SC2000: Practice Quiz 1 solutions

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1 Solutions/answers

Question 1 A research student wishes to know whether male students have more conservative attitudes than female students. A survey questionnaire is distributed at random to 50 males and 50 females students to assess their attitudes. This an example of ...

solution Inferential statistics

Question 2 For the data set: 1, 3, 7, 4, 5, 7, 14 , determine the upper and lower adjacent values of the box plot. Obtain value of the outlier (if any).

Express your answers in integer numbers only (without any space, full stop or any characters other than numbers). If there is no outlier, enter the letter N.

Upper adjacent value =

Lower adjacent value =

Outlier =

solution Sorted data: $\{1, 3, 4, 5, 7, 7, 14\}$
To calculate the 25th percentile, we have:
Lower hinge $= R_{25} = \frac{25}{100}(N-1)+1 = \frac{25}{100}(7-1)+1 = 2.5$ $P_{25} = 3+(4-3)\cdot 0.5 = 3.5$

To calculate the 75th percentile, we have: $R_{75} = \frac{75}{100}(N-1)+1 = \frac{75}{100}(7-1)+1 = 5.5$
Upper hinge= $P_{75} = 7 + (7-7)\cdot 0.5 = 7$
$Step = 1.5 \cdot IQR = 1.5 \cdot (7-3.5) = 5.25$

Upper inner fence $= 7 + 5.25 = 12.25$
Lower inner fence $= 3.5 - 5.25 = -1.75$

Upper adjacent value=7
Lower adjacent value=1
Outlier=14

Question 3 The findings of a survey conducted on a certain group of workers indicate that the correlation between the number of construction site accidents per year and the worker's age is 0.5. This means that younger workers get in more accidents.

solution False

Question 4 The universal set Ω is defined as $\Omega = \{1,2,3,4,5,6,7,8,9,10\}$. Given that A = $\{1,2,3\}$, B = $\{2,3,4,5,6,7\}$ and C = $\{7,8,9,10\}$, find $A' \cup (B \cap C')$.

solution $C' = \{1, 2, 3, 4, 5, 6\}$
$B \cap C' = \{2, 3, 4, 5, 6\}$
$A' = \{4, 5, 6, 7, 8, 9, 10\}$
$A' \cup (B \cap C') = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Question 5 Given P(A)=[a], P(B)=[b] and P(A and B)=[c]. Calculate the probability P(A' and B'). Express your answer in decimal number up to 3 decimal places.

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)) = 1 - (a + b - c)$$

Question 6 A car rental company has 4 small-size cars and 3 big-size cars. If 4 of the cars are randomly selected for a safety check, what is the probability of getting 2 of each kind? Express your answer in decimal number to 3 decimal places.

There are $\binom{4}{2}$ ways to choose 2 of the 4 small-size cars and $\binom{3}{2}$ ways to choose 2 of the 3 big-size cars. This is a total of $\binom{4}{2} \cdot \binom{3}{2} = 18$ ways out of the $\binom{7}{4} = 35$ total ways.
Thus, the probability is $18/35 = 0.514$.

Question 7

solution b,c,d are incorrect.

Question 8 A bag contains 6 marbles. Some are red and some are green. When 2 marbles are randomly picked (without replacement), the probability that both are red is 0.2. Determine the number of red marbles in the bag.

solution Let k be the number of red balls in the bag.
The probability of picking 2 red balls in a row is:

$$\frac{k}{6} \cdot \frac{k-1}{5} = 0.2$$

$$k^2 - k = 6$$
$$k^2 - k - 6 = 0$$
$$(k-3)(k+2) = 0$$

Thus $k = 3$ since it has to be a positive integer.

Question 9 A study recorded the average bedtime of students one month prior to their examinations. It was found that the correlation between their average bedtimes and their final examination scores was -0.45. This implies that students who habitually slept earlier tended to have better examination performance.

solution True

Question 10

solution Let $X_1 = \{x_{1,1}, \ldots, x_{1,n}\}, X_2 = \{x_{2,1}, \ldots, x_{2,m}\}$ represent the two survey samples on travel time and $X = \{x_{1,1}, \ldots, x_{1,n}, x_{2,1}, \ldots, x_{2,m}\}$ be the combined data. From the first row of the table, we get the following:

$$\frac{1}{20} \sum x_{1,i} = 30$$
$$s_1^2 = \frac{1}{19} \sum x_{1,i}^2 - \frac{(20 \cdot 30)^2}{20} = 100$$
$$\sum x_{1,i}^2 = 1900 + 20 \cdot 900 = 19900$$

From the second row of the table, we get the following:

$$\frac{1}{16} \sum x_{2,i} = 40$$
$$s_2^2 = \frac{1}{15} \sum x_{2,i}^2 - \frac{(16 \cdot 40)^2}{16} = 225$$
$$\sum x_{2,i}^2 = 15 \cdot 225 + 16 \cdot 1600 = 28975$$

Finally, we use again the formula $s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right]$ to compute the combined sample variance as follows:

$$s^2 = \frac{1}{(20+16)-1}\left[\sum x_{1,i}^2 + \sum x_{2,i}^2 - \frac{(20 \cdot 30 + 16 \cdot 40)^2}{20+16}\right]$$
$$s^2 = \frac{1}{35}\left[19900 + 28975 - \frac{(1240)^2}{36}\right] = 176.111$$

Question 11 The universal set Ω is defined as $\Omega = \{0,1,2,3,4,5,6,7,8,9\}$. Given that A = {1,2,3,4}, B = {3,4,5,6,7} and C = {7,8,9}, find $(A' \cup B) \cap C'$.

solution $C' = \{0, 1, 2, 3, 4, 5, 6\}$

$A' = \{0, 5, 6, 7, 8, 9\}$
$A' \cup B = \{0, 3, 4, 5, 6, 7, 8, 9\}$
$(A' \cup B) \cap C' = \{0, 3, 4, 5, 6\}$

Question 12 Given P(A)=[a], P(B)=[b] and P(A and B)=[c]. Calculate the probability $P(A \cap B')$.

Express your answer in decimal number up to 3 decimal places.

solution

$$P(A) = P(A \cap B) + P(A \cap B')$$

$$P(A \cap B') = P(A) - P(A \cap B) = a - c$$

Question 13 A chairman and a secretary are to be chosen from [x] committee members. How many different ways of selections are possible if one member said that he will serve only if he is selected to be the chairman?

solution There are $x-1$ selections in which that committee member is serving as chairman and $(x-1) \cdot (x-2)$ ways to select two other committee members for the positions. So, the total is:

$$x - 1 + (x - 1) \cdot (x - 2) = (x - 1)^2$$

Question 14 Given the data set [a], 0, [b], 2, [c], 1, [d], 10, determine the mean. Express your answer in decimal number up to 3 decimal places.

solution The mean is: $(a+0+b+2+c+1+d+10)/8 = (a+b+c+d+13)/8$

Question 15 Given that the median and the trimean of a certain data set are 20 and 19.5 respectively. If the interquartile range is 8, determine the first and the third quartile.

solution Median: $P_{50} = 20$
Trimean: $\frac{P_{25}+2P_{50}+P_{75}}{4} = 19.5$

$$P_{25} + P_{75} = 4 \cdot 19.5 - 2 \cdot 20 = 38$$

IQR: $P_{75} - P_{25} = 8$

By adding the above 2 equations, we get:

$$2 \cdot P_{75} = 46$$

$$P_{75} = 23, P_{25} = 23 - 8 = 15$$

Question 16 Figure below shows a sensor grid network consisting of 8 X 5 = 40 sensors. Each sensor is represented by a dot and located at (x,y) where x=1,2,...,8 and y=1,2,...,5. Assume that the sensor at (x,y) can only send message to the sensor at (x+1,y) or (x,y+1). Assume that all the message routes are equally likely, calculate the probability that the message sent from sensor (1,1) to sensor (8,5) will pass through sensor (7,3).

solution

- To go from (1,1) to (7,3) one needs take 6 steps to the right and 2 steps up. There are $\binom{8}{2}$ ways to do these 8 steps.
- To go from (7,3) to (8,5) one needs take 1 step to the right and 2 steps up. There are $\binom{3}{2}$ ways to do these 3 steps.
- To go from (1,1) to (8,5) one needs take 7 steps to the right and 4 steps up. There are $\binom{11}{4}$ ways to do these 11 steps.

Therefore, since all message routes are equally likely, the probability that the message goes through (7,3) is: $\frac{\binom{8}{2}\cdot\binom{3}{2}}{\binom{11}{4}} = \frac{28\cdot 3}{330} = 0.255$.

Question 17 A chairman and a secretary are to be chosen from [x] committee members. How many different ways of selections are possible if one member said that he will serve only if he is selected to be the chairman?

solution There are $x-1$ selections in which that committee member is serving as chairman and $(x-1)\cdot(x-2)$ ways to select two other committee members for the positions. So, the total is:

$$x - 1 + (x-1)\cdot(x-2) = (x-1)^2$$

Question 18 John bought one share of stock in "ABC Bank" for \$20. Each day the share price independently increases or decreases by \$1 with probability p or (1-p) respectively, where p = [p]. John will sell his one share of stock if it gains \$3 or loses \$3. Determine the probability that John will sell his share exactly 5 days after he has bought it.

solution For John to sell the stock after exactly 5 days, it has to either go up 4 times and down 1 time or down 4 times and up 1 time. However, the case where the stock either goes only up or goes only down the first 3 days has to be excluded, since in that case it would be sold on day 3.

Therefore, there are 3 combinations in which the stock can be sold with \$3 gain, as the day on which it decreases has to be one of the first 3. Each of them has probability $p^4(1-p)$. Similarly, there are 3 combinations in which the stock can be sold with \$3 loss. Each of them has probability $p(1-p)^4$. Thus, the total probability is: $3p^4(1-p) + 3p(1-p)^4$.

Question 19 A set of data shows that the mean service time at a certain service counter is much higher than the median service time, what can you say about the shape of the distribution of service times?

solution The shape of the distribution is positively skewed.

Question 20 Jessica has a safe which uses a numerical passcode to unlock. The length of the passcode can be set at minimum 4 to maximum 6 digits. Each digit is selected from {0,1,2,3,4,5,6,7,8,9} with replacement. Jessica set the passcode once and has not changed it since then. It is noted that there are 5 blurred marks over 5 digits on the key pad touch screen. Determine the number of possible distinct passcodes.

solution Due to the 5 marks, we deduce that the passcode has 5 distinct digits. Therefore, it can be 5 digits long with distinct digits or 6 digits long with 1 digit appearing twice. The number of possible 5 digit long passcodes with 5 known distinct digits is: $5! = 120$, since we know the 5 digits used, but we do not know in which order.

In the case where the passcode is 6 digit long with only 5 distinct digits, which are known to us, we have the following:

- We have 5 choices for the digit appearing twice
- We have $\binom{6}{2}$ choices for the pair of locations in which that digit appears
- We have 4! choices for the rearrangement of the remaining digits in the remaining 4 positions.

Thus, we have a total of $5 \cdot \binom{6}{2} \cdot 4! = 5 \cdot 15 \cdot 24 = 1800$ 6-digit passcodes.

The total number of possible passcodes is 1800+120=1920.

Question 21 On any day the chance of rain is 25%. The chance of rain on two consecutive days is 10%. This means that the event of rain on the first day and the event of rain on the second day is...

solution Dependent