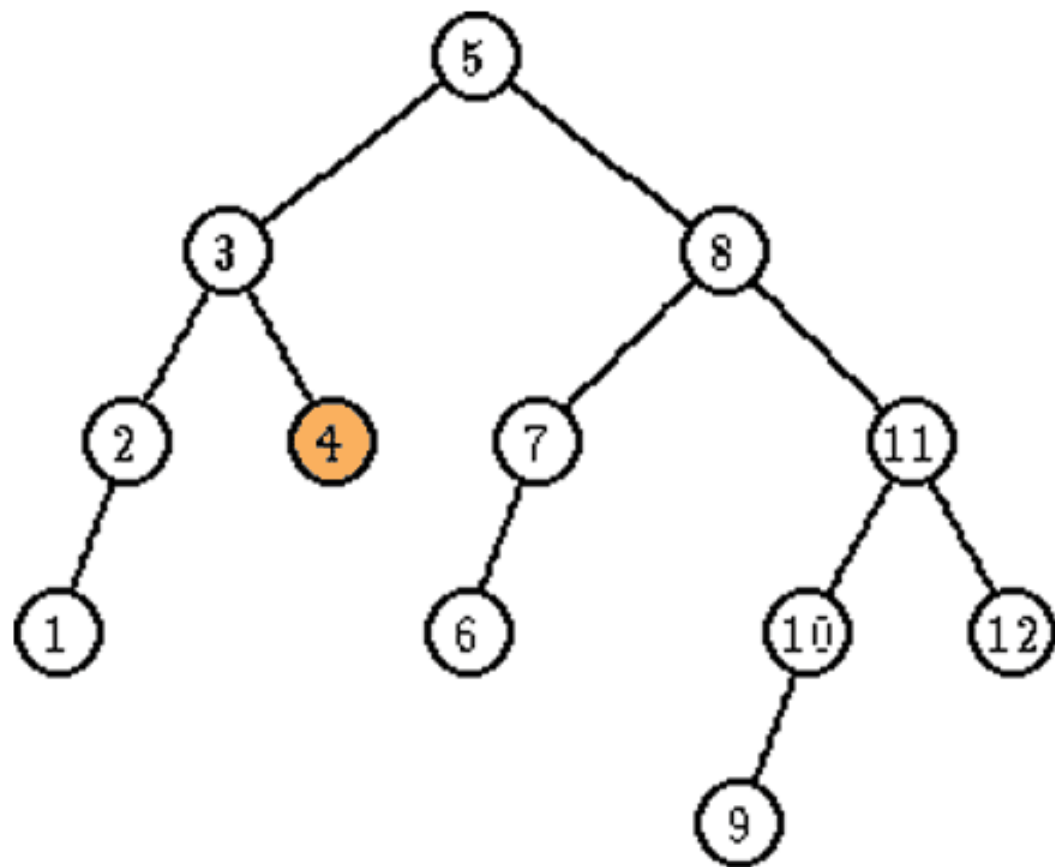


**Imbalance at 3**  
**Perform rotation with 2**



**Imbalance at 5**  
**Perform rotation with 8**





# Key Points

- AVL tree remain **balanced** by applying rotations, therefore it guarantees  **$O(\log N)$**  search time in a dynamic environment
- Tree can be re-balanced in at most  **$O(\log N)$**  time