Last Name: _______________ First Name: _______________
Student ID: _______________

Midterm ECES-512: Fundamentals of System II
Winter 2020-21

NOTE:

a. Write down all steps explicitly in a logical way. Correct answers without showing clear steps will get **no credit**.

b. Give a definite answer and circle it.

c. You have to solve the problems **manually** to earn credits, unless told to use Matlab.

d. For MATLAB simulations, in order to get **full credits**:

- Attach your Matlab code and add enough comments to your Matlab codes.
- Label your plots and axes.
- If several curves are plotted in the same figure, use different line type for each plot, and provide description in text at the bottom of the figure or by using Matlab command "legend".
- Add a brief description at the bottom of the figure to: (1) explain the figure; (2) discuss the results (if it is required).

e. Work individually. You may use your notes, course web site material and the textbook but **no internet search**.

f. This final is due End of Monday, Mar. 15^{th}, 2021

- You should provide all your work in a **single pdf** file and submit it on bblearn.
- You should also send a copy of your work to both :
  - guezal@drexel.edu
  - zw383@drexel.edu

I have read all of the instruction above:

Signature:____

Final Exam ECES-512: Fundamentals of System II
Winter 2020-21

Problem 1: Given the continuous LTI system: $\frac{dx}{dt} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & -6 & 1 \\ 0 & 0 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} r \qquad y = [0\ 1\ 1]x$

a. (4 points) Find the Resolvent matrix $\varphi(s)$ and state transition matrix $p(t)$.

b. (4 points) Find the transfer function $\frac{Y(s)}{R(s)}$.

c. (4 points) Find the zero input response $x_{zi}(t)$ and output $y_{zi}(t)$ if the initial condition is $x(0) = \begin{bmatrix} 0 \\ 1 \\ 0.5 \end{bmatrix}$.

d. (4 points) Find the zero state response $x_{zs}(t)$ and output $y_{zs}(t)$ if input $r(t) = 20u(t); u(t) = unit\ step$

e. (9 points) Use **Matlab** to simulate the time domain solution (t= [0 1]) and output (t= [0 1.5]) in part c & d.

Problem 2: Given the continuous LTI state space: $\frac{dx}{dt} = \begin{bmatrix} -4 & 5 & 3 \\ 4 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, y = [0\ -1\ \ 1]x, x(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, u = unitstep$

a. (3 points) Check the internal stability of the system.
b. (4 points) Find out if the system is BIBO stable.
c. (4 points) Check the controllability of the system.
d. (5 points) Design a state feedback controller to shift the poles to [-10 -12 -8]. Use the coefficient matching method to find K.
e. (9 points) Use **Matlab** to plot the general solution of open loop (t= [0 0.2]) and closed loop (t= [0 2]) system. Compare them and explain.

Problem 3: Given the continuous LTI system: $A = \begin{bmatrix} 0 & 1 & 0 \\ 980 & 0 & -2.8 \\ 0 & 0 & -100 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}, C = [1\ 0\ 0], x(0) = \begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix}$

a. (4 points) Check the Observability of the system.
b. (5 points) Design a controller with the poles located at $[-20 \pm 20i, -100]$. Use Ackermann method to find K.
c. (5 points) Design an observer to estimate the closed loop system with the poles at $[-100, -101, -150]$. Use a proper method to find L. Write out the combined state equations in terms of system and estimator with $x_e(0) = \begin{bmatrix} 0.03 \\ 0.5 \\ -5 \end{bmatrix}$
d. (5 points) Use **Matlab** to plot the initial condition response of open loop states, closed loop states. Compare them and explain. (Use t = 0:1E-6:0.3)
e. (6 points) Use **Matlab** to plot the initial condition response of output feedback states and errors. (Use t = 0:1E-6:0.3). Explicitly plot and compare x1 vs x_e1, x2 vs x_e2 and x3 vs x_e3. Also plot the errors independently.

Problem 4 : Given the continuous LTI system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & -1 & 1 \\ 1 & -2 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x(t)$$

a. (3 points) Consider the infinite horizon quadratic cost:

$$J = \int_0^\infty x_1(t)^2 + x_2(t)^2 + u_1(t)^2 + u_2(t)^2 \, dt$$

Solve the ARE for the cost function J using **Matlab**.

b. (3 points) Using the solution P, obtain the K matrix using **Matlab**.

c. (6 points) Use **Matlab** to plot the trajectory $x_1(t)$, $x_2(t)$ of the system for the following cases

- open-loop system with zero input and initial condition $x(0) = [1, 0, 0, 0]'$. Plot for time t ∈ [0, 15].
- closed-loop system ($u = -Kx$) with initial condition $x(0) = [1, 0, 0, 0]'$. Plot for time t ∈ [0, 10].

d. (4 points) Use **Matlab** to plot the optimal control inputs $u_1^*(t), u_2^*(t)$. Plot for time t ∈ [0, 10].

e. (9 points) Consider the modified cost function:

$$J = \int_0^\infty x_1(t)^2 + x_2(t)^2 + \boldsymbol{\alpha} * u_1(t)^2 + u_2(t)^2 \, dt$$

Write a loop in **Matlab** that finds the minimal value of $\alpha \in [0.01: 0.01: 2]$ for which the optimal control input satisfies $\max\,(u_1^*(t)) \leq 0.2$. Using the same condition in part c, plot $x_1(t)$ and $u_1^*(t)$ of the closed loop system for this value of α. Compare to the trajectories in the nominal case $\alpha = 1$.

Extra Page.