Homework 8B
ECES 512

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Problem 1

Given the state matricies $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, state $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and input u with the objective function:

$$J = \int_0^\infty x_1^2 + \beta^2 u dt$$

where m, β are some unknown constants. And $\beta > 0$.

a. Based on the given information, indicate matrix Q and R in terms of m and β.

b. Solve the ARE using Q and R. Find the matrix P in terms of m and β.

c. Find the gain vecotr K in terms of m and β.

d. Howd does the solution and cost change with β values?

e. Verify teh optimal output $F = -x_1 - 4x_2$ when $m = 8$ and $\beta = 1$.

f. Write out the time domain solution for both the open loop and closed loop system. (u = unit step)

g. Simulate both systems and plot the states using Matlab.

Problem 2

Given the state matrices $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -.4 & -4.2 & -2.1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

a. Find the open loop eigenvalues.

b. If $Q = I_3$, find P, K and the closed loop eigenvalues for $R \in \{0.01, 0.1, 1\}$.

c. Use Matlab to plot the states and optimal input with the Q and different R values given in part b.