

# **MIDTERM REVIEW**

**RECITATION WEEK 5**



# PROPERTIES OF $O(f(n))$ , $\Omega(f(n))$ , $\Theta(f(n))$

- You will need to use the definitions of the 5 asymptotic functions to prove features about them.
- Prove or disprove the following statement: For any function,  $f(n)$ ,  
 $f(n) = O\left((f(n))^2\right)$
- How would you prove that it's true? How would you prove that it's false?
- The statement is false. As a counterexample, take  $f(n) = \frac{1}{n} = n^{-1}$ .  
 $(f(n))^2 = n^{-2}$  and since  $\lim_{n \rightarrow \infty} \frac{n^{-1}}{n^{-2}} = \infty$ ,  $f(n) = \omega\left((f(n))^2\right)$ , meaning that  
 $f(n) \neq O\left((f(n))^2\right)$ .



# RANDOMIZED ANALYSIS ARGUMENTS

- You will need to find an appropriate set of indicator variables and combine them to solve a problem
- When choosing indicator variables, **think ahead** toward the goal. What **events** do I need to keep track of? How can I **combine** the values of my indicator variables?
- You have a set of  $n$  cards with labels 1 through  $n$  (with no repeats). You shuffle the deck of cards and begin flipping them over one by one, making a prediction for the next card to appear each time.
- Part (a): First you try a “memoryless” approach, that is, you uniformly at random guess a number from 1 to  $n$  without recalling what cards you have seen before. How many predictions, in expectation, do you get correct?



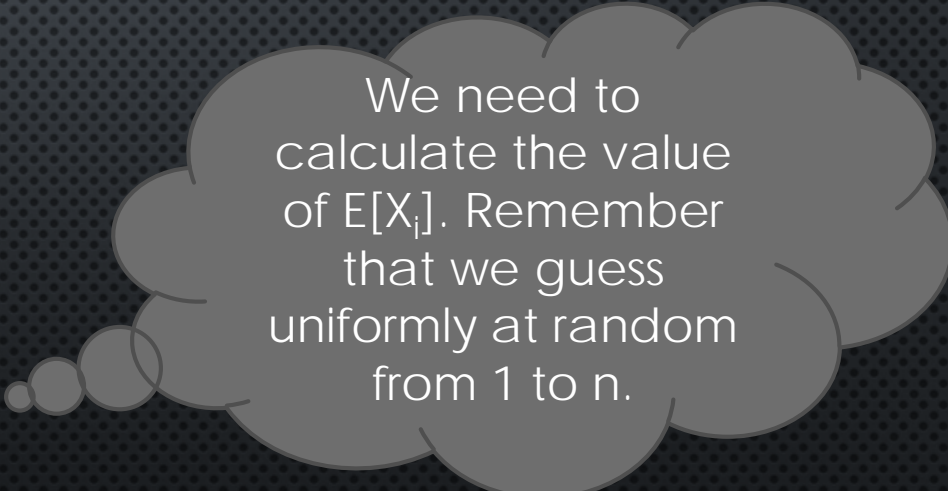
# RANDOMIZED ANALYSIS ARGUMENTS

- Let  $X_i$  be the indicator variable  $I\{\text{The } i\text{th card is correctly predicted}\}$ .
- We are looking for the total number of correct predictions, which would be  $X = \sum_{i=1}^n X_i$

$$X = \sum_{i=1}^n X_i$$

$$\begin{aligned} E[X] &= E\left[\sum_{i=1}^n X_i\right] \\ &= \sum_{i=1}^n E[X_i] \end{aligned}$$

$$= \sum_{i=1}^n \frac{1}{n} = 1$$



We need to calculate the value of  $E[X_i]$ . Remember that we guess uniformly at random from 1 to  $n$ .



- You have a set of  $n$  cards with labels 1 through  $n$  (with no repeats). You shuffle the deck of cards and begin flipping them over one by one, making a prediction for the next card to appear each time.
- Part (b): You then try an approach where you guess uniformly at random from among the set of cards you have not seen. How many predictions, in expectation, do you get correct?



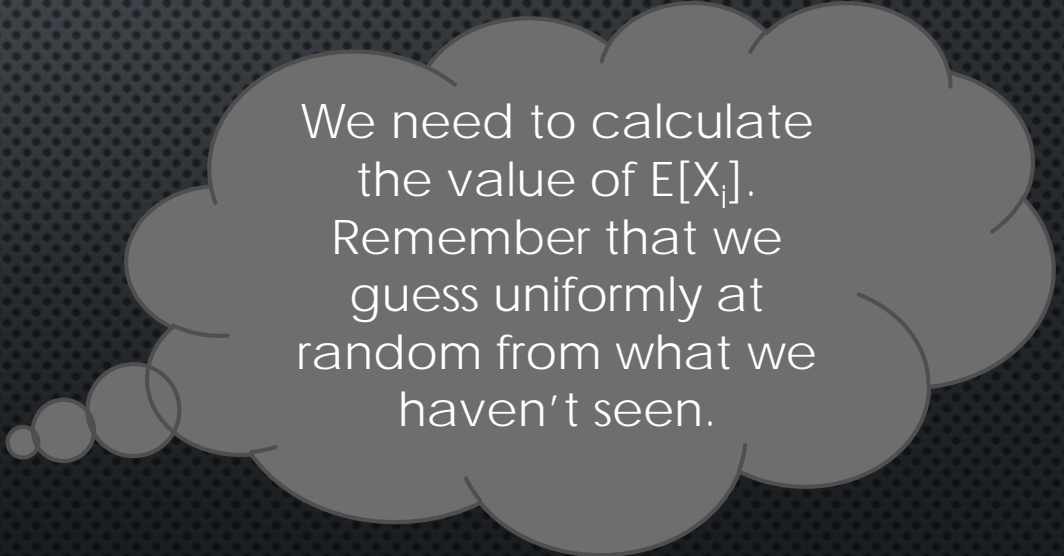
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$$= \sum_{i=1}^n \frac{1}{n-i+1} = \sum_{i=1}^n \frac{1}{i} = \Theta(\log(n))$$



We need to calculate the value of  $E[X_i]$ . Remember that we guess uniformly at random from what we haven't seen.



ADDITIONAL QUESTIONS?