

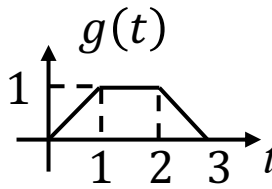
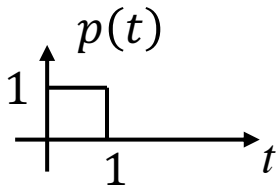
ECES511/T480 Final Exam Fall 2020

Instructions:

1. Independent work with open notes.
2. Always use MATLAB to confirm your answers.
3. Full credit will ONLY be given when you do BOTH (manual calculation & MATLAB validation) and explain and demonstrate their equivalence.
4. Please attach the published MATLAB pdf file after each solution.
5. All problems in Part I should be attached in ONE pdf file for submission

PART I:

1. Determine as the convolution of the two signals $p(t)$ and $g(t)$ below. You can either do a plot or write down as an equation for your answer. Show all your work and clearly indicate your answer.



2. Let $v_1 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$, $v_3 = \begin{bmatrix} -3 \\ 9 \\ 3 \end{bmatrix}$

- 1) Determine if $\{v_1, v_2, v_3\}$ is linearly independent?
- 2) If it is linear independent, find a, b, c where $av_1 + bv_2 + cv_3 = 0$. If not, find the relations among v_1, v_2, v_3 .

3. Given 5 equations:

$$2a + b = 20$$

$$6a + b = 18$$

$$20a + b = 10$$

$$30a + b = 6$$

$$40a + b = 2$$

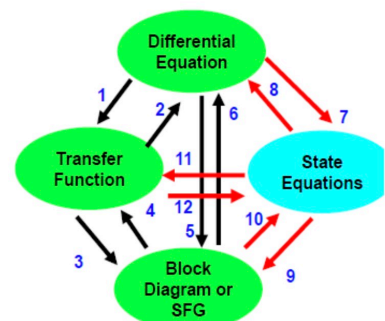
- a) Find the closest solution $\{a, b\}$ to the five equations. Use MATLAB to confirm how "close" it is.
- b) Is there any geometric meaning for $\{a, b\}$?

4. Using the transfer function $\frac{Y(s)}{R(s)} = \frac{6}{(s+2)(s+3)}$.

Note this transfer function has no zero/pole cancellation.

Using the graph below:

- 1) Follow path 2, then 7
- 2) Verify path 11, Use MATLAB to confirm.
- 3) Calculate the impulse response of the system.



5. Use the Cayley Hamilton Theorem to find A^{-1} where $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$. Validate your answer with $A A^{-1}$.

6. Given the following system with initial conditions:

$$\begin{aligned}\frac{dx}{dt} &= \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 4 \\ -1 \end{bmatrix} r(t); \\ y(t) &= [1 \ 3] x(t); \\ x(0) &= \begin{bmatrix} 4 \\ 5 \end{bmatrix}\end{aligned}$$

- a) Find the state transition matrix $\varphi(t) = e^{At}$. [Use MATLAB to confirm](#)
- b) Find the transfer function $Y(s)/R(s)$ (factor all polynomials).
- c) Find the total solution for the state vector $x(t)$ if the input $r(t) = 2u(t)$ where $u(t) = \text{unit step function}$. [Use MATLAB to confirm](#)

Clearly identify the **zero input part** of the solution

Clearly identify the **zero state part** of the solution

Then combine into a final result

Show the results in both the time domain and Laplace domain.

(**Hint:** part c is easier to solve if you work in the Laplace domain then convert to the time domain)

7. Use the same system in Problem 6

- a) Use MATLAB to plot the state variables in continuous time over the time range $[0,10]$.
- b) Calculate the A_d B_d C_d D_d for discrete system if sampled at $T = 1/2\text{s}$.
- c) Use MATLAB to plot the discrete response of the same state variable as (a) and compare the two plots.

8. Compute the singular value decomposition of $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. [Use MATLAB to confirm.](#)