1.

a) Consider the linear system whose augmented matrix is given by $\left[\begin{array}{rrr|r} 1 & 1 & 2 & 1 \\ 1 & -2 & 3 & b \\ 2 & 0 & b & 0 \end{array}\right]$,where b is a real number. For what number b will the system have a unique solution?

b) What can you say about the number of solutions to the system for other values of b?

2. Manually find the matrix Q that will diagonalized the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$ and compute the diagonalized form. Validate your solution with MATLAB using **[V, D] = eig(A)**

3.

Consider the companion-form matrix

$$\mathbf{A} = \begin{bmatrix} -\alpha_1 & -\alpha_2 & -\alpha_3 & -\alpha_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Show that its characteristic polynomial is given by

$$\Delta(\lambda) = \lambda^4 + \alpha_1\lambda^3 + \alpha_2\lambda^2 + \alpha_3\lambda + \alpha_4$$

Show also that if λ_i is an eigenvalue of $\mathbf{A}$ or a solution of $\Delta(\lambda) = 0$, then $[\lambda_i^3 \ \ \lambda_i^2 \ \ \lambda_i \ \ 1]'$ is an eigenvector of $\mathbf{A}$ associated with λ_i.

4.

Show that the companion-form matrix in Problem 3 is nonsingular if and only if $\alpha_4 \neq 0$. Under this assumption, show that its inverse equals

$$\mathbf{A}^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1/\alpha_4 & -\alpha_1/\alpha_4 & -\alpha_2/\alpha_4 & -\alpha_3/\alpha_4 \end{bmatrix}$$

5.

Manually compute the eigenvectors and generalized eigenvectors for the following matrix

a) Using the bottom up approach
b) Using the top down approach
c) Compute the Jordan form from the similarity transform $J = Q^{-1}AQ$

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$$

Validate your work with Matlab using [u,v]=jordan(A)