Dozenal Discrete Geometry: Proofs for a Base-12 Geometric Framework

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Abstract: This document presents a sequence of lemmas demonstrating exact geometric constructions in base-12 (dozenal) Cartesian geometry, culminating in a lattice-anchored regular 24-gon and discrete divisions of the circle's diameter into its circumference. Aligned with foundational explorations in quantum discreteness and π's geometry, these proofs invite verification and extension. Full details in *Understanding Base-12 Math: How to Draw the Perfect Circle Using Dozenal Geometry*.
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Introduction

Let us consider the Cartesian plane equipped with a coordinate system where the fundamental unit interval [0,1] along each axis is subdivided into twelve equal parts, rather than the conventional ten. This structure, which we shall term the *dozenal Cartesian plane*, assigns coordinates to points using fractions with denominator a power of 12 (i.e., points of the form $(m/12^k, n/12^k)$ for integers $m, n, k \geq 0$).
While the underlying Euclidean geometry remains invariant, the dozenal system enables exact lattice intersections for cyclic symmetries (e.g., 15° rotations) without decimal recurrence. We demonstrate this via a regular 24-gon anchored to the lattice $\mathbb{Z}[1/12]^2$.

Lemma 1: Equal Side Lengths in the Dozenal Lattice Polygon

Vertices of dodecagon P (base-10 equivalents):
$V_0 = (13/12, 0)$, $V_1 = (1, 7/12)$, $V_2 = (7/12, 1)$, ..., $V_{11} = (1, -7/12)$.
Claim: All sides = $5/12 \sqrt{2}$.
Proof:
$d(V_0, V_1)$: $\Delta x = -1/12$, $\Delta y = 7/12$
$d^2 = (1/144) + (49/144) = 50/144 = 25/72$
$d = \sqrt{(25/72)} = 5\sqrt{2} / 12$
By symmetry, all edges are equal.

Lemma 2: Scaling and Radial Growth

Scaling operator: $S_\lambda(V_i) = (\lambda x_i, \lambda y_i)$, $\lambda = k_{12}$.
45° ray intersects $P^{(1)}$ at (19/24, 19/24), distance $(19/24)\sqrt{2}$.
For k=4: intersection (19/6, 19/6), $r_1 = (19/6)\sqrt{2} = 3.2_{12}\ \sqrt{2}$
Perimeter $P^{(4)} = 20\sqrt{2}$

Lemma 3: 24-gon Construction

C_1: radius $r_1 = (19/6)\sqrt{2}$
Seed of Life → 24 rays at 15°
Offset from $Q = (3.2_{12}, 3.2_{12})$ by $s = 5/12\ \sqrt{2}$ → points on C_2
C_2: $r_2 = \sqrt{(r_1^2 + s^2)} = \sqrt{(185/9)}$
24 intersection points → regular 24-gon

Lemma 4: Uniform Side Lengths

Claim: All sides = $10/12\sqrt{2}$
Proof:
Distance between offset points: $\Delta x = 10/12$, $\Delta y = -10/12$
$d = \sqrt{[(10/12)^2 + (10/12)^2]} = (10/12)\sqrt{2}$
D_{24} symmetry ensures uniformity.

Lemma 5: Discrete Division

$D = 6.4_{12}\sqrt{2} = 8 \times (19/24)\sqrt{2}$
At 24 vertices: circles radius $\rho = 0.49_{12}\sqrt{2}$
Gap $= (10/12)\sqrt{2} - 2\rho = 0.06_{12}\sqrt{2}$
Total gaps: $24 \times 0.06_{12}\sqrt{2} = \sqrt{2}$
Matches: $3D = 17.0_{12}\sqrt{2}$, perimeter $= 18.0_{12}\sqrt{2} \rightarrow$ remainder $= \sqrt{2}$

Dozenal Discrete Division: Summary

Key Result: Diameter divides into circumference 3× with discrete units + balanced gaps (total $\sqrt{2}$).
Scalable to finer rings → increasing "space" → inward path to quantum-like granularity.
Verify: All coordinates in $\mathbb{Z}[1/12]^2$, no approximation.
Implication: Dozenal geometry reveals π's discrete nature naturally.
Public Access:
[QR Code Here – Insert 1.5" x 1.5" PNG]
Link: [https://yourusername.github.io/dozenal-proofs/]