MIDTERM REVIEW

Recitation Week 5

PROPERTIES OF $O(f(n)), \Omega(f(n)), \Theta(f(n))$

- You will need to use the definitions of the 5 asymptotic functions to prove features about them.
- Prove or disprove the following statement: For any function, $f(n)$, $f(n) = O\left(\left(f(n)\right)^2\right)$
- How would you prove that it's true? How would you prove that it's false?

- The statement is false. As a counterexample, take $f(n) = \frac{1}{n} = \mathrm{n}^{-1}$. $\left(f(n)\right)^2 = n^{-2}$ and since $\lim_{n\to\infty} \frac{n^{-1}}{n^{-2}} = \infty$, $f(n) = \omega\left(\left(f(n)\right)^2\right)$, meaning that $f(n) \neq O\left(\left(f(n)\right)^2\right)$.

RANDOMIZED ANALYSIS ARGUMENTS

- You will need to find an appropriate set of indicator variables and combine them to solve a problem
- When choosing indicator variables, think ahead toward the goal. What events do I need to keep track of? How can I combine the values of my indicator variables?

- You have a set of n cards with labels 1 through n (with no repeats). You shuffle the deck of cards and begin flipping them over one by one, making a prediction for the next card to appear each time.
- Part (a): First you try a "memoryless" approach, that is, you uniformly at random guess a number from 1 to n without recalling what cards you have seen before. How many predictions, in expectation, do you get correct?

RANDOMIZED ANALYSIS ARGUMENTS

- Let X_i be the indicator variable $\mathrm{I}\{$The ith card is correctly predicted$\}$.
- We are looking for the total number of correct predictions, which would be $\mathrm{X} = \sum_{i=1}^{n} X_i$

$$X = \sum_{i=1}^{n} X_i$$

$$E[X] = E\left[\sum_{i=1}^{n} X_i\right]$$

$$= \sum_{i=1}^{n} E[X_i]$$

$$= \sum_{i=1}^{n} \frac{1}{n} = 1$$

We need to calculate the value of $E[X_i]$. Remember that we guess uniformly at random from 1 to n.

- You have a set of n cards with labels 1 through n (with no repeats). You shuffle the deck of cards and begin flipping them over one by one, making a prediction for the next card to appear each time.
- Part (b): You then try an approach where you guess uniformly at random from among the set of cards you have not seen. How many predictions, in expectation, do you get correct?

RANDOMIZED ANALYSIS ARGUMENTS

- Let X_i be the indicator variable $\mathrm{I}\{$The ith card is correctly predicted$\}$.
- We are looking for the total number of correct predictions, which would be $\mathrm{X} = \sum_{i=1}^{n} X_i$

$$X = \sum_{i=1}^{n} X_i$$

$$E[X] = E\left[\sum_{i=1}^{n} X_i\right]$$

$$= \sum_{i=1}^{n} E[X_i]$$

$$= \sum_{i=1}^{n} \frac{1}{n-i+1} = \sum_{i=1}^{n} \frac{1}{i} = \Theta(\log(\mathrm{n}))$$

We need to calculate the value of $E[X_i]$. Remember that we guess uniformly at random from what we haven't seen.

Additional Questions?