Homework 1
ECES 512

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Problem 1

Given the system with state equations below with unit step input $u(t)$ and initial condition $x_1(0) = 2$, $x_2(0) = 3$:

$$\begin{aligned} \dot{x}_1 &= -2x_1 + u \\ \dot{x}_2 &= \quad x_1 - x_2 \end{aligned}$$

a. Find the state space representation of the system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

b. Find the time domain solution.
Solving for the solution using a convolution.

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)d\tau$$

First we find the matrix exponential of $\mathbf{A}$

$$e^{\mathbf{A}t} = \begin{bmatrix} e^{-2t} & 0 \\ e^{-2t}(e^t - 1) & e^{-t} \end{bmatrix}$$

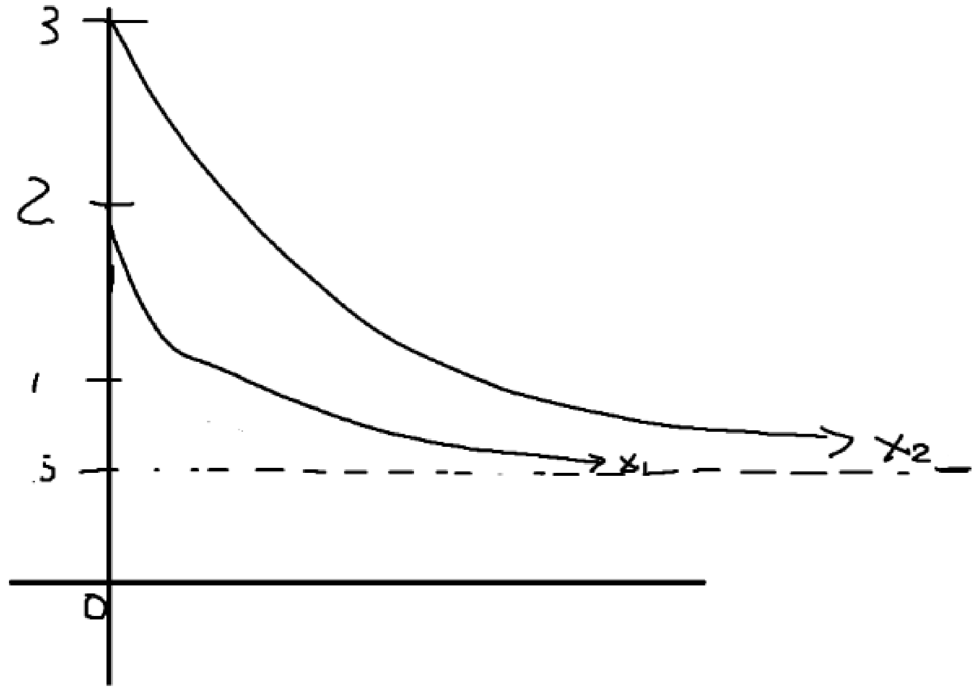
Plugging this into our equation we get

$$\mathbf{x}(t) = \begin{bmatrix} e^{-2t} & 0 \\ e^{-2t}(e^t - 1) & e^{-t} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \int_0^t \begin{bmatrix} e^{2\tau - 2t} \\ e^{\tau - 2t}(e^t - e^\tau) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} 1 d\tau$$

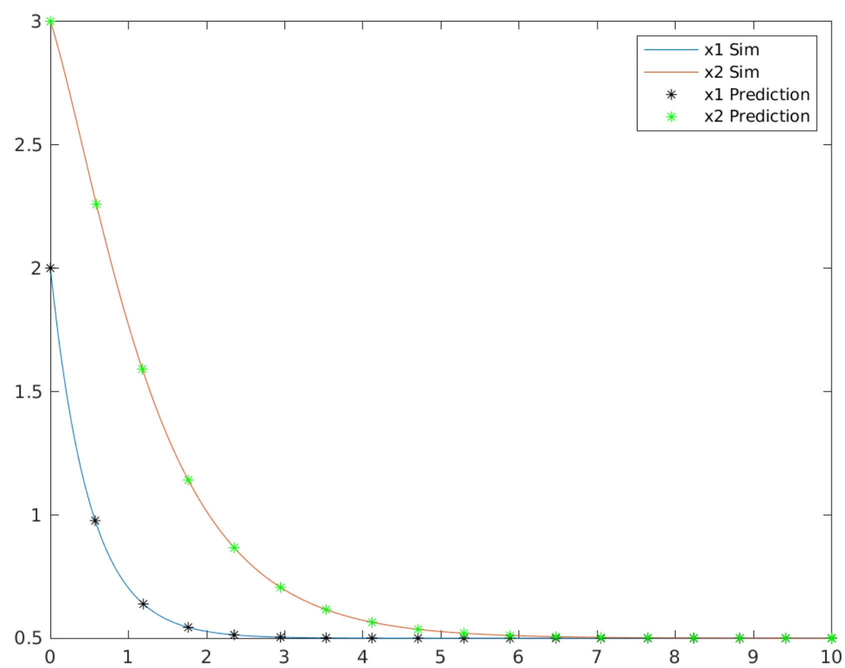
$$\mathbf{x}(t) = \begin{bmatrix} 2e^{-2t} \\ 2e^{-2t}(e^t - 1) + 3e^{-t} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} - \frac{1}{2}e^{-2t} \\ \frac{1}{2}e^{-2t}(e^t - 1)^2 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} \frac{3}{2}e^{-2t} + \frac{1}{2} \\ -\frac{3}{2}e^{-2t} + 4e^{-t} + \frac{1}{2} \end{bmatrix}$$

c. Try to sketch your solution by hand.



d. Verify your result in part b and c using Matlab(lsim).



Problem 2

Given the system with transfer function below with unit step input $u(t)$ and zero initial conditions.

$$G(s) = \frac{3s+2}{s(s^2+3s+2)}$$

a. Apply partial fraction decomposition to $G(s)$.

$$\frac{3s+2}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$3s+2 = A(s^2+3s+2) + B(s^2+2s) + C(s^2+s)$$

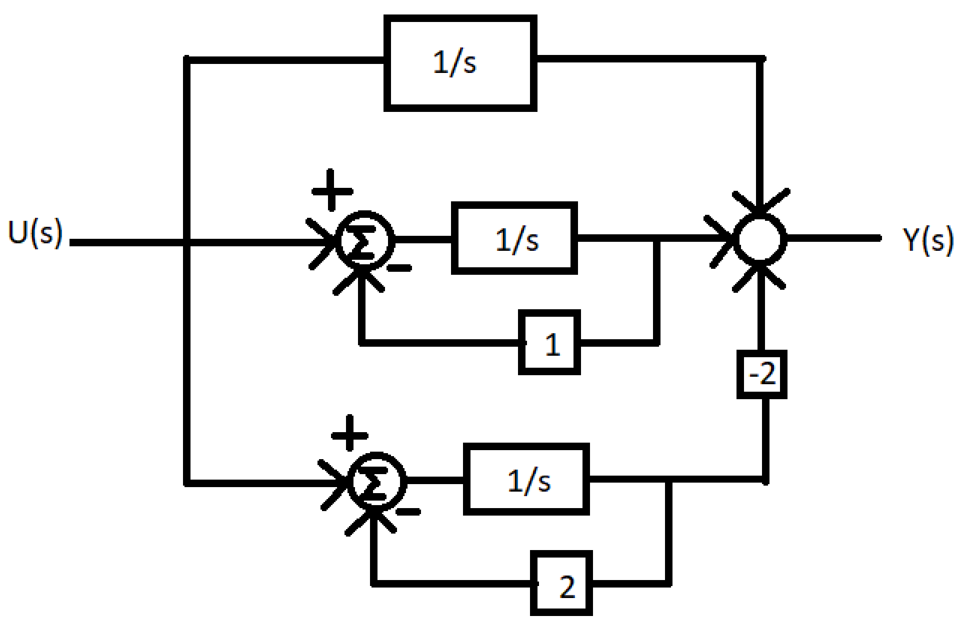
$$A = 1 \qquad A+B+C=0 \qquad 3A+2B+C=3$$

$$A=1 \qquad B+C=-1 \qquad 2B=-C$$

$$A=1 \qquad B=1 \qquad C=-2$$

$$\frac{3s+2}{s(s+2)(s+1)} = \frac{1}{s} + \frac{1}{s+1} + \frac{-2}{s+2}$$

b. Draw a block diagram of the system.



c. Find a state space representation of the system.
Using controllable Canonical Form

$$G(s) = \frac{Y(s)}{U(s)} = \frac{Z(s)(3s+2)}{Z(s)s(s^2+3s+2)}$$

$$y = 3\dot{z} + 2z$$

$$u = \dddot{z} + 3\ddot{z} + 2\dot{z}$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} z \\ \dot{z} \\ \ddot{z} \end{bmatrix}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} q_2 \\ q_3 \\ u - 3q_3 - 2q_2 \end{bmatrix}$$

$$y = 3q_2 + 2q_1$$

$$\dot{\mathbf{q}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \mathbf{q} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 3 & 0 \end{bmatrix} \mathbf{q}$$

d. Find the time domain solution.
Finding the solution using the inverse laplace transform.

$$U(s) = \frac{1}{s}$$

$$Y(s) = G(s)U(s)$$

$$Y(s) = \frac{3s+2}{s^2(s^2+3s+2)}$$

$$Y(s) = \frac{1}{s^2} + \frac{1}{s+2} - \frac{1}{s+1}$$

Take the inverse Laplace of each term since the Laplace transform is linear we get

$$y(t) = e^{-2t} - e^{-t} + t$$

e. Verify your result using Matlab(lsim).