For the following problems, **manually** calculate the expression for the response then **validate** your answer with MATLAB Simulation.
Ex. If you get the response of the system $y(t) = e^{-t}$, please build your system in MATLAB and compare the output obtained from the system with $y(t) = e^{-t}$ in one plot for verification

Problem 1

Find the response of the output variable

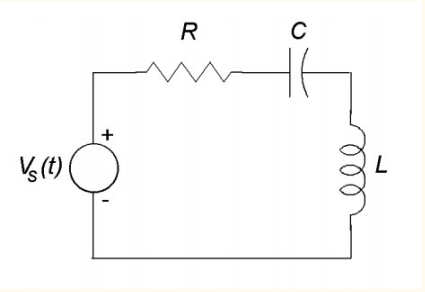
$$y = 2x_1 + x_2$$

in the system described by state equations

$$\begin{aligned} \dot{x}_1 &= -2x_1 + u \\ \dot{x}_2 &= x_1 - x_2. \end{aligned}$$

to a constant input $u(t) = 5$ for $t > 0$, if $x_1(0) = 0$, and $x_2 = 0$.

Problem 2



Given the standard RLC circuit (R = 1.2, C = 1, L = 0.2). To build the state space equation, we choose capacitor voltage $v_c(t) = x_1(t)$and inductor current $i_l(t) = x_2(t)$ as state variables and $v_c(t)$ as output.

1) Find the state space equation for the RLC system
2) If $v_c(0) = 0, i_l(0) = 5, v_s(t) = 0$, find the $\boldsymbol{x}(t)$ for the system
3) If $v_c(0) = 0, i_l(0) = 0, v_s(t) = 5$, find the $y(t)$ for the system

Problem 3

Considering the system

$$\ddot{x} + 4\dot{x} + 4x = u$$

1) Find the state space model for the system
2) Find the transfer function for the system. (Hint: you can use $\boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + D$)
3) Suppose $u(t) = e^{-2t}sin(t)$ for $t \geq 0$, and $\ddot{x}(0) = \dot{x}(0) = 0$. Find the solution $x(t)$