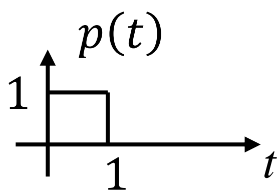
ECES511/T480 Final Exam Fall 2020

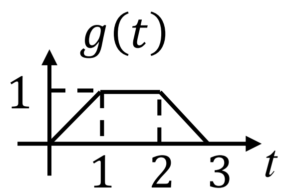
Instructions:

1. Independent work with open notes.
2. Always use MATLAB to confirm your answers.
3. Full credit will ONLY be given when you do BOTH (manual calculation & MATLAB validation) and explain and demonstrate their equivalence.
4. Please attach the published MATLAB pdf file after each solution.
5. All problems in Part I should be attached in ONE pdf file for submission

PART I:

1. Determine as the convolution of the two signals $p(t)$ and $g(t)$ below. You can either do a plot or write down as an equation for your answer. Show all your work and clearly indicate your answer.





2. Let $v_1 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}, v_3 = \begin{bmatrix} -3 \\ 9 \\ 3 \end{bmatrix}$

1) Determine if $\{v_1, v_2, v_3\}$ is linearly independent?
2) If it is linear independent, find a, b, c where $av_1 + bv_2 + cv_3 = 0$. If not, find the relations among v_1, v_2, v_3.

3. Given 5 equations:

$$\begin{aligned} 2a + b &= 20 \\ 6a + b &= 18 \\ 20a + b &= 10 \\ 30a + b &= 6 \\ 40a + b &= 2 \end{aligned}$$

a) Find the closest solution $\{a, b\}$ to the five equations. Use MATLAB to confirm how "close" it is.

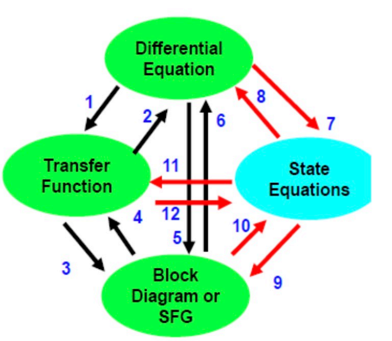
b) Is there any geometric meaning for $\{a, b\}$?

4. Using the transfer function $\frac{Y(s)}{R(s)} = \frac{6}{(s+2)(s+3)}$.

Note this transfer function has no zero/pole cancellation and is a minimal polynomial.

Using the graph below:

1) Follow path 2, then 7
2) Verify path 11, Use MATLAB to confirm.
3) Calculate the impulse response of the system.



5. Use the Cayley Hamilton Theorem to find A^{-1} where $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$. Validate your answer with $A\,A^{-1}$.

6. Given the following system with initial conditions:

$$\frac{dx}{dt} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 4 \\ -1 \end{bmatrix} r(t);$$
$$y(t) = [1\ 3]x(t);$$
$$x(0) = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

a) Find the state transition matrix $\varphi(t) = e^{At}$. Use MATLAB to confirm

b) Find the transfer function $Y(s)/R(s)$ (factor all polynomials).

c) Find the total solution for the state vector $x(t)$ if the input $r(t) = 2u(t)$ where $u(t) = unit\ step\ function$. Use MATLAB to confirm

Clearly identify the **zero input part** of the solution

Clearly identify the **zero state part** of the solution

Then combine into a final result

Show the results in both the time domain and Laplace domain.

(**Hint:** part c is easier to solve if you work in the Laplace domain then convert to the time domain)

7. Use the same system in Problem 6

a) Use MATLAB to plot the state variables in continuous time over the time range [0,10].

b) Calculate the $A_d\ B_d\ C_d\ D_d$ for discrete system if sampled at T = 1/2s.

c) Use MATLAB to plot the discrete response of the same state variable as (a) and compare the two plots.

8. Compute the singular value decomposition of $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. Use MATLAB to confirm.