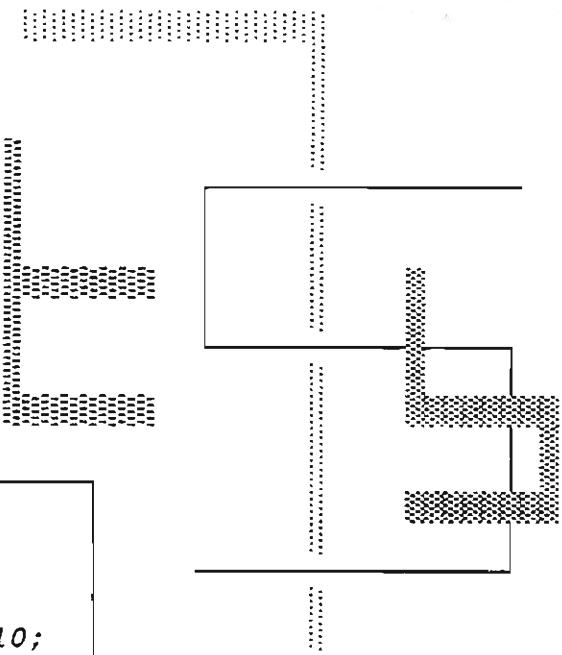


THE DUODECIMAL BULLETIN 5#;



DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

Volume 31;
Number 3;
Fall 1988
1198;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: *The Duodecimal Society of America*)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are \$12.00 (US) for one calendar year. Student membership is \$3.00 per year, and a Life membership is \$144.00 (US).

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., c/o Math Department, Nassau Community College, Garden City, LI, NY, 11530.

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THE DUODECIMAL BULLETIN

Whole Number Five Dozen El

Volume 31; Number 3;

Fall 1198;



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1944

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DOZENAL SOCIETY OF AMERICA

SCHEDULE OF THE ANNUAL MEETING -- 1988;

Friday and Saturday

*October 14 and 15, 1988
(12; and 13; October 1988;)*

*Nassau Community College
Garden City, LI, NY 11530*

Schedule

Friday Evening, October 14, 1988:

A social evening with spouses and friends. Tentative plans include dinner at BEST WOK in Bethpage, followed by an evening of theatre at the Plainedge Playhouse, also in Bethpage. The featured presentation is "Cole" -- the music of Cole Porter.

Saturday, October 15, 1988:

I BOARD OF DIRECTORS MEETING -- Tentative Agenda

10 A.M. (Administrative Tower, Nassau Community College, Garden City, LI, NY)

1. Call to order - Fred Newhall, Chair

Continued . . .

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve. Thus $1/2 = 0.5 = 0;6$.

SCHEDULE, 1988 ANNUAL MEETING, Continued

2. Report of the Nominating Committee (J. Malone, L. Aufiero, A. Catania), and proposal of a slate of Officers as follows:

Board Chair	Dr. Angelo Scordato
President	Fred Newhall
V. President	Gene Zirkel
Secretary	Larry Aufiero
Treasurer	Anthony Catania

3. Election of Officers. (These new Officers will be installed later in the meeting. The business of the Annual Meeting is conducted by the outgoing Officers and Committee Chairs, etc.)

4. Remarks

5. Appointments to various Committees. (For current Committee appointments and Chairs, see Bulletin number 59; pages 5 and 6.)

Committees of the Dozenal Society of America:

Annual Meeting
Finance
Awards
Constitution and By-Laws
Video

Other positions include:

Parliamentarian
Editor of the *Duodecimal Bulletin*
Reviewers of articles for the Bulletin

6. Further affairs and business of the Board

7. Adjournment

Continued . . .

SCHEDULE, 1988 ANNUAL MEETING, Continued

- II ANNUAL MEMBERSHIP MEETING -- Tentative Agenda**
1. Call to order - Gene Zirkel, President
Attendance
 2. Minutes of the 1987 Annual Meeting - L. Aufiero
 3. President's Report - G. Zirkel
 4. Treasurer's Report - J. Malone
 5. Reports of other Officers, as called for.
 6. Annual Meeting - Smith, Catania, Berridge (Consultant)
 7. Finance - Scordato, George, Malone, Razziano, P. Zirkel
 8. Awards - Scordato, Impagliazzo, Malone
 9. Reports of other Committees, as called for.
 - *. Nominating - Malone, Aufiero, Catania
 - a) Nomination and election to the Board of the Class of 1991:

Anthony Catania	Seaford, NY
Carmine De Santo	Merrick, NY
Dr. Angelo Scordato	Valley Stream, NY
Patricia McCormick Zirkel	West Islip, NY
 - b) Election of a Nominating Committee for 1988-1989.
The following are proposed:

L. Aufiero
K. McKiernan
G. Zirkel
 - #. New business.

*Continued . . .**SCHEDULE, 1988 ANNUAL MEETING, Continued***III LUNCHEON****IV AFTERNOON PRESENTATIONS**

1. Fred Newhall - Topic to be announced. Last year Fred has an interest in the concept of Frequency, and last year he spoke on "Frequency in Music".
2. Other speakers to be announced. Last year's topics included presentation of a new Dozenal clock and a stimulating discussion of divisibility tests.

V EVENING BANQUET

We will gather at the John Peel Room Restaurant at the Island Inn, Old Country Road, Westbury for cocktails and dinner in an English pub atmosphere. The approximate cost per person will be \$25.00.



Alice Berridge and John Impagliazzo share a lighter moment at the 1987 DSA Annual Meeting.

SQUARE INTEGERS, N²

Charles W. Trigg
San Diego, California

In Table 1 the square integers $\langle \#^2 \rangle$; are exhibited in such fashion that certain relationships are easily perceived. For example:

The units digit of each number in the table is a square.

If the integer N ends in 0, 1, 4 or 9, then N^2 ends in the same digit as N.

The units' digits of consecutive squares form a repetitive sequence with a period of 0 1 4 9 4 1. Thus the sequence consists of the palindromes 1 4 9 4 1 separated by zeros.

The penultimate digits of consecutive squares form a repetitive sequence with a 6-dozen-digit period. This period consists of a zero and a palindrome. Thus

Penulti-	0	0	0	0	1	2	3	4	5	6	8	*
Digits:	0	2	4	6	9	0	3	6	9	0	4	8
	0	4	8	0	5	*	3	8	1	6	0	6
	0	6	0	6	1	8	3	*	5	0	8	4
	0	8	4	0	9	6	3	0	9	6	4	2
	0	*	8	6	5	4	3	2	1	0	0	0

Common Differ-	0	2	4	6	8	*	0	2	4	6	8	*
-------------------	---	---	---	---	---	---	---	---	---	---	---	---

Text continued on page *; . . .

SQUARE INTEGERS, Continued

Table 1. Square Integers, N²

	0	1	2	3	4	5	6	7	8	9	*	#
0	0	1	4	9	14	21	30	41	54	69	84	*1
1	100	121	144	169	194	201	230	261	294	309	344	381
2	400	441	484	509	554	5*1	630	681	714	769	804	861
3	900	961	*04	*69	#14	#81	1030	10*1	1154	1209	1284	1341
4	1400	1481	1544	1609	1694	1761	1830	1901	1994	1*69	1#44	2021
5	2100	21*1	2284	2369	2454	2541	2630	2721	2814	2909	2*04	2#01
6	3000	3101	3204	3309	3414	3521	3630	3741	3854	3969	3*84	3#*1
7	4100	4221	4344	4469	4594	4701	4830	4961	4*94	5009	5144	5281
8	5400	5541	5684	5809	5954	5**1	6030	6181	6314	6469	6604	6761
9	6900	6*61	7004	7169	7314	7481	7630	77*1	7954	7#09	8084	8241
*	8400	8581	8744	8909	8*94	9061	9230	9401	9594	9769	9944	9#21
#	*100	*2*1	*484	*669	*854	**41	#030	#221	#414	#609	#804	#*01

Continued . . .

Why not give some of our literature to a friend? Brochures, Excursions and Bulletins are available.

*

SQUARE INTEGERS, Continued

As written above in six rows, each column is a cyclic arithmetic progression. The common differences themselves form an arithmetic progression as shown below the columns.

The digital root of an integer is found by adding its digits, adding the digits of that sum, and continuing the process until there is obtained a single digit, the digital root. The digital roots of consecutive squares form a repetitive sequence with a period of

1 4 9 5 3 3 5 9 4 1 #

which has a digit sum of 47;. The sequence of digital roots consists of like palindromes separated by #'s.

Would a gambler consider the field of square numbers smaller than 28^2 unlucky? There is nary a seven or eleven among its digits!

End

HOW MANY FINGERS???

In Dean Koontz' novel *Demon Seed*, the computer, Proteus, designs the perfect human. It should be easy for you to guess how many fingers were on each hand!

(See page 167, Bantam Books Edition)

NEW

MAILING

LABELS



We have recently updated our production of mailing labels. Please check your new label to see if your name is spelled correctly, and if all the information is as it should be.

The format is as follows:

1. Your name & DSA number if any.

Your DSA number should be followed by F, L, H and/or S if you are:

a Fellow,
a Life Member,
an Honorary Member, or
a Student Member of our Society.

2. Your address including your Zip code.

Foreign countries have many styles of placing their Zip codes. Is yours correct?

We regret any errors which may have occurred during the transition to the new labels. Please let us know of any mistakes so that we may correct them. Thanks.

End

THE DOZENAL CLOCK

Dr. Paul Rapoport
 McMaster University
 Hamilton, Ontario, Canada

(The following is a condensed version of a talk presented at the 1987 annual meeting.)

For 25 years I have been interested in creating a good dozenal timepiece. Now when I say this to the non-initiated, they respond, "But our current clock is based on twelve." Fair enough, but only up to a point. The day is currently divided first into 2 (AM/PM), then into 12 (hours), then into 60 (minutes), then another 60 (seconds). This is hardly dozenal at all, and actually rather difficult to use. Those who don't agree might simply try subtracting the following times:

$$\begin{array}{r} 2.30.08 \text{ PM} \\ - 11.36.19 \text{ AM} \end{array}$$

A truly dozenal metric timepiece contains "nested dozens," i.e. each unit contains 10; of the next smaller unit. Accordingly, in this system, the day is first divided into 10; parts (2 hours in the conventional system), then into 10; again (dek minutes), 10; again (42; seconds), and 10; again (4 1/6 seconds), ... with as many such divisions as one wishes, although these four are sufficient for most purposes.

In the dozenal system, then, the above subtraction problem is very easy:

$$\begin{array}{r} 730;2 \\ - 597;7 \\ \hline 154;7 \end{array}$$

Continued . . .

DOZENAL CLOCK, Continued

For a truly dozenal timepiece, therefore, we need either a digital device with three or (preferably) four digits, or an analog one with the same number of hands. It is possible to slow down a conventional electric clock to half-speed (twice as slow) by cutting the 60 Hz A.C. down to 30. The slowest hand then completes one circle every day and passes by 1 of the 10; numerals on the clock face every 2 hours (twice as slow as the 1 hour for the current hour-hand); the next slowest hand completes one circle every 2 hours and passes by 1 of the 10; numerals on the clock face every * minutes (twice as slow as the 5 minutes for the current minute-hand). But the current second-hand is of no use: at half-speed it would go around the face not once every 42; seconds as required, but once every 26; seconds. A half-speed electric clock is thus limited to two hands, the second-hand must be removed.

What I have just described was in fact created in 1971, with a huge clock whose face I redrew in a rather crude manner. Until the box cutting the A.C. in half conked out (in 1984), I had a reasonable analog dozenal clock, although I wanted four hands really. Several attempts to fiddle with analog watches failed. They are very fussy characters. Cutting gears with limited equipment and less skill is not recommended.

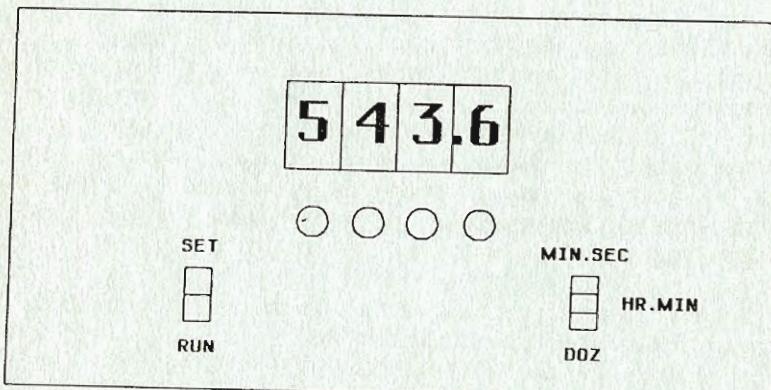
I should point out that dozenal time is in no way my invention. Over the years George Terry, Kingsland Camp, Henry Churchman, and others have all written about dozenal time in the Society's bulletin and elsewhere. All I did was put the obvious into practice.

Continued . . .

THINK 12 . . . twelve . . . 10; . . . do

DOZENAL CLOCK, Continued

The latest excursion in this practice is an electric dozenal clock with four digits. The face of this recently created clock is simple: it looks like the illustration below. On the left is a two-position switch, on the right a three-position switch.



To set the time, you move the left switch up (to SET) and the right switch down (to DOZ). (The display will be 000.0.) Then press in and hold each of the four buttons under the display to advance each digit to the desired numeral. To run the clock from the set time, move the left switch down (to RUN).

To see the time in a conventional display, move the right switch to either the central position (HR.MIN) or the up position (MIN.SEC). Supposing the time to be 10:42:55 AM, the former displays it as hours-plus-minutes (10.42), the latter as minutes-plus-seconds (42.55). If the time is PM, a dot flashes to the left of the leftmost digit in the HR.MIN mode.

Continued . . .

DOZENAL CLOCK, Continued

That is all there is to it, although there are a few subtleties involved. For example, you cannot set the time in either of the conventional modes, only in dozenal. And the flashing dot, indicating PM, is annoying--deliberately. This is to detract from the running of the clock in a conventional mode. Far better to run it as a dozenal clock. When people come along and ask you the time, you may tell them the dozenal time and the conventional time. When they ask how you know the conventional time only by looking at the dozenal display, simply flip the right switch up one position.

The clock measures approximately 19.5 cm. wide x 11 cm. high x 6 cm. deep, and weighs approximately 0.7 kg. (including the A.C. power supply/adapter). It operates with a quartz crystal vibrating at 8 MHz.; counters in milliseconds run and synchronize the digits.

Designer: Dr. Paul Rapoport
Department of Music
McMaster University
1280 Main Street West
Hamilton, Ontario, Canada L8S 4M2

(Phone area 416, 648-2181 weekends between 10 AM and 10 PM, or nights up to 10 PM.)

Manufacturer: Philip R. Pottier and Associates, Canada

Continued . . .

HAVE YOU MARKED YOUR CALENDAR???

The DSA Annual Meeting will be held on October 14 and 15, 1988 at Nassau Community College...

...and it won't be the same without you!

DOZENAL CLOCK. Continued

Cost:

To members of the DSA: U.S. \$119 ea. Quotation on request for 12 or more.

To nonmembers: U.S. \$125 ea.; this includes a one-year membership in the Society (essentially at half-price: regular membership is \$12). Quotation on request for 12 or more clocks.

Postage and packing extra. Prices subject to change without notice.

Quoted prices include the creation of digits (within the limitations of the 7-segment display) according to the specifications of the purchaser. Anyone ordering a clock should specify what is desired for ten (dek) and eleven (el), and for any other digits if their form is nonstandard.

This digital dozenal-metric clock is dedicated to the memory of Tom Linton, former President of the Dozenal Society of America, pioneer in creation of dozenal artifacts. The second specimen was first set up to run at the Annual Meeting of the Dozenal Society of America on October 17, 1987 at 826.0 (4:25 PM). Dozenal reckoning of time is now a practical reality. Others are invited to observe and use it.

End

*
* ON THE COVER
*
* Our cover is a fanciful representation of the
* 7-segment display of the dozenal clockface,
* including Dr. Rapoport's digits for 5, 7, *dek*
* and *el*.
*

FUNDAMENTAL OPERATIONS IN THE DUODECIMAL SYSTEM - PART I

*Jay Schiffman
Math Dept
Jersey City State College
Jersey City, NJ 07305*

Introduction: A standard assignment in elementary mathematics consists of having the student master the addition and multiplication tables where the computations are performed in the base ten system of numeration. In this paper, we endeavor to demonstrate how one performs addition, subtraction, multiplication, and division in the duodecimal system with and without resorting to the addition and multiplication tables. In addition, we solve a number of algebraic equations in base twelve. We initiate our discussion by presenting the standard duodecimal addition and multiplication facts via tables where the symbols T and E denote the numerals ten and eleven in base ten.

[See the Tables on pages 16; and 17;]

Before proceeding, a few preliminary remarks are in order. The duodecimal system of numeration utilizes twelve symbols:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E.

At first there appear to be numerous errors in these tables which non dozenalists generally interpret in terms of base ten. Let us check some of the number facts. Consider

(T) + (E) = 19; (see Table I).

Converting to the decimal system,

$$(T) + (E) = (10) + (11) = (21).$$

Now (21) = (19;), meaning one group of twelve plus nine units. Similarly, (E) + (E) = (11) + (11) = (22). Now (22) = (1T;), that is, one group of twelve plus ten units.

Text continued on page 18; . . .

FUNDAMENTAL OPERATIONS (I), Continued

+	0	1	2	3	4	5	6	7	8	9	T	E
0	0	1	2	3	4	5	6	7	8	9	T	E
1	1	2	3	4	5	6	7	8	9	T	E	10
2	2	3	4	5	6	7	8	9	T	E	10	11
3	3	4	5	6	7	8	9	T	E	10	11	12
4	4	5	6	7	8	9	T	E	10	11	12	13
5	5	6	7	8	9	T	E	10	11	12	13	14
6	6	7	8	9	T	E	10	11	12	13	14	15
7	7	8	9	T	E	10	11	12	13	14	15	16
8	8	9	T	E	10	11	12	13	14	15	16	17
9	9	T	E	10	11	12	13	14	15	16	17	18
T	T	E	10	11	12	13	14	15	16	17	18	19
E	E	10	11	12	13	14	15	16	17	18	19	1T

*Continued . . .**FUNDAMENTAL OPERATIONS (I), Continued*Table II - The Duodecimal Multiplication Table

X	0	1	2	3	4	5	6	7	8	9	T	E
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	T	E
2	0	2	4	6	8	T	10	12	14	16	18	1T
3	0	3	6	9	10	13	16	19	20	23	26	29
4	0	4	8	10	14	18	20	24	28	30	34	38
5	0	5	T	13	18	21	26	2E	34	39	42	47
6	0	6	10	16	20	26	30	36	40	46	50	56
7	0	7	12	19	24	2E	36	41	48	53	5T	65
8	0	8	14	20	28	34	40	48	54	60	68	74
9	0	9	16	23	30	39	46	53	60	69	76	83
T	0	T	18	26	34	42	50	5T	68	76	84	92
E	0	E	1T	29	38	47	56	65	74	83	92	T1

Continued . . .

If one next considers multiplication in the usual sense as repeated addition

then multiplication problems are performed with relative ease.

For example, consider $(7; \times 8;) = \frac{(8+8+8+8+8+8+8)}{7 \text{ addends of } 8}$.

Let us convert to the decimal system.

$(8 \times 7) = (56) = (48;)$, which is four groups of twelve plus eight units (see Table II). Observe that decimal $(56) = (4 \times 12) + (8 \times 1)$ in duodecimals. Note:

<u>Decimals</u>	<u>Duodecimals</u>
$(8+8) = (16)$	= (14), which is one group of twelve and four units.
$(8+8+8) = (24)$	= (20), two groups of twelve and no units.
$(8+8+8+8) = (32)$	= (28), two groups of twelve and eight units.
$(8+8+8+8+8) = (40)$	= (34), three groups of twelve and four units.
$(8+8+8+8+8+8) = (48)$	= (40), four groups of twelve and no units.
$(8+8+8+8+8+8+8) = (56)$	= (48), four groups of twelve and eight units.

Similarly, in duodecimals we compute

$$(ExT) = \frac{(T+T+T+T+T+T+T+T+T+T+T)}{E \text{ addends of } T}.$$

Converting to the decimal system, $(TxE) = (110) = (92;)$, which is nine groups of twelve and two units (see Table II). Observe that decimal $(110) = (9 \times 12) + (2 \times 1)$ in duodecimals.

Note:

<u>Decimals</u>	<u>Duodecimals</u>
$(T+T) = (20)$	= (18), which is one group of twelve and eight units.
$(T+T+T) = (30)$	= (26), two groups of twelve and six units.
$(T+T+T+T) = (40)$	= (34), three groups of twelve and four units.
$(T+T+T+T+T) = (50)$	= (42), four groups of twelve and two units.
$(T+T+T+T+T+T) = (60)$	= (50), five groups of twelve and no units.
$(T+T+T+T+T+T+T) = (70)$	= (5T), five groups of twelve and ten units.
$(T+T+T+T+T+T+T+T) = (80)$	= (68), six groups of twelve and eight units.
$(T+T+T+T+T+T+T+T+T) = (90)$	= (76), seven groups of twelve and six units.
$(T+T+T+T+T+T+T+T+T+T) = (100)$	= (84), eight groups of twelve and four units.
$(T+T+T+T+T+T+T+T+T+T+T) = (110)$	= (92), nine groups of twelve and two units.

The reader is invited to verify other entries in the table.

At this juncture one is now ready to discuss the arithmetic operations in the duodecimal system. All of our operations will be illustrated by examples.

Continued . . .

Do you have an idea to share with our members? Why not submit an article to the *Bulletin*?

Continued . . .

FUNDAMENTAL OPERATIONS (I), Continued**I. Addition In Base Twelve**

Example 1: Add 467;
 $\underline{+238;}$

Solution: Note that decimally $(7+8) = (15)$, that is, duodecimal 13;. We place the 3 in the units column and carry the 1 over to the twelves column.

Decimal $(1+6+3) = (10) = \text{duodecimal } (T)$.

Next place the T in the second column. There is no carrying over into the 12^2 (i.e. 144, or one gross) column as T is less than the base 12. Now $4+2=6$ in the leftmost column. Hence, our sum consists of 6 groups of 144, 10 groups of 12, and 3 units.

Thus 467;
 $\underline{+238;}$
 $6T3;$

Alternatively, one could convert each addend in the sum to the decimal system of numeration, add using base ten, and then convert the sum to the duodecimal system:

$$\begin{array}{rcl} 467; & =(4 \times 12^2) + (6 \times 12) + (7 \times 1) & = 576 + 72 + 7 = 655. \\ +238; & =(2 \times 12^2) + (3 \times 12) + (8 \times 1) & = 288 + 36 + 8 = +332. \\ & & 987. \end{array}$$

Continued . . .

"Each one teach one."

-- Ralph Beard, Founder of the DSA

FUNDAMENTAL OPERATIONS (I), Continued

Now convert 987 to a duodecimal number.

Let us note that the place values in base 12 are ... 12^3 , 12^2 , 12, 1, or ... 1728, 144, 12, 1. The highest power of the base less than 987 is $12^2 = 144$. Successive divisions by the respective powers of the base 12 yields

$$\begin{array}{r} 6 \\ 144) 987 \\ 864 \\ \hline 123 \\ 864 \\ \hline 3 \\ 3 \\ \hline 0 \\ 0 \\ \hline 3 \end{array} \quad \begin{array}{r} 10 \text{ (or } T\text{)} \\ 12) 123 \\ 12 \\ \hline 3 \\ 3 \\ \hline 0 \end{array} \quad \begin{array}{r} 3 \\ 1) 3 \\ 3 \\ \hline 0 \end{array}$$

Hence decimal 987 equals duodecimal 6T3.

One can utilize an alternative method for changing a number in the decimal system to the duodecimal system. We illustrate by converting decimal 987 to base twelve. Dividing 987 by 12 yields a quotient of 82 and a remainder of 3. Write the quotient above the dividend and the remainder on the right, as illustrated below:

$$\begin{array}{r} 82 \\ 12) 987 \\ 96 \\ \hline 27 \\ 24 \\ \hline 3 \end{array} \quad \text{R. 3}$$

We continue the process of division:

$$\begin{array}{r} 0 \\ 12) 6 \\ 12) 82 \\ 12) 987 \\ 84 \\ \hline 47 \\ 48 \\ \hline -1 \\ 1 \end{array} \quad \begin{array}{l} \text{R. 6} \\ \text{R. T} \\ \text{R. 3} \end{array}$$

Answer = 6T3

(Since the dividend, 6, is smaller than the divisor, 12, the remainder is 6.) Note that the division continues until the dividend is zero.

Thus decimal 987 equals duodecimal 6T3;.

Continued . . .

FUNDAMENTAL OPERATIONS (I), Continued

Example 2: Add 470;
 +347;

Solution: $0+7=7$ which is less than the base 12. $7+4=E$, less than the base 12. (Hence place 7 in the units column and E in the twelves column.) Finally $4+3=7$ and we place 7 in the gross (one hundred forty-four) column. There is no carrying involved in this problem.

Hence 470;
 +347;
 7E7;

One may resort to the ideas provided in our first example to obtain an alternative solution.

This article will be continued in a future issue of the Bulletin.

The human mind through the ages has resorted to binary division (dividing in halves) as the easiest form of division. Next to halves is thirds for simplicity, and from the combination of these is derived the common multiple 12, which is found in the division of time and the circle. These common and simple forms of division and multiplication are not found in the metric system. France, the birthplace of the metric system, recognized the logic and convenience of binary division and adopted a modular unity of 1.2 meters, a deviation from 10, because it is divisible into subunits of 200, 300, 400, and 600 millimeters.

--From a General Accounting Office "Report to the Congress", as found in the *American Metric Journal*.

DOZENAL JOTTINGS

. . . from members and friends . . . News of Dozens and Dozenalists . . .

FRED NEWHALL addressed a sixth-grade math class in Valley Stream, LI, NY on the history of number systems. He has spoken to many elementary and secondary school students at Math Fairs over the past several years. The students learned about Egyptian, Babylonian, Roman and Chinese symbols. Fred explained how awkward these antiquated systems were and how much more useful our current Arabic (base ten) system is than these. He went on to show that counting and figuring could be further facilitated by using a base other than ten -- namely twelve...

DUDLEY GEORGE wrote from his summer home in Vashon, WA, to say that he would try to attend the DSA Annual Meeting in NY in October. We hope to see you, Dudley...

DON HAMMOND, Secretary of the DSGB, wrote voicing dissatisfaction with the current British government's endorsement of metric measuring: "We are still suffering from cut-off dates here. Another one has just taken effect which ends dual weight-marking on pre-packaged foods and condiments, which are now in grammes only. You should think yourselves lucky that your system of government does not permit such secrecy! We have reached the dangerous stage where even Conservative governments enact fundamental constitutional changes with no reference to the general public and, indeed, which are contrary to the wishes of most people. We are becoming a mere North European Province..."

Continued . . .

Remember -- your gift to the DSA is tax deductible.

DOZENAL JOTTINGS, Continued

HAYDEN STEELE wrote to us from Arataki, Mt. Maunganui, New Zealand. Mt. Maunganui is a rapidly growing area and is now the third busiest export point in New Zealand, following only Auckland and Wellington...

RICHARD TRELFA (St. Johnsbury, VT) recently purchased a Hewlett-Packard HP-18C "Business Consultant". He feels the HP "Solve" program could be usable for many dozenal calculations and conversions. Perhaps some of you dozenalists out there would like to work on this...

The Missouri Council of Teachers of Mathematics will publish an interview with "A Dozenalist" in a future issue of their Bulletin. This is a great way to share news about our Society and its purposes with their readers...

Continued . . .

MORE ON ANTIQUITY

Don Hammond, Secretary of the DSGB, makes the following comment in our ongoing debate on Duodecimals in Antiquity...

In reply to Gerard Brost's point about the Babylonians' value of 3 for pi: It is more likely that this arose from dividing the circle into twelve, rather than just six. (This can be done with a square grid. See the illustration on the contents page of the DSGB Journal, number 6.) The area of a regular dodecagon in a circle is exactly $3r^2$, and this is very close to the circle itself!

End

DOZENAL JOTTINGS, Continued

Thanks to NELSON GRAY of Camden, NY and Arizona we now have a copy of the "Conversion Rules" which originally accompanied Duodecimal Test IV (see last issue). If anyone would like to see a copy, please write to us...

Welcome to our newest member:

Number 2#8 Charles David Ashbacher, an Instructor of Mathematics at Mount Mercy College in Cedar Rapids, Iowa...

End

THE FOLLOWING ARE AVAILABLE FROM THE SOCIETY

1. Our brochure. (Free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, October 1934. (Free)
3. *Manual of the Dozen System* by George S. Terry. (\$1.00)
4. *New Numbers* by F. Emerson Andrews. (\$10.00)
5. *Douze: Notre Dix Futur* by Jean Essig. In French. (\$10.00)
6. Dozenal Slide Rule, designed by Tom Linton. (\$3.00)
7. Back issues of the *Duodecimal Bulletin*, as available, 1944 to present. (\$4.00 each)

MEDITATION

12 ÷ 1

12 ÷ 2

12 ÷ 4

12 ÷ 6

12 ÷ 8

12 ÷ 9

12 ÷ 12

1 x 12

2 x 6

3 x 4 and

2 x 2 x 3

Furthermore

Π 3;184809

e 2;875236

DON'T KEEP THIS MAGAZINE

Do you discard your copies of the *Bulletin* after you have read them? Or do they gather dust on a shelf in your attic? Why not pass them along to your local library, or to a school library. Perhaps some nearby math teacher would appreciate a copy. You can also just leave them in a dentist's office or other waiting area.

Help spread the word!

(If you ever need a back copy, we'd be glad to help.)

WHY CHANGE?

This same question was probably ripe in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took 500 years, and despite much opposition—"Who needs a symbol for nothing?"—the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used, and as early as 1585, Simon Stevin stated that the duodecimal base was to be preferred to the base ten.

The parallel seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatisfactory because it has NOT ENOUGH FACTORS.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurements. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions ($1/3 = 0;4$) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written 10, and is called *do*, for dozen. The quantity *one gross* is written 100, and is called *gro*. 1000 is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called 2 *gro* 6 *do* 5, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3#2	Two ft. eight in.	2;8'
19#	1000	Eleven ft. seven in.	#;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so set down 53. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only 2#, which $12 \frac{1}{2} \underline{365}$
is two dozen and eleven. For larger numbers, $12 \frac{1}{2} \underline{30} + 5$
keep dividing by 12, and the successive re- $12 \frac{1}{2} \underline{2} + 6$
mainders are the desired dozenal numbers. $0 + 2$ Answer: 265

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* (\$1.00).

We extend an invitation to membership in our society.
dues are only \$12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name LAST FIRST MIDDLE

Mailing Address (for DSA items)

(See below for alternate address)

Telephone: Home Business

Date & Place of Birth

College Degrees

Business or Profession

Annual Dues \$12.00 (US)

Student (Enter data below) \$3.00 (US)

Life \$144.00 (US)

School

Address

Year & Math Class

Instructor Dept.

Other Society Memberships

Alternate Address (indicate whether home, office, school, other)

Signed Date

My interest in duodecimals arose from

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530