Problem 1.

For the matrix $\mathbf{A}$ given as

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Compute the state transition matrix $\varphi(t) = e^{\mathbf{A}t}$ by

1. using the Laplace transform;
2. using the Cayley Hamilton theorem;
3. determining the Jordan form $\mathbf{J}$ of $\mathbf{A}$ and computing $e^{\mathbf{A}t} = \mathbf{Q}e^{\mathbf{J}t}\mathbf{Q}^{-1}$

Problem 2.

For the matrix $\mathbf{J}$ given as

$$\mathbf{J} = \begin{matrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{matrix}$$

1. Using the relationships in the notes, find $e^{\mathbf{J}t}$.
2. Identify each Jordan block and its order.

Problem 3.

Suppose we are given a set of points $\{(1,1), (2,2), (3,1.7), (3.5,2.5), (4,3.6), (5,3.6)\}$. For linear model and parabola model, which one fit these points better? Why?