Final
ECES 512

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Problem 1

Given the continuous LTI system:

$$\frac{dx}{dt} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & -6 & 1 \\ 0 & 0 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} x$$

a. Find the Resolvent matrix $\varphi(s)$ and the state transition matrix $p(t)$.

b. Find the transfer function $\frac{Y(s)}{R(s)}$.

c. Find the zero input response $x_{zi}(t)$ and the output $y_{zi}(t)$ if the initial condition is $x(0) = \begin{bmatrix} 0 \\ 1 \\ 0.5 \end{bmatrix}$.

d. Find the zero state response $x_{zs}(t)$ and the output $y_{zs}(t)$ if the input $r(t) = 20u(t);u(t) = unitstep$

e. Use **Matlab** to simulate the time domain solution ($t = [0 \quad 1]$) and output ($t = [01.5]$) in part c & d.

Problem 2

Given the continuous LTI state space:

$$\frac{dx}{dt} = \begin{bmatrix} -4 & 5 & 3 \\ 4 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} x$$

$$x(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad u(t) = unitstep$$

a. Check the internal stability of the system.

b. Find out if the system is BIBO stable.

c. Check the controllability of the system.

d. Design a state feedback controller to shift the poles to $[-10 \quad 12 \quad -8]$. Use the coefficient matching method to find K.

e. Use **Matlab** to plot the general solution of open loop ($t = [0 \quad 0.2]$) and closed loop ($t = [0 \quad 2]$) system. Compare them and explain.

Problem 3

Given the continuous LTI system:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 980 & 0 & -2.8 \\ 0 & 0 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad x(0) = \begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix}$$

a. Check the Observability of the system.

b. Design a controller with the poles located at $[-20 \pm 20i \quad -100]$. Use Ackermann's method to find K.

c. Design an observer to estimate the closed loop system with the poles at $[-100 \quad -101 \quad -150]$. Use a proper method to find L. Write out the combined state equation in terms of system and estimator with $x_e(0) = \begin{bmatrix} 0.03 \\ 0.5 \\ -5 \end{bmatrix}$

d. Use **Matlab** to plot the initial condition response of open loop states and closed loop states. Compare them and explain. (Use $t = 0 : 1E-6 : 0.3$)

e. Use **Matlab** to plot the inital condition response of output feedback states and errors. (Use $t = 0 : 1E-6 : 0.3$). Explicitly plot and compare x_1 vs x_{e1}, x_2 vs x_{e2}, x_3 vs x_{e3}. Also plot the error independently.

Problem 4

Given the continuous LTI system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -1 & 1 \\ 1 & -2 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x(t)$$

a. Consider the infinite horizon quadratic cost:

$$J = \int_0^\infty x_1(t)^2 + x_2(t)^2 + u_1(t)^2 + u_2(t)^2 dt$$

Solve the ARE for the cost function J using **Matlab**.

b. Using the solution P, obtain the K matrix using **Matlab**.

c. Use **Matlab** to plot the trajectory $x_1(t), x_2(t)$ of the system for the following cases

- Open-loop system with zero input and initial condition $x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Plot for time $t \in [0, 15]$.
- Closed-loop system ($u = -Kx$) with initial condition $x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ Plot for time $t \in [0, 10]$.

d. Use **Matlab** to plot the optimal control inputs $u_1^*(t), u_2^*(t)$. Plot for time $t \in [0, 10]$.

e. Consider the modified cost function:

$$J = \int_0^\infty x_1(t)^2 + x_2(t)^2 + \alpha u_1(t)^2 + u_2(t)^2 dt$$

Write a loop in **Matlab** that finds the minimal value of $\alpha \in [0.01 : 0.01 : 2]$ for which the optimal control input satisfies $max(u_1^*(t)) \leq 0.2$. Using the same condition in part c, plot $x_1(t)$ and $u_1^*(t)$ of the closed loop system for this value of α. Compare trajectories in the nominal case $\alpha = 1$.