partitions-leanblueprint

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0.1 Definitions

Definition 1 (Sequence)**.** A sequence, denoted a or $\{a_n\}$, is a function $a : \mathbb{N} \to \mathbb{R}$.

Definition 2 (Convergence)**.** A sequence $\{a_n\}$ converges to $L \in \mathbb{R}$ if for all $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that for all $n \geq N$, $|a_n - L| < \varepsilon$. We say $\{a_n\}$ converges if there exists $L \in \mathbb{R}$ such that $\{a_n\}$ converges to L.

0.2 Theorems

Theorem 3 (Limit Laws)**.**
Let $C \in \mathbb{R}$. Suppose $\{a_n\}$ converges to L and $\{b_n\}$ converges to K. Then

(i) $\{Ca_n\}$ converges to CL
(ii) $\{a_n + b_n\}$ converges to $L + K$.

Proof. Trivial. $\square$ $\square$

Lemma 4.
Suppose that there exists an $N \in \mathbb{N}$ such that for all $n \geq N$, $a_n \geq 0$. Then $\lim_{n\to\infty} a_n \geq 0$.

Theorem 5 (Order Limit Theorem)**.**
Let $\{a_n\}$ and $\{b_n\}$ be sequences. Suppose that there exists an $N \in \mathbb{N}$ such that for all $n \geq N$, $a_n \leq b_n$. Then $\lim_{n\to\infty} a_n \leq \lim_{n\to\infty} b_n$.