Unit II

UNIT II Transformations: Basic 2D & 3D transformations - Translation, Scaling, Rotation, Reflection, Shearing, Multiple Transformations, Rotation about an axis parallel to a coordinate axis, Rotation about an arbitrary axis in space, Affine and Perspective Geometry, Orthographic projections and Axonometric projections

Transformations:

Definition

Transformation means changing some graphics into something else by applying rules. Transformations play an important role in computer graphics to reposition the graphics on the screen and change their size or orientation.

There are two types of transformation:

1) **Geometric Transformation**: Geometric transformations change the position, orientation, or shape of an object. The object itself is transformed relative to the coordinate system or background. For example, Moving the automobile while keeping the background fixed. Video game developers leverage geometric transformation to bring characters and environments to life, providing immersive gameplay experiences. Additionally, geometric transformation is vital in medical imaging, architectural design, and industrial simulations.
2) **Coordinate Transformation**: The object is held stationary while the coordinate system is transformed relative to the object. This effect is attained through the application of coordinate transformations. For example , We can keep the car fixed while moving the background scenery.

   Coordinate transformation plays a critical role in navigation systems, robotics, and GPS technology. It enables precise positioning and movement calculations, facilitating accurate path planning for autonomous vehicles and robotic arms. Coordinate transformation also contributes to satellite imagery, cartography, and geographical analysis, aiding in mapping and geospatial data processing.

   Representation of point :

   A "row-major" order would represent the point coordinates as a single row in a matrix, but this is less common in most mathematical contexts..

   [2, 3]

   A point is typically represented as a column vector in a matrix, meaning the coordinates of the point are listed vertically in a single column, making it a "column-major" order representation.

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

   Since column vector representation is standard mathematical notation and since many graphics package like GKS and PHIGS uses column vector we will also follow column vector representation.

2D Transformation in Computer Graphics :

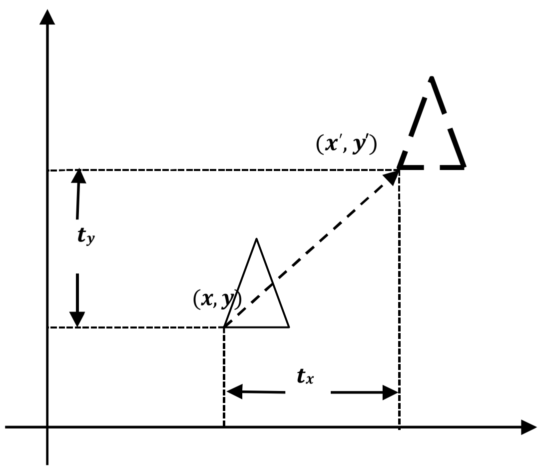
When a transformation takes place on a 2D plane, it is called 2D transformation. 2D Transformation is utilized to modify the position, orientation, or size of objects within a two-dimensional space. These transformations involve applying mathematical operations to the coordinates of points or vertices in order to achieve the desired changes.

- Rotation
- Translation
- Scaling
- Reflection
- Shearing

Translation

Translation in computer graphics is a transformation that moves an object without changing its shape or orientation in a specified direction..

It is rigid body transformation so we need to translate whole object. We translate two dimensional point by adding translation distance tx and ty to the original coordinate position (x, y) to move at new position (x', y') where translation in the x-direction is represented using tx , the translation in y-direction is represented using ty.as:



$$x' = x + tx \quad \&$$

$$y' = y + ty$$

 Translation distance pair (tx, ty) is called a Translation Vector or Shift Vector.

- We can represent it into single matrix equation in column vector as;

P′* = *P* + *T

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Translation Matrix

Problems on translation transformation:

1)Given a circle C with radius 10 and centre coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Given-

Old centre coordinates of C = (Xold, Yold) = (1, 4)

Translation vector = (Tx, Ty) = (5, 1)

Let the new centre coordinates of C = (Xnew, Ynew).

Applying the translation equations, we have-

Xnew = Xold + Tx = 1 + 5 = 6

Ynew = Yold + Ty = 4 + 1 = 5

Thus, New centre coordinates of C = (6, 5).

- Example2: Translate a polygon with coordinates A(2,5), B(7,10) and C(10,2) by 3 units in x direction and 4 unit in y direction.
- Solution:

$$A' = A + T = \begin{bmatrix}2\\5\end{bmatrix} + \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}5\\9\end{bmatrix}$$

$$B' = B + T = \begin{bmatrix}7\\10\end{bmatrix} + \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}10\\14\end{bmatrix}$$

$$C' = C + T = \begin{bmatrix}10\\2\end{bmatrix} + \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}13\\6\end{bmatrix}$$

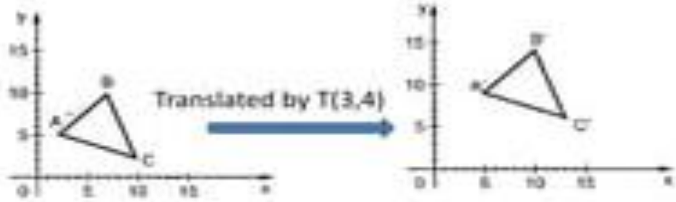


Fig: Before translation   Fig: After translation

3)Given a square with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the translation with distance 1 towards X axis and 1 towards Y axis. Obtain the new coordinates of the square.

Solution-

Given-

Old coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)

Translation vector = (Tx, Ty) = (1, 1)

A(0, 3)

Applying the translation equations, we have-

Xnew = Xold + Tx = 0 + 1 = 1

Ynew = Yold + Ty = 3 + 1 = 4

Thus, New coordinates of corner A = (1, 4).

B(3, 3)

Applying the translation equations, we have-

Xnew = Xold + Tx = 3 + 1 = 4

Ynew = Yold + Ty = 3 + 1 = 4

Thus, New coordinates of corner B = (4, 4).

C(3, 0)

Applying the translation equations, we have-

Xnew = Xold + Tx = 3 + 1 = 4

Ynew = Yold + Ty = 0 + 1 = 1

Thus, New coordinates of corner C = (4, 1).

For Coordinates D(0, 0)

Applying the translation equations, we have-

Xnew = Xold + Tx = 0 + 1 = 1

Ynew = Yold + Ty = 0 + 1 = 1

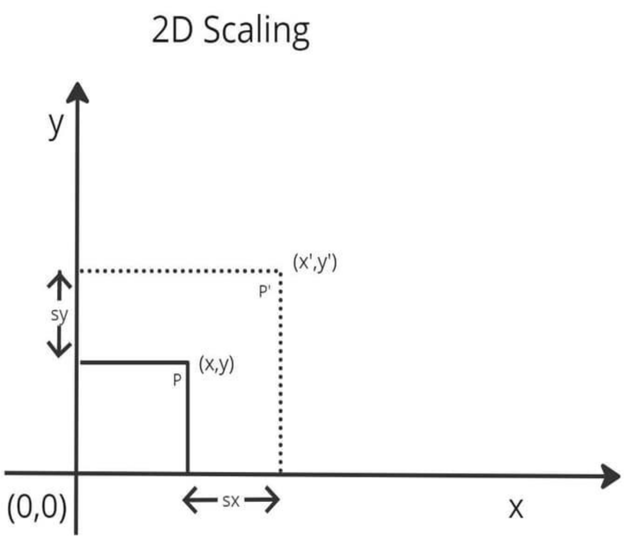
Thus, New coordinates of corner D = (1, 1).

Thus, New coordinates of the square = A (1, 4), B(4, 4), C(4, 1), D(1, 1).

(Use any one method to solve problem)

4) (H W)Translate the triangle [A (10, 10), B (15, 15), C (20, 10)] 2 unit in x direction and 1 unit in y direction.

Scaling



In computer graphics, scaling is a process of modifying or altering the size of objects.

• Scaling may be used to increase or reduce the size of object . This operation is carried out by multiplying coordinate value (x, y) with scaling factor (sx, sy) for x and y axis respectively or you can say Sx scaling factor in x direction Sy scaling factor in y-direction

So equation for scaling is given by:

$$x' = x \cdot sx$$
$$y' = y \cdot sy$$

• Scaling factor determines whether the object size is to be increased or reduced.

• If scaling factor > 1, then the object size is increased.
• If scaling factor < 1, then the object size is reduced.
• If scaling factors are equal ,then the object remains unchanged.

Same values of sx and sy will produce Uniform Scaling. And different values of sx and sy will produce Differential Scaling.
We can control the position of object after scaling by keeping one position fixed called Fix point (xf, yf) that point will remain unchanged after the scaling transformation.

The matrix used in the representation of scaling is known as the scaling matrix.

- These equation can be represented in column vector matrix equation as:
  $P' = S \cdot P$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Scaling Matrix

Numerical on scaling

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)

Scaling factor along X axis Sx= 2

Scaling factor along Y axis Sy = 3

Applying the scaling equations on A(0,3)

we have-

Xnew = Xold x Sx = 0 x 2 = 0

Ynew = Yold x Sy = 3 x 3 = 9

Thus, New coordinates of corner A after scaling = (0, 9).

For Coordinates B(3, 3)

Xnew = Xold x Sx = 3 x 2 = 6

Ynew = Yold x Sy = 3 x 3 = 9

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0)

Xnew = Xold x Sx = 3 x 2 = 6

Ynew = Yold x Sy = 0 x 3 = 0

Thus, New coordinates of corner C after scaling = (6, 0).

For Coordinates D(0, 0)

Xnew = Xold x Sx = 0 x 2 = 0

Ynew = Yold x Sy = 0 x 3 = 0

Thus, New coordinates of corner D after scaling = (0, 0).

Thus, New coordinates of the square after scaling = A (0, 9), B(6, 9), C(6, 0), D(0, 0).

2)Consider square with left-bottom corner at (2, 2) and right-top corner at (6, 6) apply the transformation which makes its size half.

As we want size half so value of scale factor are sx = 0.5, sy = 0.5 and Coordinates of square are [A (2, 2), B (6, 2), C (6, 6), D (2, 6)].

(solve this by coloum matrix representation)

Rotation

It is a transformation that used to involves rotating an object around a fixed point in a two-dimensional plane.

To generate a rotation we specify a rotation angle $\boldsymbol{\theta}$ and the position of the Rotation Point (Pivot Point) (X$\boldsymbol{r}$,Y$\boldsymbol{r}$) about which the object is to be rotated.

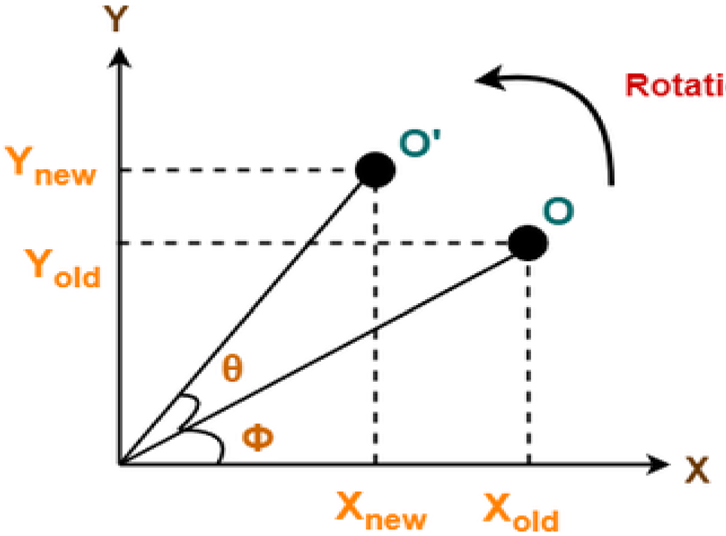
We first find the equation of rotation when pivot point is at coordinate origin($\mathbf{0}$, $\mathbf{0}$). There are two types of rotations according to the direction of the movement of the object.

These are:

- Anti-clockwise rotation
- Clockwise rotation

The positive value of the rotation angle rotates an object in an anti-clockwise direction while the negative value of the rotation angle rotates an object in a clockwise direction. When we rotate any object, then every point of that object is rotated by the same angle. For example, a straight line is rotated by the endpoints with the same angle and the line is re-drawn between the new endpoints. Also, the polygon is rotated by shifting every vertex with the help of the same rotational angle.

From the following figure, we can see that the point P(X, Y) is located at angle φ from the horizontal X coordinate with distance r from the origin. Let us suppose you want to rotate it at the angle θ. After rotating it to a new location , you will get a new point P' (X', Y').



2D Rotation in Computer Graphics

- From figure we can write for anticlockwise rotation,

  $x = r \cos \emptyset$

  $y = r \sin \emptyset$

  and

  $x' = r \cos(\theta + \emptyset) = r \cos \emptyset \cos \theta - r \sin \emptyset \sin \theta$

  $y' = r \sin(\emptyset + \theta) = r \cos \emptyset \sin \theta + r \sin \emptyset \cos \theta$
- Now replace $r \cos \emptyset$ with x and $r \sin \emptyset$ with y in above equation.

  $x' = x \cos \theta - y \sin \theta$

  $y' = x \sin \theta + y \cos \theta$
- We can write it in the form of column vector matrix equation as;
- $P' = R \cdot P$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

For clockwise rotation we have

$x' = x \cos(-\theta) - y \sin(-\theta)$

$y' = x \sin(-\theta) + y \cos(-\theta)$

Put $\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta$ we get

$$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}}$$

Numerical on rotation:

1)Given a triangle with corner coordinates (0, 0), (1, 0) and (1, 1). Rotate the triangle by 90 degrees anticlockwise direction and find out the new coordinates.

Rotate a line CD whose endpoints are (3, 4) and (12, 15) about origin through a 45° anticlockwise direction.

2)Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos30 & -\sin30 \\ \sin30 & \cos30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos30 - 4 \times \sin30 \\ 4 \times \sin30 + 4 \times \cos30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos30 - 4 \times \sin30 \\ 4 \times \sin30 + 4 \times \cos30 \end{bmatrix}$$

Solve by putting values

Reflection

In 2D transformation, reflection is a process that involves flipping an object or image over a line and creating its mirror image. In 2D, the rotation is also described as the rotation transformation by θ = 180. Reflection occurs around a specific line called the axis of reflection.

We can represent Reflection by using four ways-

Reflection along X-axis:

For reflection about x axis x co ordinate is not changed and sign of y coordinate is changed.

i.e. X’=X

and Y’= -Y

thus transformation matrix is represented as:

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

Reflection along Y-axis:

A reflection transformation about the Y-axis in computer graphics means that a point is mirrored across the Y-axis, where the Y-coordinate remains the same, and the X-coordinate becomes its opposite sign, resulting in a new point with coordinates (-x, y) for the original point (x, y).

i.e. X'= - X

and Y'= Y

thus transformation matrix is represented as:

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

Reflection along origin :

A "reflection transformation about the origin" in computer graphics means flipping a 2D object across the origin point (0, 0), essentially creating a mirror image where the x and y coordinates of each point are negated,

If P(x, y) is the point on x-y plane then P'(x', y') is the reflection about origin given as

i.e. X'= - X ,

Y'= -Y

thus transformation matrix is represented as:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Reflection about the straight line Y=X:

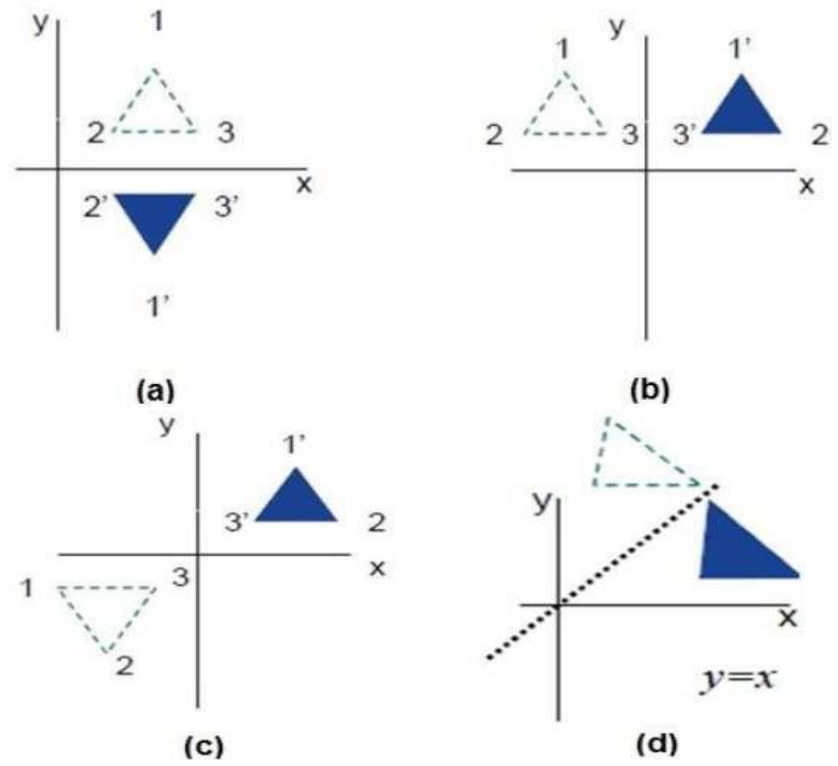
When you reflect a point across the line y = x, the x-coordinate and y-coordinate change their places. The reflection of the point (x, y) across the line y = x is (y, x).

i.e. X'=Y

and Y’=X

thus transformation matrix is represented as:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



In above figure a) represent the reflection along x axis b) represent reflection along y axis c) reflection along origin d) reflection along y=x axis

Shearing :

The transformation shearing when applied to any object results only in distortion of shape. It is also called twist or torque or tilt transformation .

Shearing can be done either in x direction or y direction.

X shear or horizontal shearing :

In x shear, the y co-ordinates remain the same but the x co-ordinates changes. In horizontal shearing, the x-coordinates of points change proportionally to their y-coordinates.

If P(x, y) is the point then the new points will be P’(x’, y’) given as –

X’= X+Shx∗Y;

Y’=Y

The transformation can be written as:

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

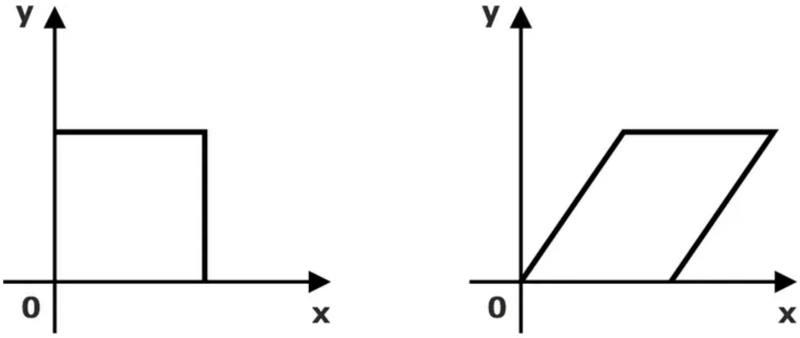


fig. shearing about the x-axis

Y shear or vertical shearing :

In this, we can store the x coordinate and only change the y coordinate. It is also called **“Vertical Shearing.”**

The equation is now:

X’ = X

Y’= X.shy + Y

The transformation can be written as:

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

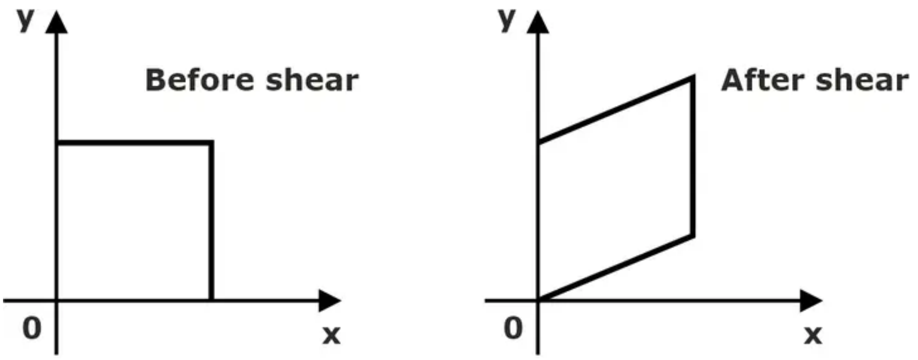


fig. shear in y direction

Shearing in both direction can be written as :

X'= X+Shx∗Y;

Y'= X*Shy + Y

The transformation can be written as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous Transformation

We have matrix representation of basic transformation and we can express it in the general matrix form as:

$\boldsymbol{P' = M1 * P + M2}$

Where

$\boldsymbol{P}$ and $\boldsymbol{P'}$ are initial and final point position,

$\boldsymbol{M1}$ is multiplicative matrix and $\boldsymbol{M2}$ is additive matrix

In some cases, however desired orientation of an object may require more than one transformation to be applied. For example let us consider a case which requires 90 degree rotation of a point about origin followed by reflection about the line y = x.

For rotation put cos 90 = 0 and sin 90 = 1 we get

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$$

Now perform reflection about y= x axis

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -y \\ x \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

These successive matrix operation can be expressed as first rotation matrix multiplied with reflection matrix gives

$$[X''] = [T_M]_{y=x}\,[X']$$

$$[X''] = [T_M]_{y=x}\,\{[T_R]_{90°}[X]\}$$

$$[T_M]_{y=x}[T_R]_{90°} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[X''] = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

which shows same result as before.

This process of calculating the product of matrix of a number of different transformation in a sequence is known as concatenation or combination of transformations and the resultant matrix is referred to as composite or concatenated transformation matrix. But the real problem arises when there is a translation or rotation or scaling a point other than the origin as it involved among several successive transformations. For efficient utilization we must calculate all sequence of transformation in one step and for that reason we reformulate above equation to eliminate the matrix addition associated with translation terms in matrix $\boldsymbol{M2}$.

This is done with the homogeneous coordinate.

- We can combine that thing by expanding 2 X 2 matrix representation into 3X3 matrices.
- It will allows us to convert all transformation into matrix multiplication but we need to represent vertex position $(\boldsymbol{x}, \boldsymbol{y})$ with homogeneous coordinate triple $(\boldsymbol{xh}, \boldsymbol{yh}, \boldsymbol{h})$ Where $\boldsymbol{x} = \boldsymbol{xh}/h$, $\boldsymbol{y} = \boldsymbol{yh}/h$ thus we can also write triple as $(\boldsymbol{h}\cdot\boldsymbol{x},\ \boldsymbol{h}\cdot\boldsymbol{y},\ \boldsymbol{h})$.
- For two dimensional geometric transformation ,we can take value of $\boldsymbol{h}$ is any positive number so we can get infinite homogeneous representation for coordinate

value (x, y). These points are "homogeneous" because they represent the same point in Euclidean space (or Cartesian space). In other words, Homogeneous coordinates are scale invariant. (1,2,3) (2,4,6) and (3,6,9) are the same Cartesian point (1/3, 2/3).

- But convenient choice is set $h = 1$ as it is multiplicative identity, than (x, y) is represented as $(x, y, 1)$.
- Expressing coordinates in homogeneous coordinates form allows us to represent all geometric transformation equations as matrix multiplication. Thus they form the basis for geometry which is used extensively to display three-dimensional objects on two-dimensional image planes.

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Translation Matrix

(Homogeneous Coordinates Representation)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Rotation Matrix

(Homogeneous Coordinates Representation)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Scaling Matrix

(Homogeneous Coordinates Representation)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

(Homogeneous Coordinates Representation)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

(Homogeneous Coordinates Representation)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous co-ordinates representation for shearing .

Composite Transformation

- We can set up a matrix for any sequence of transformations as a composite transformation matrix by calculating the matrix product of individual transformation.
- we form composite transformations by multiplying matrices in order from right to left.
- If a transformation of the plane T1 is followed by a second plane transformation T2, then the result itself may be represented by a single transformation T which is the composition of T1 and T2 taken in that order. This is written as T = T2·T1 in **column matrix form**

Composite transformation can be achieved by concatenation of transformation matrices to obtain a combined transformation matrix.

A combined matrix –
[T] = [Tn] …. [T3] [T2] [T1] [X]
Where [Ti] is any combination of

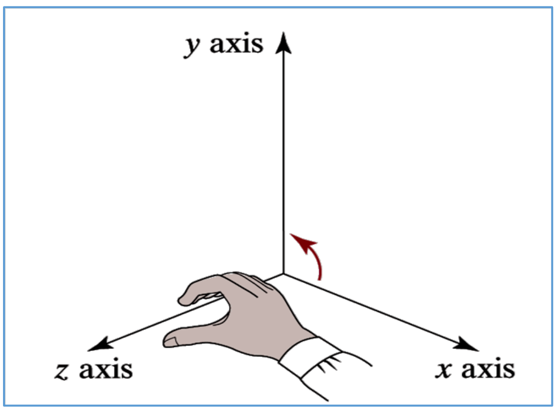
- Translation
- Scaling
- Shearing
- Rotation
- Reflection

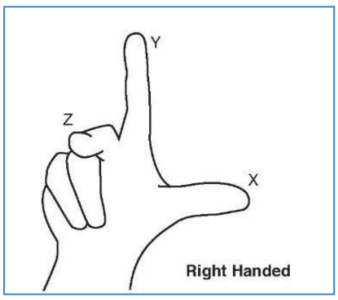
The change in the order of transformation would lead to different results, as in general matrix multiplication is not cumulative, that is [A] . [B] ≠ [B] . [A] and the order of multiplication. The basic purpose of composing transformations is to gain efficiency by applying a single composed transformation to a point, rather than applying a series of transformation, one after another.

3D transformation

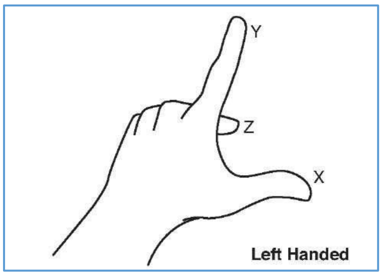
When the transformation takes place on a 3D plane, it is called 3D transformation.3D transformation in homogeneous coordinates manipulates the position, orientation, and scale of 3D objects in three-dimensional space. 3D transformations are used to create realistic and dynamic visual scenes

In a Right-Handed (RH) coordinate system as shown in fig., open the first 3 fingers of your right hand, orient them such that the thumb points at +ve X-direction (to the right side), Index finger points at the +ve Y-direction (upside), middle finger points at the +ve Z-direction (to yourself). In such an alignment, it is always that we see the world along the –ve Z-direction, as the +ve Z-direction is pointing towards self as shown in the figure below.





If we use our left-hand for representing a 3D coordinate system, as similar to that of the RH coordinate system, we call that a Left-Handed (LH) coordinate system, as shown below, where the Z- direction is away from the self and points in the opposite direction, with X and Y directions still being the same as that of the RH coordinate system.



With RH being the convention, that we shall follow, it should be always remembered that we see the world along the –ve Z-direction.

Basic Transformations in 3D:

Homogeneous coordinates representation of 3D transformation:

Homogeneous coordinates extend the traditional Cartesian coordinates (X, Y, Z) with an additional coordinate (X, Y, Z, W), enabling the perspective projections using matrix multiplication.

3D transformations using homogeneous coordinates are typically represented as 4×4 matrices, known as transformation matrices.

Translation in 3D:

The translation moves an object in 3D space along the x, y, and z axes. Suppose (x,y,z,1) is a point in 3D space and (x′,y′,z′,1) is a transformed point. Translation in the x-direction is represented using Tx. The translation is y-direction is represented using Ty. The translation in the z- direction is represented using Tz .

As similar to that of 2D Translation, 3D translation is represented using the following equations, Here, (tx,ty,tz,1) represents the translation vector. Three-dimensional transformations are performed by transforming each vertex of the object. If an object has five corners, then the translation will be accomplished by translating all five points to new locations

x’ = x + tx

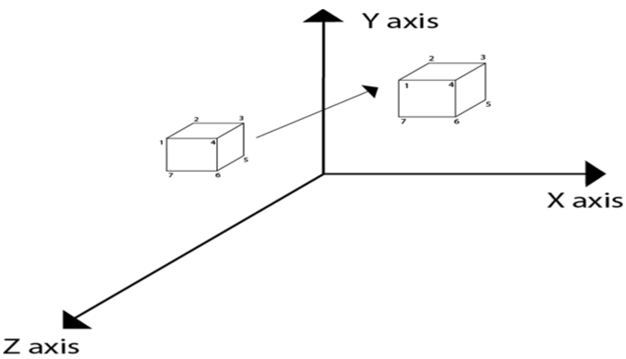
$y' = y + ty$

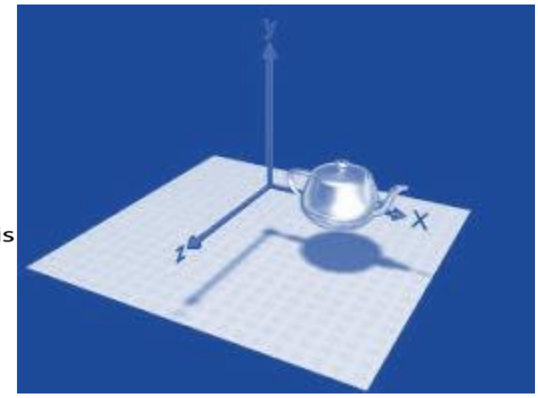
$z' = z + tz$

$1=1$

The corresponding matrix representation in homogeneous form is given by

$$\begin{pmatrix} x^1 \\ y^1 \\ z^1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$





Rotation in 3D

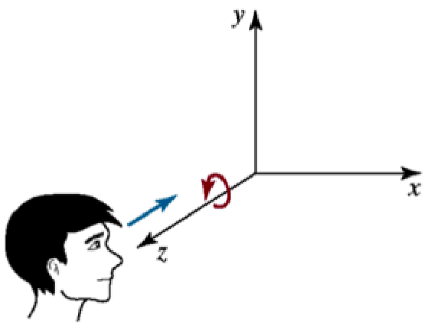
Rotation in 3D is complex, because, rotation is performed with respect to an axis of rotation (unlike 2D rotation which is about a pivot point) and the possible orientations of the objects and the axes about which the objects are rotated. It is also important to know the conventions for rotation for whether it is +ve or –ve. Rotations about an axis in counter clock- wise i.e. anti-clock wise direction are considered +ve. Rotations about an axis in clock wise direction are considered -ve.

3D rotation can be classified as 'coordinate-axes rotations', which are rotations performed about the standard coordinate axes also called canonical rotation. Observe that, rotations about any axis, does not alter the corresponding coordinate, but alters the other coordinates, i.e., if we perform rotation about X-axis, the x-coordinate remains same after rotation, but y, z coordinates get altered.

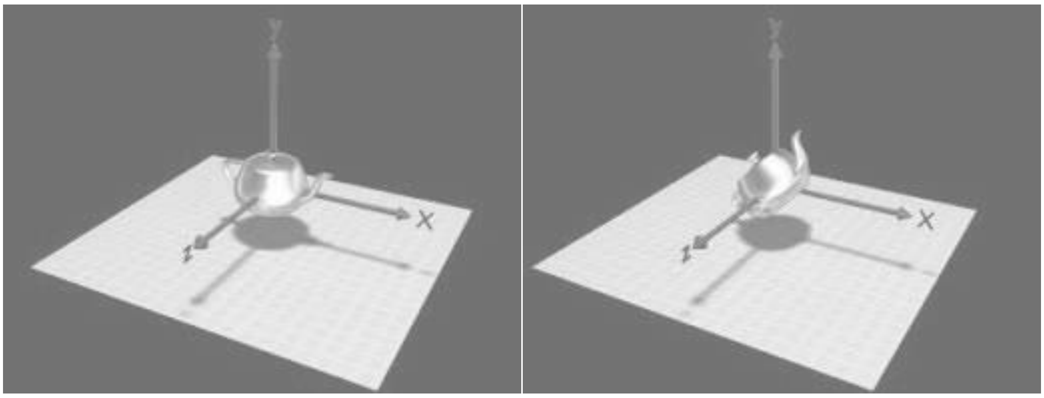
- Coordinate-Axes Rotations
- – X-axis rotation (pitch)
- – Y-axis rotation (yaw)
- – Z-axis rotation (roll)

Rotation about Z – axis:

If we perform rotation about Z-axis, the Z-coordinate remains same after rotation, but X, Y coordinates get altered.



(a)



The equations for rotation about Z-axis are given below.

$$z' = z$$

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

$P' = R_z(\theta).P$ can be represented

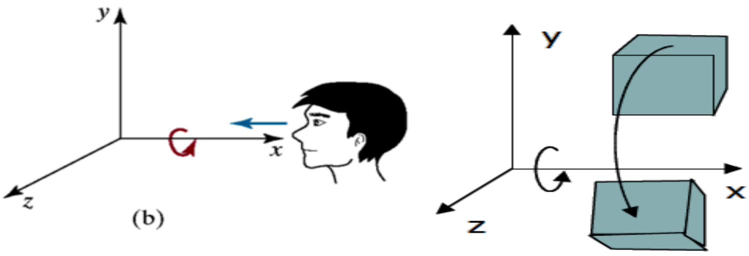
$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Z-Axis Rotation)

Rotation about X – axis :

If we perform rotation about X-axis, the x-coordinate remains same after rotation, but y, z coordinates get altered.



Follow this thumb rule to write eqs. for rotations with respect to X and Y axes.

$X \rightarrow Y \rightarrow Z \rightarrow X$

Equations for rotation about *X*-axis can thus be obtained using the above thumb rule as

$$y' = y\cos\theta - z\sin\theta$$

$$z' = y\sin\theta + z\cos\theta$$

$$x' = x$$

$P' = R_x(\theta).P$ *can be represented*

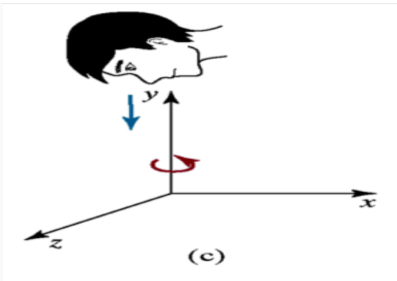
$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

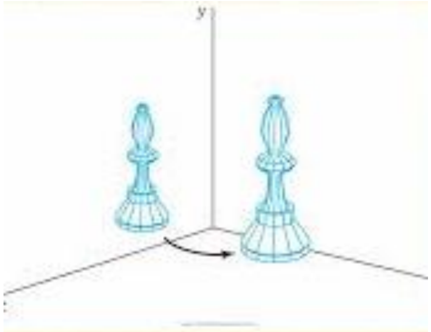
3D Rotation Matrix

(For X-Axis Rotation)

Rotation about Y– axis :

If we perform rotation about Y-axis, the Y-coordinate remains same after rotation, but X, Z coordinates get altered.





Rotation of an object about the y axis

The equations for rotation about Z-axis are given below.

$y' = y$
$z' = z \cos\theta - x \sin\theta$
$x' = z \sin\theta + x \cos\theta$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Y-Axis Rotation)