Problem 1.

Consider the inverted pendulum as depicted in Fig. 1.
For small rotation angle θ, the equation of motion is given
by

$$M\ell\ddot{\theta} = (M + m)g\theta - u$$
$$M\ddot{x} = u - mg\theta$$

Assume M = 3kg, m = 0.75kg, and ℓ = 1.5m and the state
$x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = x$, and $x_4 = \dot{x}$, with output $y_1 = \theta$, and $y_2 = x$.
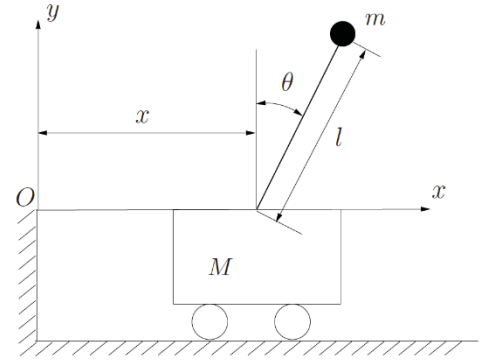

Figure 1: The inverted pendulum on cart.

a. Create a state space model for the system.

b. Is the system stable? Verify with plotting initial condition response in Matlab, with x(0)=[0.05 0 0 0]'. If unstable, under what condition should we be able to stabilize the system. How do we figure whether we can design an observer to the system?

c. Find the state feedback gain K to put the closed-loop poles at $-4 \pm 4j$, -25, and -25. Evaluate the performance of your controller by simulation via the initial condition response for x(0)=[0.05 0 0 0]'; Also simulate and plot the step response of the controlled system.

d. Design and simulate a full order closed loop observer with x(0)=[0.05 1 -1 2]' but use [0 0 0 0]' for the observer's initial states. Put all the observer poles at (-50 -50 -100 -100). Plot the state estimation error.

e. Design an output feedback controller using the observer designed in d). Simulate and plot the step responses to different initial conditions for the state of the plant and the state of the observer. Compare with c) and discuss.

Problem 2.

Consider the mechanical system given in Figure 1, where ξ_1, ξ_2 are displacements of the masses, u_1 is the force acting on the masses. Take the masses to be $m_1 = m_2 = 1$ kg, the spring stiffness to be $k1 = k2 = k3 = 1$ N/m, and the damping coefficient to be $v = 1$ Ns/m.
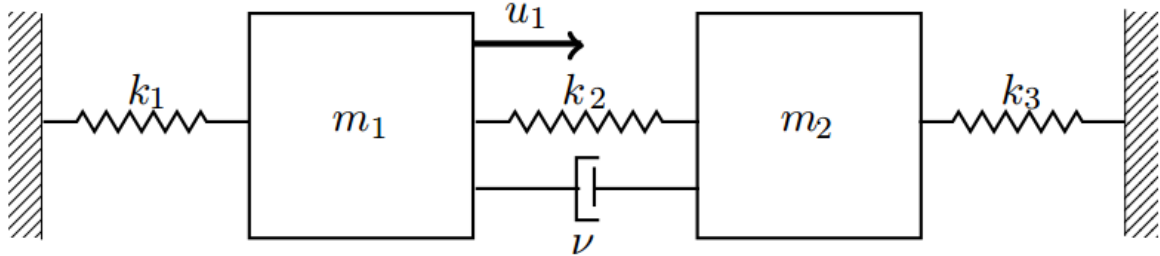

Figure 2: Simple spring-damper system

$$x(t) = [\xi_1, \xi_2, \dot{\xi}_1, \dot{\xi}_2]' \in \mathbb{R}^4.$$

$$\ddot{\xi}_1 = -\xi_1 + (\xi_2 - \xi_1) + (\dot{\xi}_2 - \dot{\xi}_1) + u_1$$

$$\ddot{\xi}_2 = -\xi_2 + (\xi_1 - \xi_2) + (\dot{\xi}_1 - \dot{\xi}_2)$$

a. Create a state space model for the system. Consider both displacement as output y_1, y_2.

b. Is the system stable? Verify with plotting initial condition response in Matlab, with x(0)=[0.05 0 0.15 0]'. If unstable, under what condition should we be able to stabilize the system. How do we figure whether we can design an observer to the system?

c. Find the state feedback gain K to put the closed-loop poles at $-4 \pm 4j$, -25, and -25. Evaluate the performance of your controller by simulation via the initial condition response for x(0)=[0.05 0 0.15 0]'; Also simulate and plot the step response of the controlled system.

d. Design and simulate a full order closed loop observer with x(0)=[0.05 0.01 0.15 0.01]', but use [0 0 0 0]' for the observer's initial states. Put all the observer poles at (-50 -50 -100 -100). Plot the state estimation error.

e. Design an output feedback controller using the observer designed in d). Simulate and plot the step responses to different initial conditions for the state of the plant and the state of the observer. Compare with c) and discuss.