Markov Process

Markov Property

"The future is independent of the past given the present"
미래는 현재가 주어지면 과거로부터 독립이다.

Markov Property

Definition
A state S_t is Markov if and only if

$$\mathbb{P}[S_{t+1}|S_t] = \mathbb{P}[S_{t+1}|S_1, \cdots, S_t].$$

The state captures all relevant information from the history.
state는 history에서 모든 관련 정보를 수집한다.
Once the state is known, the history may be thrown away.
state가 알려지면, history는 버려질 수 있다. (?)
The state is a sufficient statistic of the future.
state는 미래에 대한 충분 통계량이다.

State Transition Matrix

For a Markov state s and successor state s', the state transition probability is defined by

$$\mathcal{P}_{ss'} = \mathbb{P}[S_{t+1} = s'|S_t = s].$$

State transition matrix $\mathcal{P}$ defines transition probabilities from all states s to all successor states s',

$$\mathcal{P} = from \begin{bmatrix} \mathcal{P}_{11} & \cdots & \mathcal{P}_{1n} \\ \vdots & \ddots & \vdots \\ \mathcal{P}_{n1} & \cdots & \mathcal{P}_{nn} \end{bmatrix},$$

where each row for the matrix sums to 1.
행렬의 행의 합이 1이어야 한다.

Markov Process

A Markov Process is a memoryless random process, i.e. a sequence of random states $S_1, S_2, \cdots$ with the Markov property.

Definition
A Markov Process (or Markov Chain) is a tuple $\langle \mathcal{S}, \mathcal{P} \rangle$

- $\mathcal{S}$ is a (finite) set of states
- $\mathcal{P}$ is a state transition probability matrix,
  $\mathcal{P}_{ss'} = \mathbb{P}[S_{t+1} = s' | S_t = s]$

> in Graph Neural Networks,
> adjacency matrix가 state transition matrix와 개념이 동일하므로,
> adaptive adjacency matrix에 활용할 수 있을 것으로 예상