10. Inference from Small Samples

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10.1 Student's *t* Distribution

Student's t Distribution (1 of 3)

W.S. Gosset in 1908 derived a complicated formula for the density function of

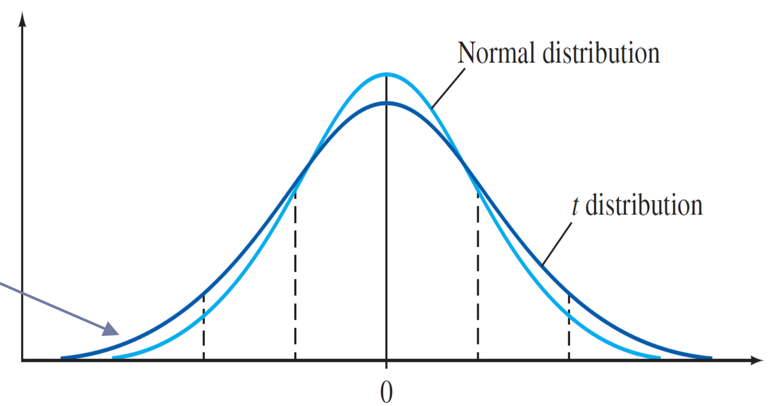
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

for random samples of size *n* from a normal population, and he published his results under the pen name "Student." Ever since, the statistic has been known as **Student's *t***.

Note that both $\bar{x}$ and s are random variables.

Student's *t* Distribution (2 of 3)

- Student's *t* distribution and z are similar.
- Student's *t* distribution has "heavier tails" than z. This is because the *t* statistic involves two random quantities, $\bar{x}$ and s whereas the z statistic only the sample mean, $\bar{x}$.
- The *t*-distribution approaches the normal distribution with mean 0 and variance 1, when n is large.



Standard normal z and the *t* distribution with 5 degrees of freedom

Figure 10.1

Student's *t* Distribution (3 of 3)

The shape of the *t* distribution depends on the sample size *n*. As *n* increases, *t* distribution is more similar to z because the estimate s is closer to σ.

The divisor (*n* − 1) in the formula for the sample variance s^2 is called the number of degrees of freedom (*df*) associated with s^2. It determines the *shape* of the *t* distribution.

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Note that the term degrees of freedom is also used in other areas and methods.

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Example 10.2

Suppose you have a sample of size n = 10 from a normal distribution. Find a value of t such that only 1% of all values of t will be smaller.

Example 10.2 – Solution (1 of 2)

The degrees of freedom that specify the correct *t* distribution are $df = n - 1 = 9$, and the necessary *t*-value must be in the lower portion of the distribution, with area .01 to its left, as shown in Figure 10.3.

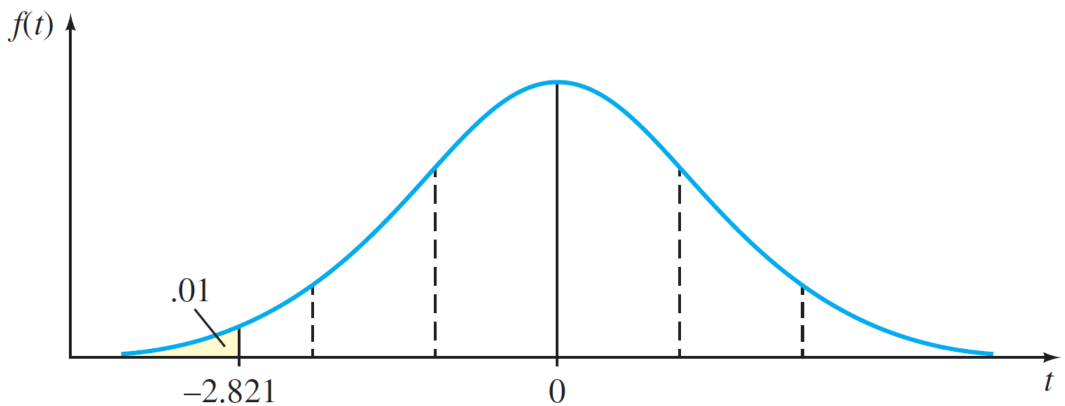


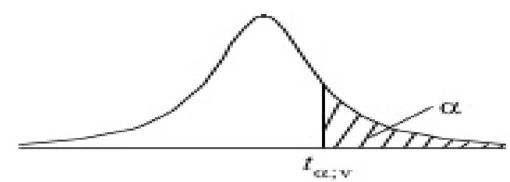
Figure 10.3

Example 10.2 – Solution (2 of 2)

Since the t distribution is symmetric about 0, this value is simply the negative of the value on the right-hand side with area .01 to its right, or $-t_{.01} = -2.821$.

Table of the Student's t-distribution

The table gives the values of $t_{\alpha;\,\nu}$ where $\Pr(T_\nu > t_{\alpha;\,\nu}) = \alpha$, with ν degrees of freedom



ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587

Assumptions behind Student's *t* Distribution (1 of 1)

The critical values of *t* allow you to make reliable inferences *only if* you follow all the rules; that is, your sample must meet these requirements specified by the *t* distribution:

- The sample must be randomly selected.
- The population from which you are sampling must be normally distributed.

Student's t Distribution vs Normal distribution

- x_i follows a normal distribution.
- n<30
- σ is unknown.
- $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$ follows a t-distribution with degrees-of-freedom n-1.

- x_i follows an unknown distribution.
- n≥30
- z= $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \approx \frac{\bar{x}-\mu}{s/\sqrt{n}}$ follows the standard normal distribution (CLT).

- x_i follows a normal distribution
- n<30
- σ is known.
- z= $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ follows the standard normal distribution.

10.2 Small-Sample Inferences Concerning a Population Mean

Small-Sample Hypothesis Test for μ (n<30)

Assumption: The sample is randomly selected from a normally distributed population.

1. Null hypothesis: $H_0 : \mu = \mu_0$

2. Alternative hypothesis:

One-Tailed Test

$H_a : \mu > \mu_0$

(or, $H_a : \mu < \mu_0$)

Two-Tailed Test

$H_a : \mu \neq \mu_0$

3. Test statistic: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

4. Rejection region: Reject H_0 when

One-Tailed Test

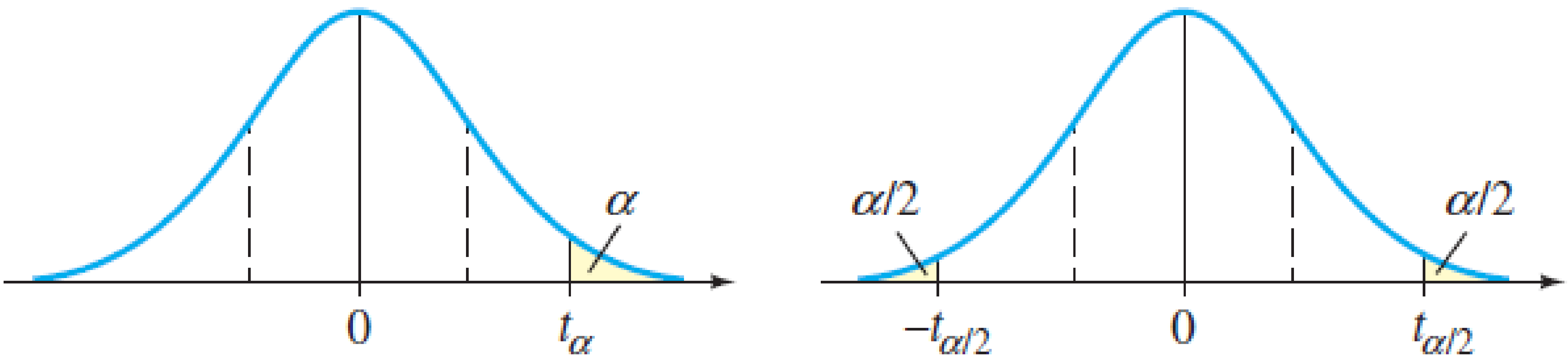
$t > t_\alpha$

(or $t < -t_\alpha$ when the alternative hypothesis is $H_a: \mu < \mu_0$)

Two-Tailed Test

$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$

or when p-value $< \alpha$

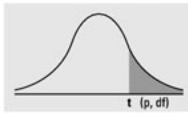


The critical values of t, t_α, and $t_{\alpha/2}$ are based on $(n - 1)$ degrees of freedom. These tabulated values can be found using Table in the next page.

Small-Sample Inferences Concerning a Population Mean (4 of 10)

Student's t Distribution Table

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

Online t-distribution calculator
https://homepage.stat.uiowa.edu/~mbognar/applets/t.html

https://www.dummies.com/education/math/statistics/how-to-use-the-t-table-to-solve-statistics-problems/

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Small-Sample $(1 - \alpha)100\%$ Confidence Interval for μ

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where $s/\sqrt{n}$ is the estimated standard error of $\bar{x}$ often referred to as the **standard error of the mean**.

Example 10.3

A new process for producing synthetic diamonds can be operated at a profitable level only if the average weight of the diamonds is greater than .5 karat. To evaluate the profitability of the process, six diamonds are generated, with recorded weights .46, .61, .52, .48, .57, and .54 karat.

Do the six measurements present sufficient evidence to indicate that the average weight of the diamonds produced by the process is in excess of .5 karat?

Example 10.3 – Solution (1 of 5)

The population of diamond weights produced by this new process has mean μ. Assume that x follows a normal distribution.

Null and alternative hypotheses:

$$H_0 : \mu = 0.5 \text{ versus } H_a : \mu > 0.5$$

Test statistic: The mean and standard deviation for the six diamond weights are

$$\bar{x} = 0.53$$
$$s = 0.0559$$

Example 10.3 – Solution (2 of 5)

The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{.53 - .5}{.0559/\sqrt{6}} = 1.32$$

As with the large-sample tests, the test statistic provides evidence for either rejecting or not rejecting H_0 depending on how far from the center of the t distribution it lies.

Example 10.3 – Solution (3 of 5)

Rejection region: If you choose a 5% level of significance (0.05), the right-tailed rejection region is found using the critical values of *t* from Table.

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	*df*
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
100	1.290	1.660	1.984	2.364	2.626	100
200	1.286	1.653	1.972	2.345	2.601	200
300	1.284	1.650	1.968	2.339	2.592	300
400	1.284	1.649	1.966	2.336	2.588	400
500	1.283	1.648	1.965	2.334	2.586	500
inf.	1.282	1.645	1.96	2.326	2.576	inf.

The Student's *t* Table from Table 4 in Appendix I

Table 10.1

Example 10.3 – Solution (4 of 5)

With $df = n - 1 = 5$, you can reject H_0 if $t > t_{.05} = 2.015$, as shown in Figure 10.5.

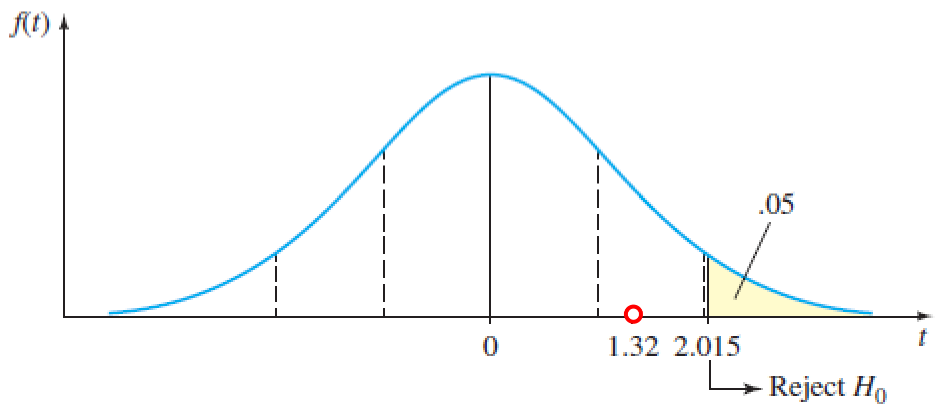


Figure 10.5

Example 10.3 – Solution (5 of 5)

Conclusion: Since the calculated value of the test statistic, 1.32, does not fall in the rejection region, you cannot reject H_0.

The data do not present sufficient evidence to indicate that the mean diamond weight exceeds 0.5 karat.

Remember that there are two ways to conduct a test of hypothesis:

- **The critical value approach:** Set up a rejection region based on the critical values of the statistic's sampling distribution. If the test statistic falls in the rejection region, you can reject H_0. (e.g., $t > t_\alpha$, reject H_0)
- **The *p*-value approach:** Calculate the *p*-value based on the observed value of the test statistic. If the *p*-value is smaller than the significance level, α, you can reject H_0. (e.g., $p < \alpha$, reject H_0)

Example 10.4 (1 of 2)

Labels on 1-gallon cans of paint usually indicate the drying time and the area that can be covered in one coat. One manufacturer claims that a gallon of its paint will cover 400 square feet of surface area. To test this claim, a random sample of ten 1-gallon cans of white paint were used to paint 10 identical areas using the same kind of equipment. The actual areas (in square feet) covered by these 10 gallons of paint are:

310	311	412	368	447
376	303	410	365	350

Example 10.4 (2 of 2)

Do the data present sufficient evidence to indicate that the average coverage differs from 400 square feet? Find the p-value for the test, and use it to evaluate the statistical significance of the results. (Assume that x follows a normal distribution.)

Example 10.4 – Solution (1 of 5)

To test the claim, the hypotheses to be tested are

$$H_0 : \mu = 400 \quad \text{versus} \quad H_a : \mu \neq 400$$

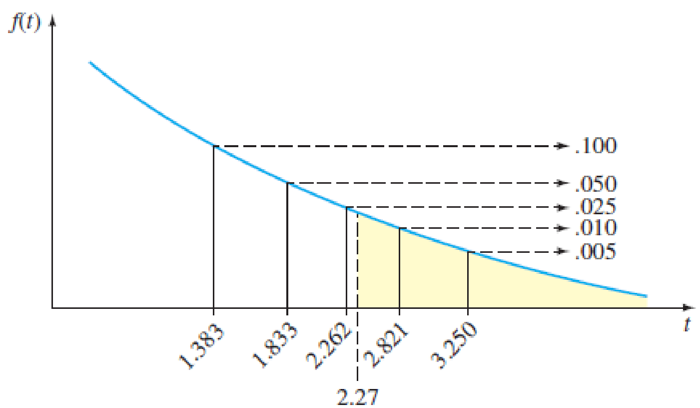
The sample mean and standard deviation for the recorded data are

$$\bar{x} = 365.2 \qquad s = 48.417$$

and the test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{365.2 - 400}{48.417/\sqrt{10}} = -2.27$$

Since this is a two-tailed test, the p-value is the probability that either $t \leq -2.27$ or $t \geq 2.27$.



(shaded area = $\frac{1}{2}$ p-value)

Figure 10.6

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
100	1.290	1.660	1.984	2.364	2.626	100
200	1.286	1.653	1.972	2.345	2.601	200
300	1.284	1.650	1.968	2.339	2.592	300
400	1.284	1.649	1.966	2.336	2.588	400
500	1.283	1.648	1.965	2.334	2.586	500
inf.	1.282	1.645	1.96	2.326	2.576	inf.

Example 10.4 – Solution (3 of 5)

The value $t = 2.27$ falls between $t_{.025} = 2.262$ and $t_{.010} = 2.821$. Therefore, the right-tail area corresponding to the probability that $t > 2.27$ lies between .01 and .025.

Since this area represents only half of the *p*-value, you can write

$$.01 < \frac{1}{2}(p\text{-value}) < .025 \quad \text{or} \quad .02 < p\text{-value} < .05$$

If $\alpha = 0.05$, H_0 *will be rejected, but if* $\alpha = 0.02$, H_0 *will not be rejected.*

Example 10.4 – Solution (4 of 5)

For this test of hypothesis, H_0 is rejected at the 5% significance level. There is sufficient evidence to indicate that the average coverage differs from 400 square feet.

Within what limits does this average coverage *really* fall? A 95% confidence interval gives the upper and lower limits for μ as

$$\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$365.2 \pm 2.262\left(\frac{48.417}{\sqrt{10}}\right)$$

$$365.2 \pm 34.63$$

Example 10.4 – Solution (5 of 5)

The average area covered by 1 gallon of this brand of paint lies in the interval 330.6 to 399.8.

A more precise interval estimate (a shorter interval) can generally be obtained by increasing the sample size.

Notice that the upper limit of this interval is very close to the value of 400 square feet, the coverage claimed on the label.

10.3 Small-Sample Inferences for the Difference Between Two Population Means: Independent Random Samples

- x_1 follows a normal distribution with mean μ_1 and σ.
- x_2 follows a normal distribution with mean μ_2 and σ.
- The numbers of sample of x_1 and x_2 are respectively n_1 and n_2. They are randomly and independently drawn from the distribution.
- To test the difference between the two population means e.g.,

$$Ho: \mu_1 - \mu_2 = 0$$
$$H_A: \mu_1 - \mu_2 \neq 0$$

- The statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

- follows a *Student's t distribution with degrees of freedom* $n_1 + n_2 - 2$.

- where s^2 is *pooled estimate* of the common variance:

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Remarks:

- The two distributions have the same σ.
- s^2 is an unbiased estimator of the common population variance σ^2.

Test of Hypothesis Concerning the Difference Between Two Means: Independent Random Samples

1. Null hypothesis: $H_0 : (\mu_1 - \mu_2) = D_0$,

2. Alternative hypothesis:

One-Tailed Test

$H_a : (\mu_1 - \mu_2) > D_0$

[or $H_a : (\mu_1 - \mu_2) < D_0$]

Two-Tailed Test

$H_a : (\mu_1 - \mu_2) \neq D_0$

Remarks: D_0 is some specified difference that you wish to test. For many tests, you will hypothesize that there is no difference between μ_1 and μ_2 so that $D_0 = 0$.

3. Test statistic: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{s^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

4. Rejection region: Reject H_0 when:

One-Tailed Test

$t > t_\alpha$

[or $t < -t_\alpha$ when the alternative hypothesis is $H_a : (\mu_1 - \mu_2) < D_0$]

or when p-value $< \alpha$

Two-Tailed Test

$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$

The critical values of t, t_{α}, and $t_{\alpha/2}$ are based on $(n_1 + n_2 - 2)$ *df*. The tabulated values can be found using *t*-Table.

Assumptions: The samples are randomly and independently selected from normally distributed populations. The variances of the populations σ_1^2 and σ_2^2 are equal.

Small-Sample $(1-\alpha)100\%$ Confidence Interval for $(\mu_1-\mu_2)$ Based on Independent Random Samples

$$(\bar{x}_1-\bar{x}_2)\pm t_{\alpha/2}\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

where s^2 is the pooled estimate of σ^2

$$s^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$

Example 10.5 (1 of 3)

A course can be taken for credit either by attending lecture sessions in a classroom, or by doing online sessions.

The instructor wants to know if these two ways of taking the course resulted in a significant difference in achievement as measured by the final exam for the course.

Example 10.5 (2 of 3)

Table 10.2 gives the scores on an examination with 45 possible points for one group of $n_1 = 9$ students who took the course online, and a second group of $n_2 = 9$ students who took the course by attending classroom lecture sessions.

Online	Classroom
32	35
37	31
35	29
28	25
41	34
44	40
35	27
31	32
34	31

Test Scores for Online and Classroom Presentations

Table 10.2

Example 10.5 (3 of 3)

Do these data present sufficient evidence to indicate that the average grade for students who take the course online is significantly higher than for those who attend classroom lecture sessions? Use $\alpha=0.05$ in the test.

Example 10.5 – Solution (1 of 6)

Let μ_1 and μ_2 be the mean scores for the online group and the classroom group, respectively. Then, because you want to support the theory that $\mu_1 > \mu_2$, you can test the null hypothesis

$$H_0 : \mu_1 = \mu_2 \qquad [\text{or} : H_0 : (\mu_1 - \mu_2) = 0]$$

versus the alternative hypothesis

$$H_a : \mu_1 > \mu_2 \qquad [\text{or} : H_a : (\mu_1 - \mu_2) > 0]$$

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Example 10.5 – Solution (2 of 6)

To conduct the *t*-test for these two independent samples, you must assume that the sampled populations are both normal and have the same variance σ^2. Is this reasonable? Stem and leaf plots of the data in figure show at least a "mounding" pattern, so that the assumption of normality is not unreasonable.

Online		Classroom	
2	8	2	579
3	124	3	1124
3	557	3	5
4	14	4	0

Test Scores for Online and Classroom Presentations

Figure 10.8

Example 10.5 – Solution (3 of 6)

Furthermore, the standard deviations of the two samples, calculated as

$$s_1 = 4.9441 \quad \text{and} \quad s_2 = 4.4752$$

are not different enough for us to doubt that the two distributions may have the same shape. If you make these two assumptions and calculate (using full accuracy) the pooled estimate of the common variance as

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{8(4.9441)^2 + 8(4.4752)^2}{9 + 9 - 2} = 22.2361$$

Example 10.5 – Solution (4 of 6)

The test statistic,

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{35.22 - 31.56}{\sqrt{22.2361\left(\frac{1}{9} + \frac{1}{9}\right)}} = 1.65$$

The alternative hypothesis $H_a : \mu_1 > \mu_2$ or, equivalently, $H_a : (\mu_1 - \mu_2) > 0$ implies that you should use a one-tailed test in the upper tail of the t distribution with $(n_1 + n_2 - 2) = 16$ degrees of freedom.

Example 10.5 – Solution (5 of 6)

Since α=0.05, degrees of freedom=16, $t_{0.05} = 1.746$.

Since 1.65<1.746, cannot reject H_o.

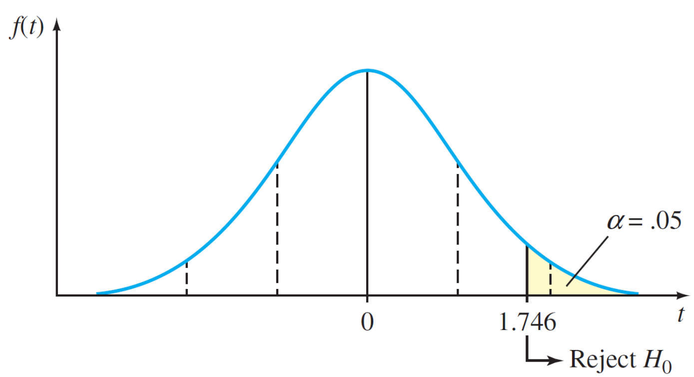

Figure 10.9

df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216

Example 10.5 – Solution (6 of 6)

There is insufficient evidence to indicate that the average online course grade is higher than the average classroom course grade at the 5% level of significance.

Example 10.6

Find the p-value that would be reported for the statistical test in Example 10.5.

Solution:

The observed value of t for this one-tailed test is $t = 1.65$. Therefore,

$$p\text{-value} = P(t > 1.65)$$

Example 10.6 – Solution

- $t_{0.05}$= 1.746, $t_{0.1}$=1.337 and t = 1.65
- Since $t_{0.05}$>t> $t_{0.01}$
- The *p*-value for this test would be reported as

0.05 < *p*-value <0.10

Because the *p*-value is greater than .05, most researchers would report the results as *not significant.*

df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
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7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
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18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216

If the population variances are far from equal, there is an alternative procedure for estimation and testing that has an *approximate t* distribution in repeated sampling.

As a rule of thumb, you should use this procedure if the ratio of the two sample variances,

$$\frac{\text{Larger } s^2}{\text{Smaller } s^2} > 3$$

When the population variances are not equal, the pooled estimator s^2 is no longer appropriate, and each population variance must be estimated by its corresponding sample variance.

The familiar test statistic is

$$\frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

However, when the sample sizes are *small*, critical values for this statistic are found using degrees of freedom approximated by the formula

$$df \approx \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{(s_1^2/n_1)^2}{(n_1 - 1)} + \dfrac{(s_2^2/n_2)^2}{(n_2 - 1)}}$$

The degrees of freedom are taken to be the integer part of this result.