partitions-leanblueprint

Carter Hastings Quinn Oveson

September 11, 2025

0.1 Definitions

Definition 1 (Modular Form)**.** In lean, A modular form of weight $k \in \mathbb{N}$ is a function $f : \mathbb{C} \to \mathbb{C}$ such that :

(1) f is holomorphic on $\mathbb{H}$
(2) For all $z \in \mathbb{H}, f(z) = f(z+1)$
(3) For all $z \in \mathbb{H}, f(z) = z^{-k} f(-1/z)$
(4) f is bounded as $\mathrm{Re}(z) \to \infty$

Definition 2 (Integer Modular Form)**.** An integer modular form of weight $k \in \mathbb{N}$ is a sequence $a : \mathbb{N} \to \mathbb{Z}$ such that $\sum_{n=0}^{\infty} a(n) q^n$ is a modular form of weight k, where $q = e^{2\pi i z}$.

Definition 3 (ModularFormMod ℓ)**.** A modular form mod ℓ of weight $k \in \mathbb{Z}/(\ell-1)\mathbb{Z}$ is a sequence $a : \mathbb{N} \to \mathbb{Z}/\ell\mathbb{Z}$ such that there exists an integer modular form b of weight k' where $b \equiv a (\mod \ell)$ and $k' \equiv k (\mod (\ell - 1))$.

Definition 4 (Theta)**.** Θ sends modular forms mod ℓ of weight k to weight $k+2$ by
$(\Theta a) n = n a(n)$.

Definition 5 (U Operator)**.** The operator U sends modular forms mod ℓ of weight k to weight k by
$(a|U) n = a(\ell n)$.

Definition 6 (hasWeight)**.** A modular form mod ℓ called a has weight $j \in \mathbb{N}$ if there exists an integer modular form b of weight j such that $b \equiv a (\mod \ell)$.

Definition 7 (Filtration)**.** The filtration of a modular form mod ℓ called a is defined as
The minimum natural number j such that a has weight j.
The filtration of the zero function is 0.