Project Control

ECES 512

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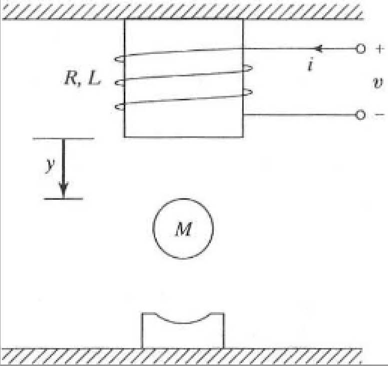
Electromechanical Magnetic-Ball Suspension

- Make an object levitate by controlling the current



Mathematical Model

Variables

- R - Resistance
- L - Inductance
- v - Voltage
- m - Mass
- K - Coefficient that relates force to the magnetic field
- g - Gravity
- i - Current
- y - Distance of Mass M to electromagnet

$$v(t) = Ri(t) + L\frac{di(t)}{dt}$$

$$m\frac{d^2y(t)}{dt^2} = mg - K\frac{i^2(t)}{y(t)}$$

I/O and State Variables

- We control the voltage v
- Goal is to control distance y

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} i \\ y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \text{Current} \\ \text{Distance} \\ \text{Velocity} \end{bmatrix}$$

Linearization about the Equilibrium

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \frac{u(t)-Rx_1(t)}{L} \\ x_3(t) \\ -\frac{Kx_1^2(t)}{mx_2(t)} + g \end{bmatrix}$$

$$A = \frac{\partial h}{\partial x} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{2Kx_1(t)}{mx_2(t)} & \frac{K}{m}(\frac{x_1(t)}{x_2(t)})^2 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{2Kx_{01}}{mx_{02}} & \frac{K}{m}(\frac{x_{01}}{x_{02}})^2 & 0 \end{bmatrix}$$

$$B = \frac{\partial h}{\partial u} = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \end{bmatrix}$$

$$v_0 = 7 \qquad x_0 = \begin{bmatrix} 7 \\ .00998 \\ 0 \end{bmatrix} \qquad A = \begin{bmatrix} -100 & 0 & 0 \\ 0 & 0 & 1 \\ -2.803 & 982 & 0 \end{bmatrix}$$

Stability

Internal stability: Look at the eigenvalues of our A matrix

$$\begin{bmatrix} -100 & 0 & 0 \\ 0 & 0 & 1 \\ -2.803 & 982 & 0 \end{bmatrix}$$

$$\lambda_1 = -100 \quad \lambda_2 \approx 31.34 \quad \lambda_3 \approx -31.34$$

Our system is not internally stable

Stability

BIBO stability: Look at the transfer function

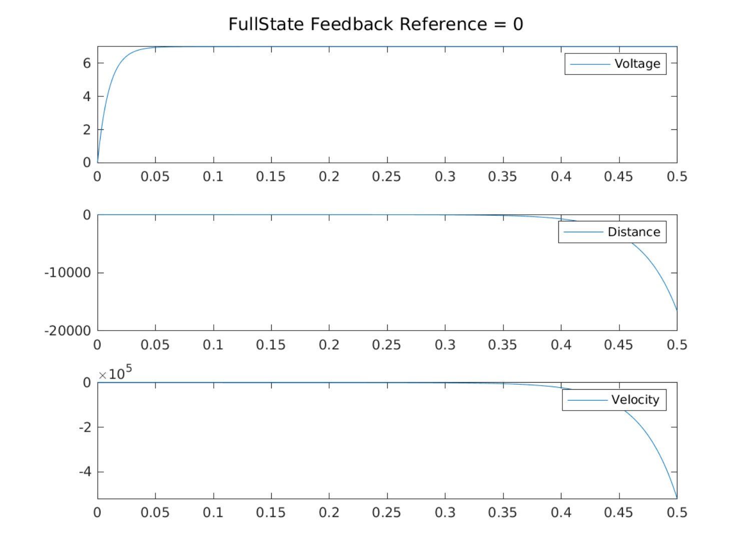
$$C(sI - A)^{-1}B$$

$$A = \begin{bmatrix} -100 & 0 & 0 \\ 0 & 0 & 1 \\ -2.803 & 982 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$G(s) = \frac{-280}{(s - 31.34)(s + 31.34)(s + 100)}$$

Since we have a pole in the right half plane our system is also not BIBO stable.

Open Loop


FullState Feedback Reference = 0
Voltage
Distance
Velocity
6
4
2
0
-10000
-20000
×10^5
-2
-4
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5

Controllability

Controllability Matrix

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$A = \begin{bmatrix} -100 & 0 & 0 \\ 0 & 0 & 1 \\ -2.803 & 982 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} 100 & -10000 & 10^6 \\ 0 & 0 & -280.3 \\ 0 & -280.3 & 28030 \end{bmatrix}$$

$$\text{rank}(\mathcal{C}) = 3$$

The system is controllable.

Observability

Observability Matrix

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

$$A = \begin{bmatrix} -100 & 0 & 0 \\ 0 & 0 & 1 \\ -2.803 & 982 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\mathcal{O} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2.803 & 982 & 0 \end{bmatrix}$$

$$\text{rank}(\mathcal{O}) = 3$$

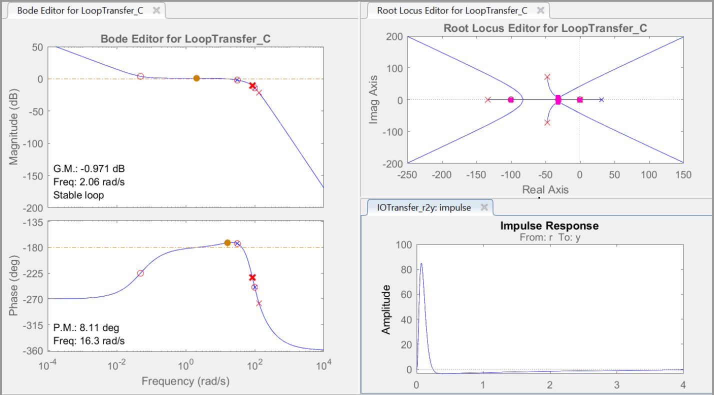
The system is Observable.

Desired behavior

- Can't move too far from linearization range
- Low to no overshoot (5% ¡)
- Fast response (¡.05s)

Used bode plot and root locus tools in MATLAB.

Choosing Poles



$$p_1 = -20 + 20i \qquad p_2 = -20 - 20i \qquad p_3 = -100$$

$$(s - (-20 + 20i))(s - (-20 + 20i))(s - (-100))$$

$$s^3 + 140s^2 + 4800s + 80000$$

Selecting Gain Matrix

Using Ackermann's formula we can find the K values to select our desired poles.

$$k^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathcal{C}^{-1} \Delta(A)$$

$$\Delta(x) = x^3 + 140x^2 + 4800x + 80000$$

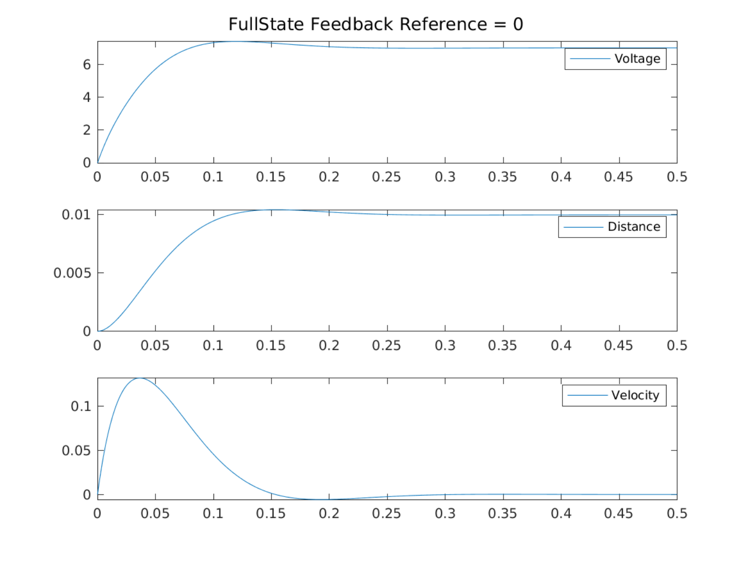
$$k^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (C)^{-1} (A^3 + 140A^2 + 4800A + 80000)$$

$$k = \begin{bmatrix} -.1 & -458.01 & -13.49 \end{bmatrix}$$

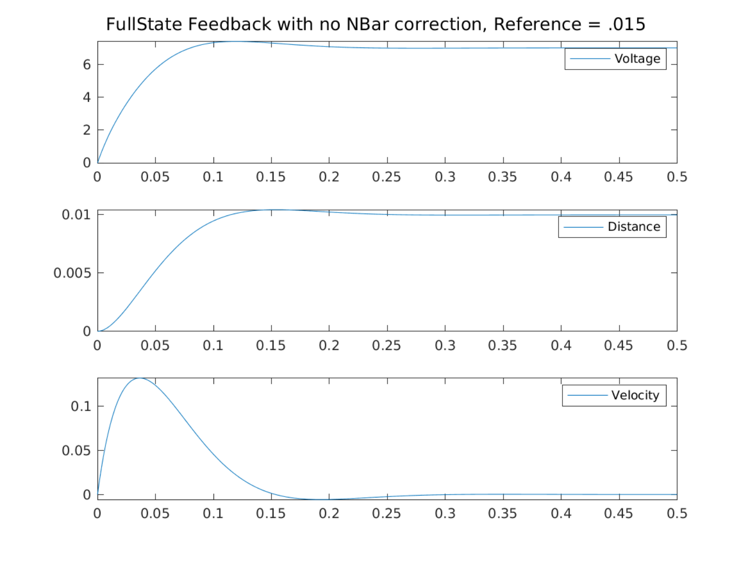
Modified System

$$\dot{x} = \begin{bmatrix} -100 & 0 & 0 \\ 0 & 0 & 1 \\ -2.803 & 982 & 0 \end{bmatrix} x + \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -.1 & -458.01 & -13.49 \end{bmatrix} u$$

Full State Feedback No reference



Full State Feedback without $\bar{N}$



Choosing $\bar{N}$

$$\bar{N}\frac{Y(S)}{R(S)}_{s->0} = 1$$

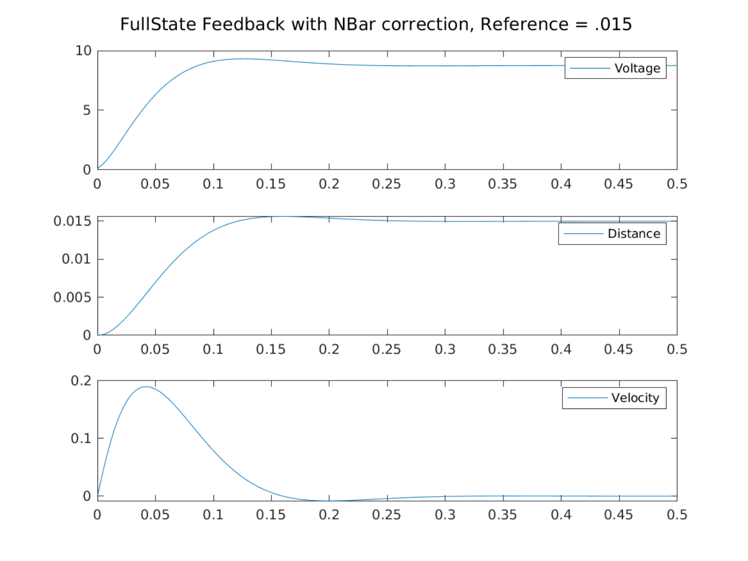
$$\bar{N} = \frac{R(S)}{Y(S)}_{s->0}$$

$$\frac{Y(s)}{R(s)} = C(sI - (A - BK))^{-1}B = -\frac{280.3}{s^3 + 140s^2 + 4800s + 80000}$$

Evaluating at $s- > 0$ we get $\frac{-280.3}{80000}$ Solving for $\bar{N}$ we get

$$\bar{N} = \frac{-80000}{280} \approx -285.72$$

Full State Feedback with $\bar{N}$



Observer Gain Matrix

Choose our poles to be 5 times further than the poles of the control matrix.

$$p_1 = -100 + 100i \qquad p_2 = -100 - 100i \qquad p_3 = -500$$

$$(s - (-100 + 100i))(s - (-100 + 100i))(s - (-500))$$

$$s^3 + 700 * s^2 + 120000 * s + 10000000$$

Using Ackermann's formula we can find the L values to select our desired poles.

$$l^T = \Delta(A)\mathcal{O}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Delta(x) = x^3 + 90x^2 + 2800x + 40000$$

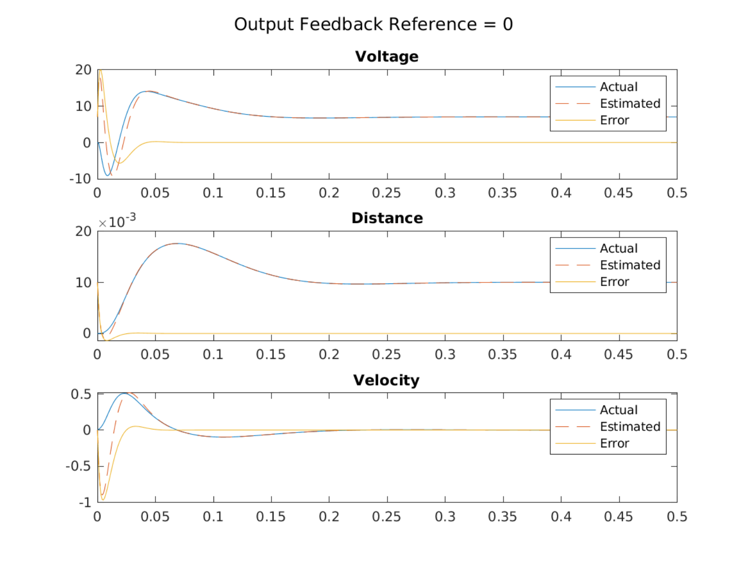
$$l = \begin{bmatrix} -1.427 \times 10^6 \\ 600 \\ 60892 \end{bmatrix}$$

Output Feedback System

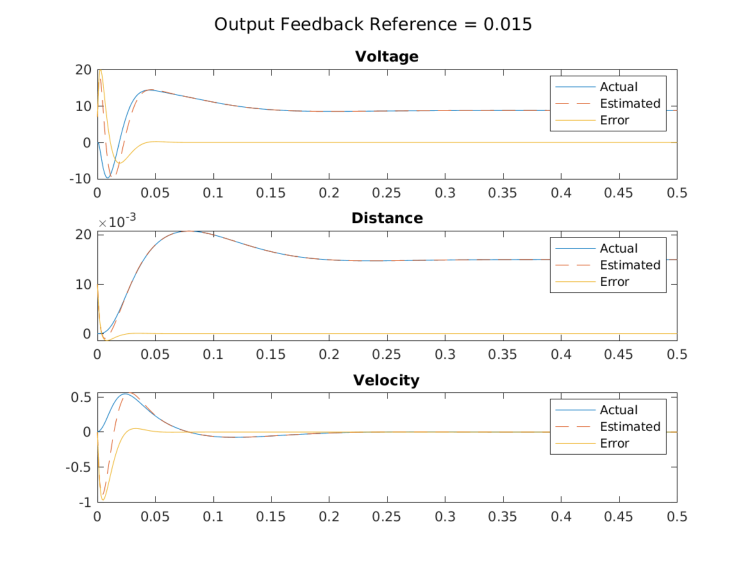
$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -Bk \\ lC & A - Bk - lC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} u$$

$$\begin{bmatrix} y \\ \hat{y} \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + 0$$

Output Feedback no Reference



Output Feedback with Reference



References

1. http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlStateSpace
2. https://elec3004.uqcloud.net/laboratories/LeviLab/Levitating%20Magnet%20Modelling%20Example.pdf