ECE T480/ECES 511- Homework 2

Problem 1. Find the basis of the range spaces and null spaces of the matrices below.

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{A}_2 = \begin{bmatrix} 4 & 1 & -1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} \qquad \mathbf{A}_3 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 2.

Let $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -2 \\ 3 \end{bmatrix}, \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 5 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \boldsymbol{v_3} = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 5 \\ -5 \end{bmatrix}, \boldsymbol{v}_4 = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -4 \\ 1 \end{bmatrix}, \boldsymbol{v}_5 = \begin{bmatrix} 5 \\ 6 \\ 1 \\ -1 \\ -1 \end{bmatrix}$

Let $\boldsymbol{W} := \{\boldsymbol{w} \in \boldsymbol{R}^5 \mid \boldsymbol{w} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + a_3\boldsymbol{v}_3 + a_4\boldsymbol{v}_4 + a_5\boldsymbol{v}_5\}$

1. Find bases $\boldsymbol{Q}$ of $\boldsymbol{W}$
2. What is the dimension of $\boldsymbol{W}$
3. What is the representation of the vector $x = \begin{bmatrix} -5 \\ 6 \\ -5 \\ 11 \\ -1 \end{bmatrix}$ with respect to the basis $\boldsymbol{Q}$
4. Orthogonalize $\boldsymbol{Q}$ using Gram-Schmidt orthogonalization
5. Let $\boldsymbol{B} = \begin{bmatrix} 1 & 2 & 1 & 3 & 5 \\ 2 & 5 & 3 & 1 & 6 \\ 1 & -1 & -2 & 2 & 1 \\ -2 & 3 & 5 & -4 & -1 \\ 3 & -2 & -5 & 1 & -1 \end{bmatrix}$, find the range($\boldsymbol{B}$), the columns space of $\boldsymbol{B}$, the rank of $\boldsymbol{B}$ and the null space of $\boldsymbol{B}$