ECE T480/ECES 511 – Fundamental of systems I **Drexel University**
Special Assignment 1 **10/02/2020**

This special assignment is designed for linearization and simulation of single link planar arm with DC motor. The model equations which were given in the class:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{J}(Nk_t\, x_3 - bx_2 - mglsin(x_1))$$
$$\dot{x}_3 = \frac{1}{L}(u(t) - Rx_3 - k_bNx_2)$$

where

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ i \end{bmatrix}$$

Input $u(t)$ to the system is motor supply voltage.

The system parameters,

$$J = 0.02, k_t = k_b = 0.01, N = 10, R = 2, L = 0.5, b = 0.2, m = 1, g = 10, l = 0.3$$

a) Derive equilibrium states given input voltage $u = 15, u = 21.2, u = 30, u = 40$ and $u = 75$. What do you observe? Does the equilibrium exist for each input? Justify your answers.
b) Derive linear models around equilibrium points for u = 15 volts and u = 45 volts. This will give you two separate linearized systems which you will need to convert into state equations. Hint: Look up the ss command in MATLAB.
c) Simulate the linearized system using MATLABS *lsim* command. Plot (you will need your state space models from part (b) for this) the state trajectories over a variety of different initial conditions for each of your linearized models.
   i. $u = 10, x_0 = \left[\frac{\pi}{2}, 0, 0\right]$
   ii. $u = 21.2, x_0 = [0, 0, 0]$
   iii. $u = 21.2, x_0 = \left[\frac{\pi}{4}, 0, 0\right]$
   iv. $u = 21.2, x_0 = \left[\frac{\pi}{3}, 0, 0\right]$
   v. $u = 21.2, x_0 = \left[\frac{\pi}{2}, 0, 0\right]$
   vi. $u = 30, x_0 = \left[-\frac{\pi}{2}, 0, 0\right]$
   vii. $u = 30, x_0 = [0, 0, 0]$
d) Repeat the exact same input and initial conditions used in part (c) but this time using the non-linear model and MATLAB's *ode45* command. Compare your results from parts C and D by plotting the trajectories on the same figure. Play around with this and try different inputs with your three systems (non-linear and two linear models). How are each of the models behaving? For example, what happens if you use the 45 volt linearized model and give an input of 5 volts Vs 50 volts? What about 80 volts? Compare this to the non-linear model on the same plot.
e) Compare (by plotting the trajectories on the same figure so that it is easy to compare) and discuss the results explaining the benefits and risks of the local linearization method.
f) Repeat the exact same input and initial conditions simulation for 4-5 seconds duration using the

Rectangular, Trapazoidal, and Runge Kutta numerical approximation techniques. Compare and plot all three techniques on the same graph Vs the non-linear model using *ode45.*

g) Compare (by plotting the trajectories on the same figure so that it is easy to compare) and discuss the results explaining the difference between numerical methods such as accuracy and efficiency.