AVL trees

Today

- AVL Deletes and rotations, then testing your knowledge of these concepts!

- Before I get into details, I want to show you some animated operations in an AVL tree.
- I think it's important to just get the gears turning in your mind.
- We'll look at some animations *again* after we study (some) details.
- Interactive web applet

AVL tree

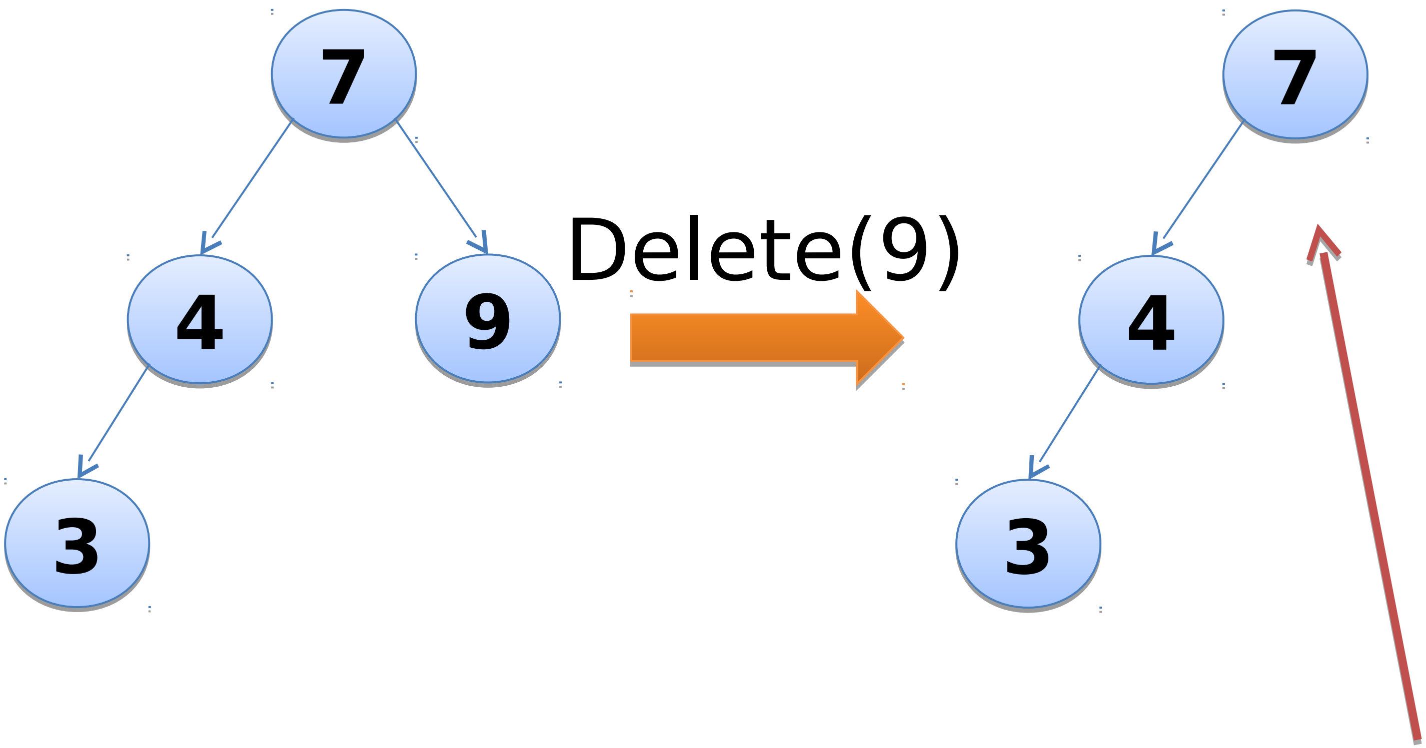
- Is a binary search tree
- Has an additional ***height constraint***:
  - For each node x in the tree, Height(x.left) differs from Height(x.right) by at most 1
- I promise:
  - If you satisfy the ***height constraint***, then the **height of the tree is O(lg n)**.
  - (Proof is easy, but no time! =])

AVL tree

- To be an AVL tree, must **always:**
  - (1) Be a ***binary search tree***
  - (2) Satisfy the ***height constraint***

- Suppose we start with an AVL tree, then delete as if we're in a regular BST.
- Will the tree be an AVL tree after the delete?
  - (1) It will still be a BST… that's one part.
  - (2) Will it satisfy the ***height constraint***?
- (Not covering insert, since you already did in class)

BST Delete breaks an AVL tree

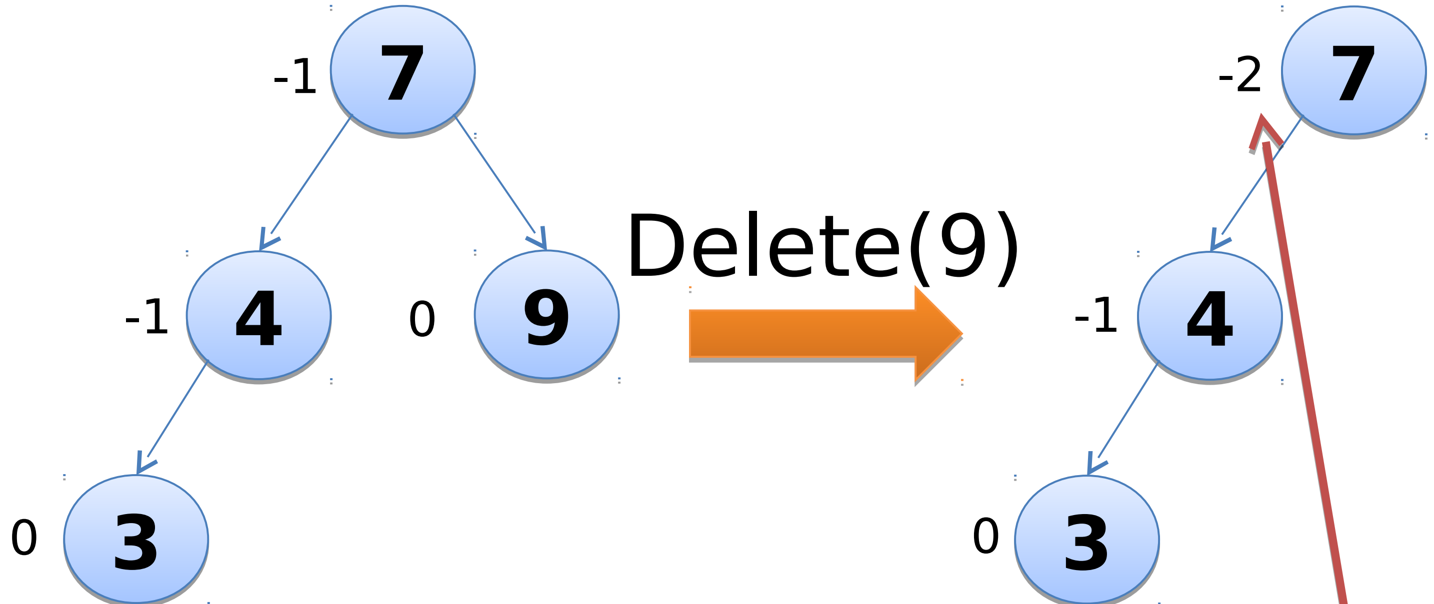


h(left) > h(right)+1

Balance factors

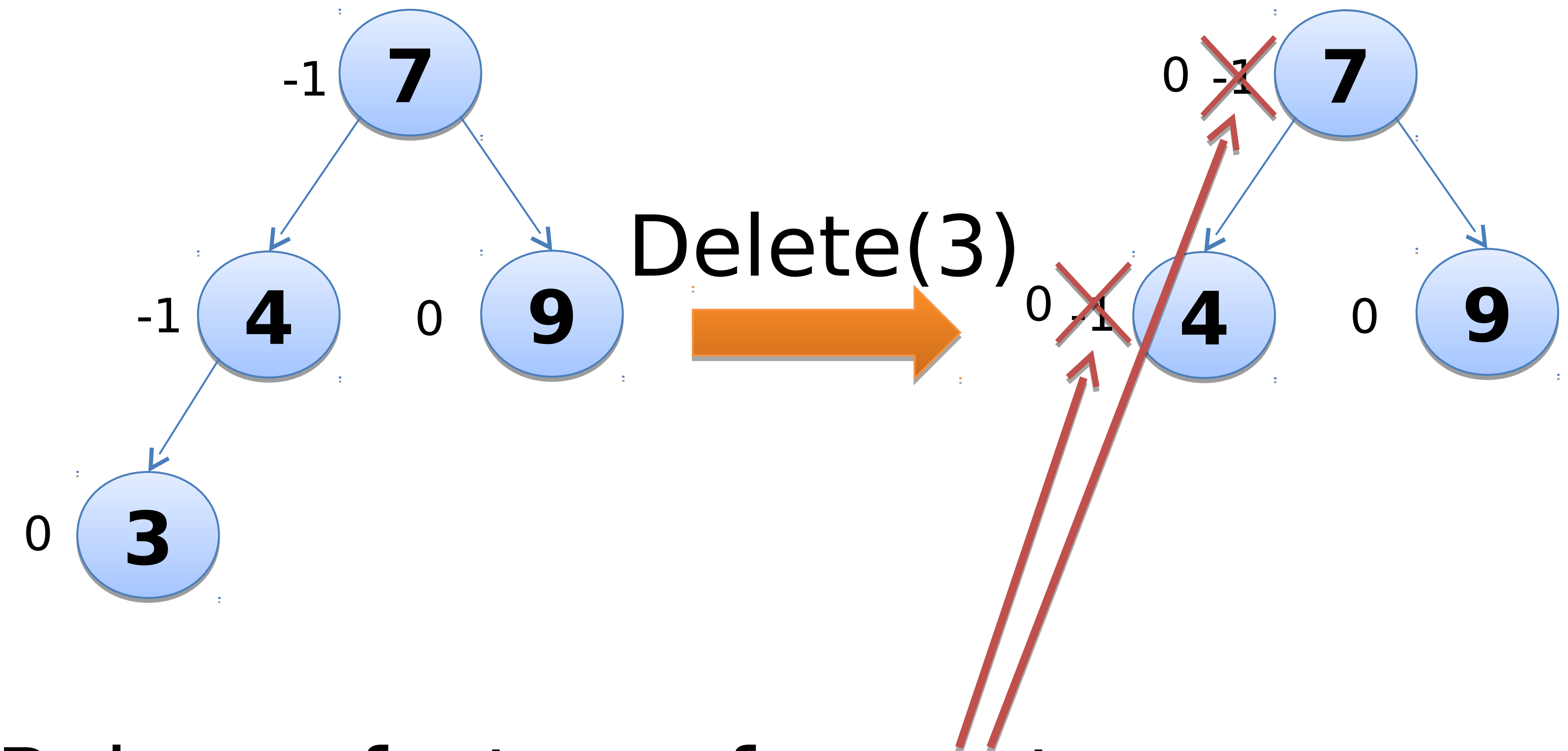
- To check the **balance constraint**, we have to know the ***height h*** of each node
- Or do we?
- In fact, we can store ***balance factors*** instead.
- The balance factor bf(x) = h(x.right) – h(x.left)
  - bf(x) values -1, 0, and 1 are allowed.
  - If bf(x) < -1 or bf(x) > 1 then tree is **NOT AVL**

Same example with **bf(x),** **<u>not</u> h(x)**



bf < -1
so **<u>NOT</u>** an AVL tree

What else can BST Delete break?



- Balance factors of ancestors…

Need a new Delete algorithm

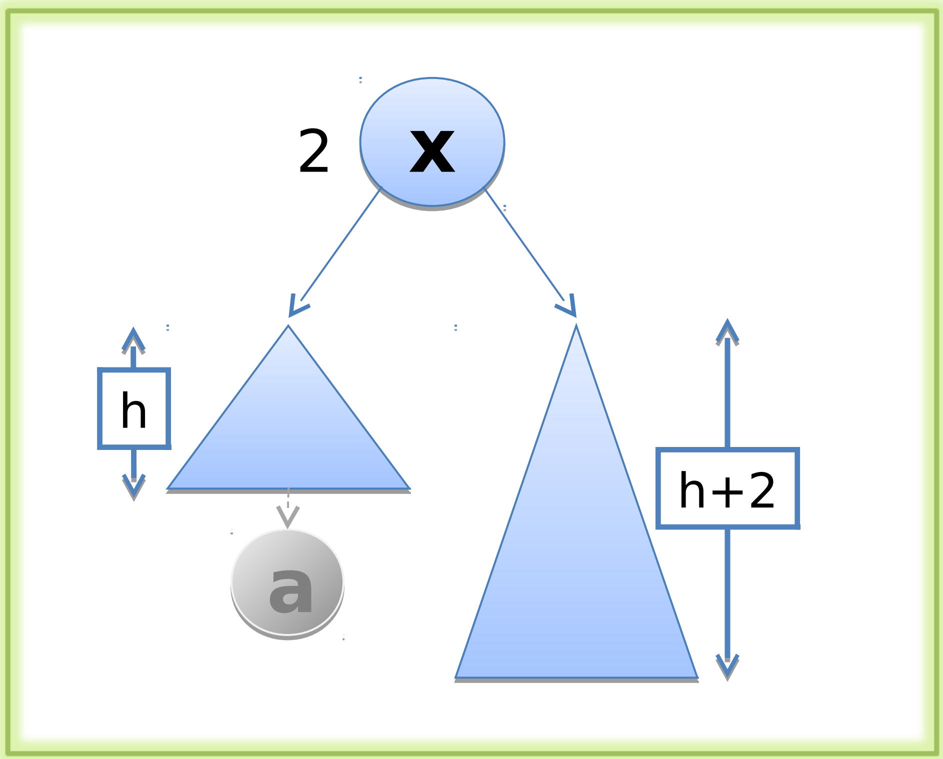
- We are starting to see what our delete algorithm must look like.
- **Goal:** if tree is AVL before Delete, then tree is AVL after Delete.
- **Step 1:** do BST delete.
  - This maintains the *BST property*, but can BREAK the *balance factors of ancestors!*
- **Step 2:** fix the balance constraint.
  - Do something that maintains the BST property, but fixes any balance factors that are < -1 or > 1.

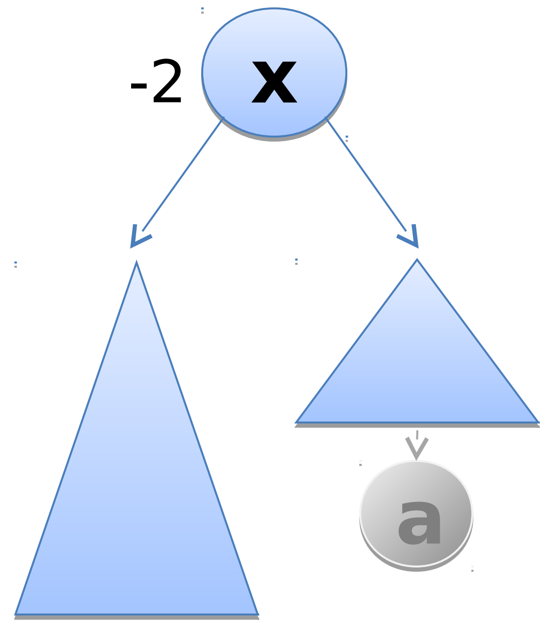
Bad balance factors

- Start with an AVL tree, then do a BST Delete.
- What bad values can bf(x) take on?
  - Delete can reduce a subtree's height by 1.
  - So, it might increase or decrease h(x.right) – h(x.left) by 1.
  - So, bf(x) might increase or decrease by 1.
  - This means:
    - if bf(x) = 1 before Delete, it might bec
    - If bf(x) = -1 before Delete, it might become -2. **BAD!**
    - If bf(x) = 0 before Delete, then it is still -1, 0 or 1. **OK.**

2 cases

Problematic cases for Delete(a)





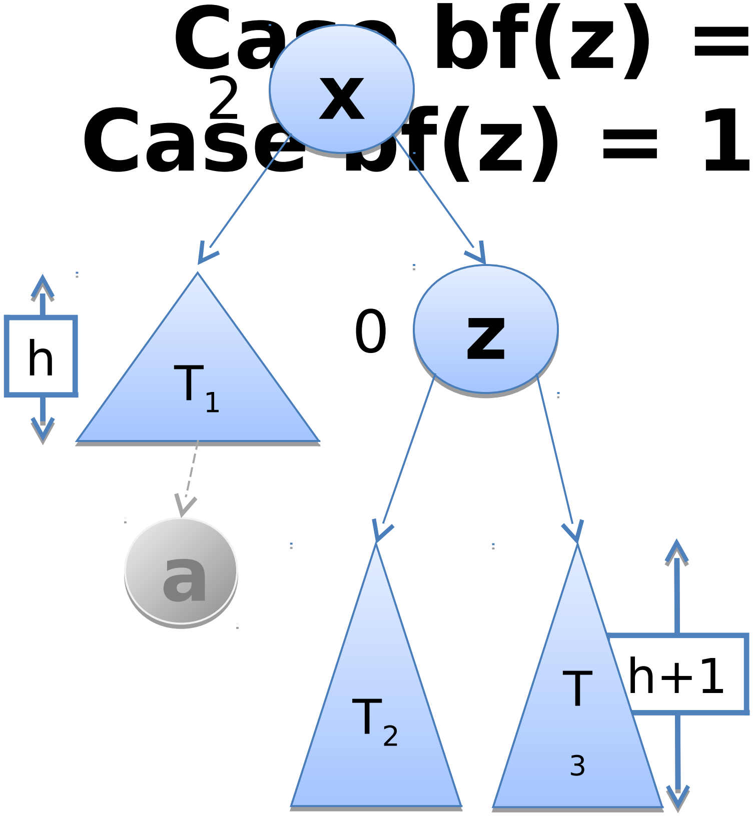
- bf(x) = -2 is just **symmetric** to bf(x) = 2.
- So, we just look at bf(x) = 2.

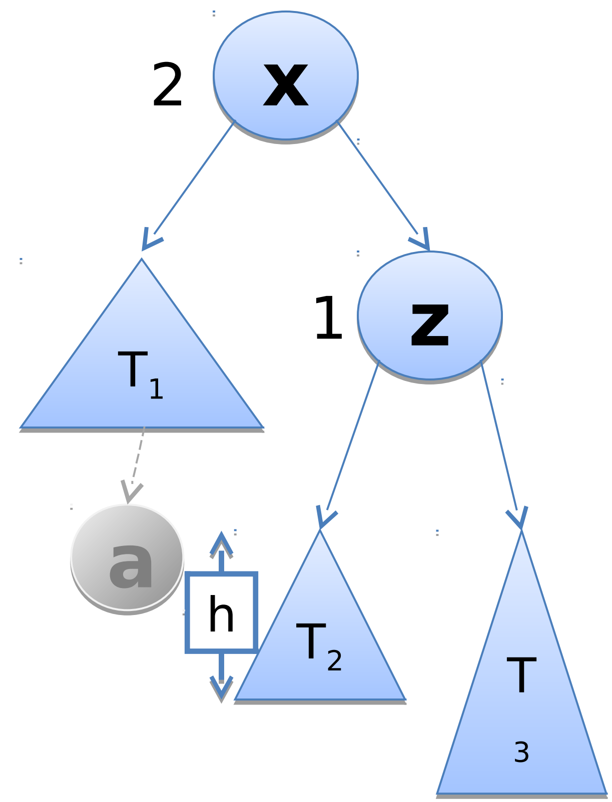
Delete(a): 3 subcases for bf(x)=2

- Since tree was AVL before, **bf(z) = -1, 0 or 1**

Case bf(z) = 0

Case bf(z) = 1





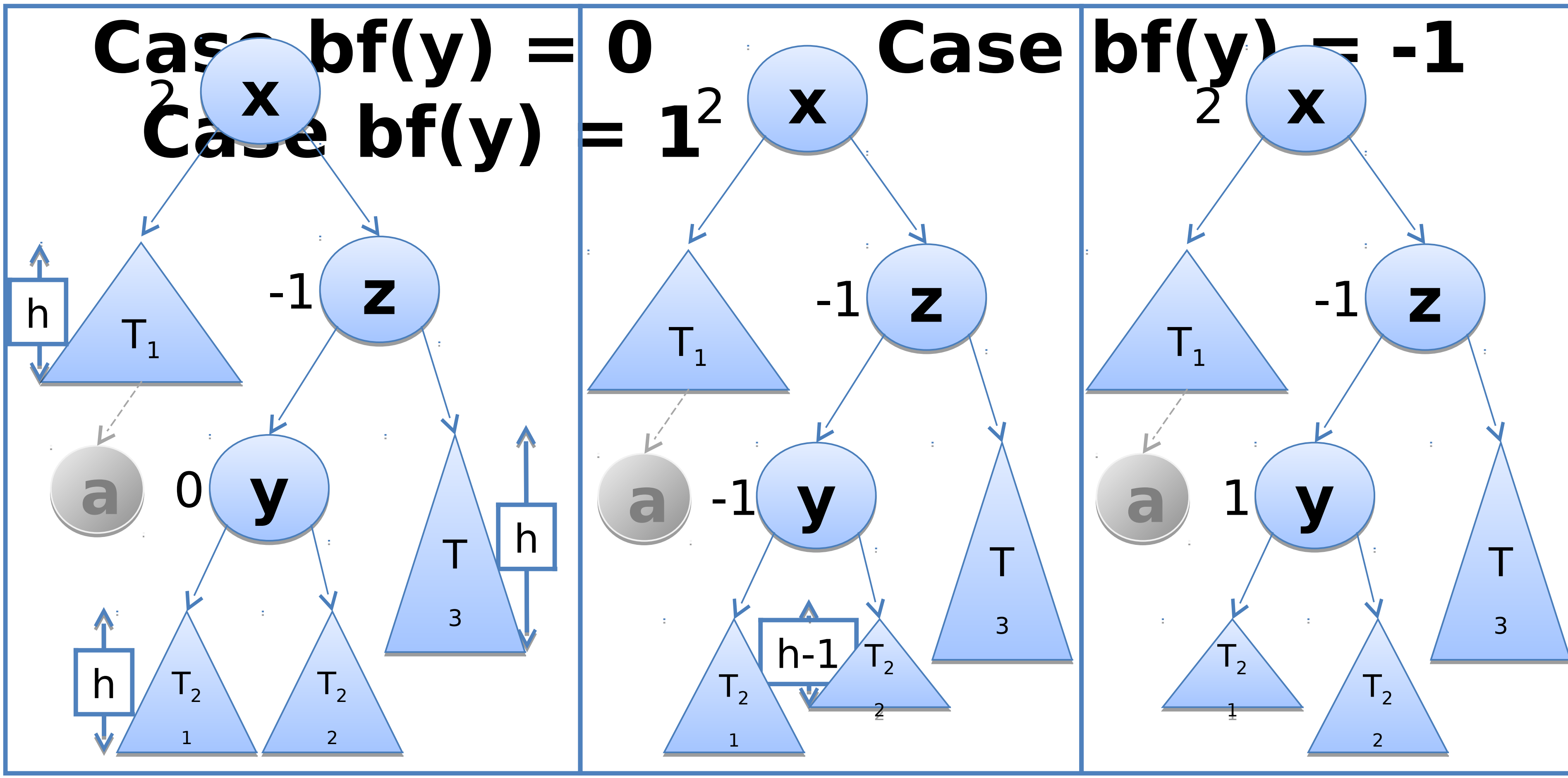
Delete(a): final subcase for bf(x)=2

Case bf(z) = -1: we have 3 subcases.
(More details)

Case bf(y) = 0

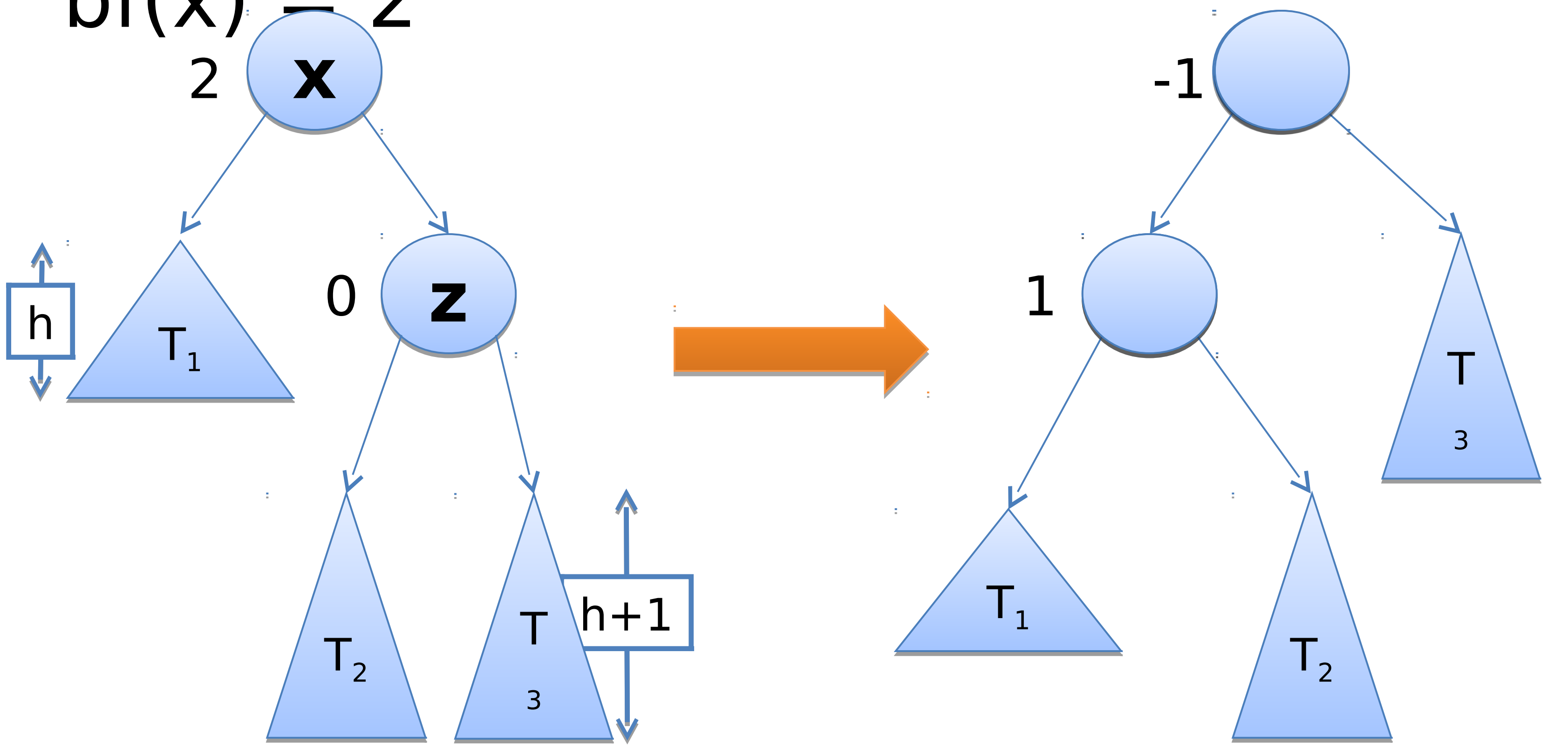
Case bf(y) = 1

Case bf(y) = -1



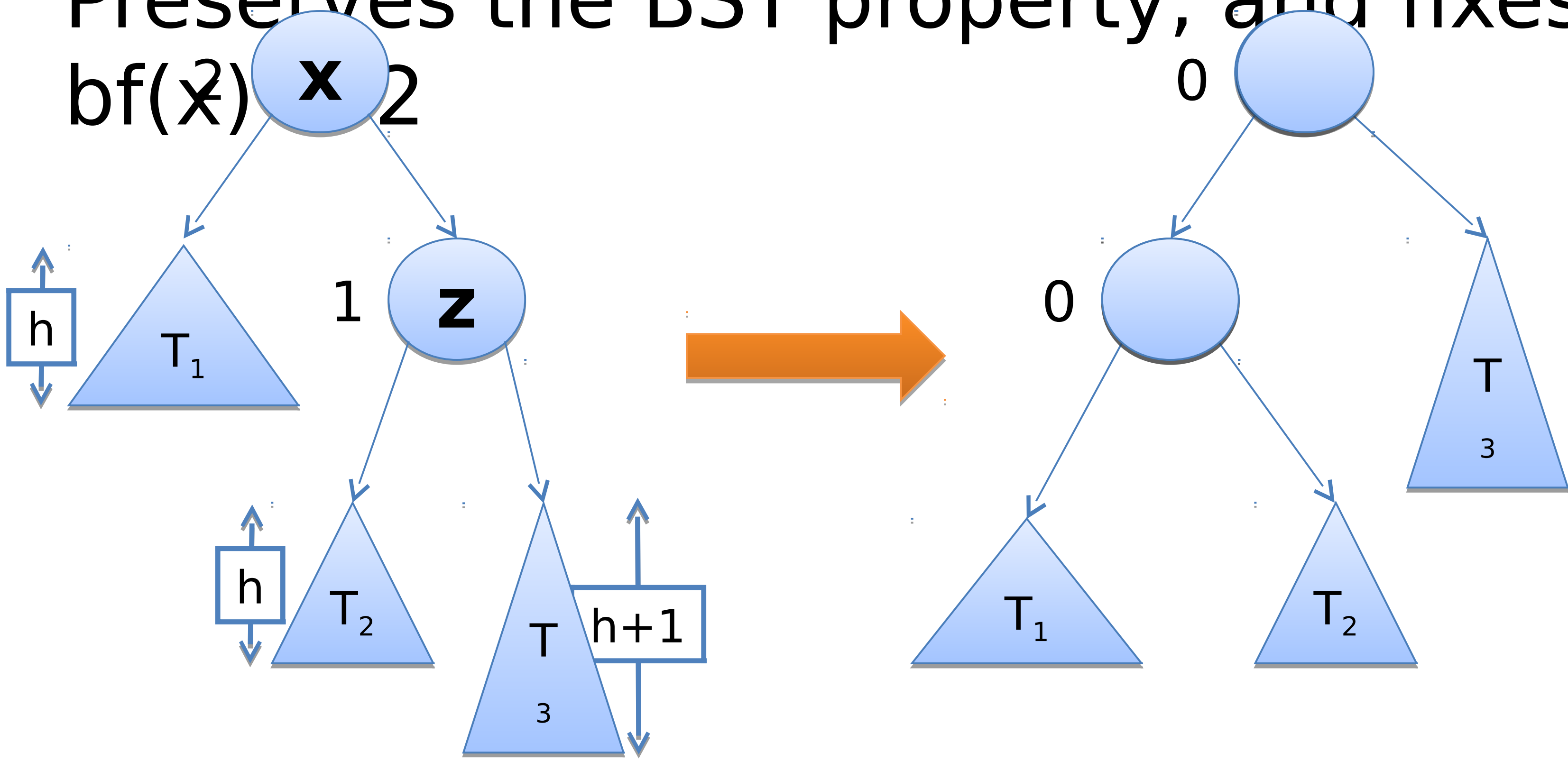
Fixing case bf(x) = 2, bf(z) = 0

- We do a ***single left rotation***
- Preserves the BST property, and fixes bf(x) = 2



Fixing case bf(x) = 2, bf(z) = 1

- We do a ***single left rotation*** (same as last case)
- Preserves the BST property, and fixes bf(x) = 2



Interactive AVL Deletes

- Interactive web applet
- Video of this applet being used to show most cases for insert / delete