Problem 1: Given the SISO LTI system

$$\frac{dx}{dt} = \begin{bmatrix} -1 & 4 \\ 4 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [1\ 1]x$$

a. Show why the system is not controllable.
b. Reduce the state equation to a controllable one.
c. Is the reduced equation observable?

Problem 2: Given the LTV system $\dot{x} = A(t)x + B(t)u, y = C(t)x$ where:

$$A(t) = \begin{bmatrix} -2 & t \\ 0 & -3 \end{bmatrix} B(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} C(t) = [0 \quad e^{5t}] \quad \text{and} \quad t_0 > 0$$

Check the controllability and Observability of the linear time varying system. Remember the necessary and sufficient conditions of the tests in the text.

Problem 3: Given the DTS system $x[k+1] = Ax[k] + Bu[k],\ y[k] = Cx[k]$ where:

$$A = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.5 \end{bmatrix} B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C = [0\ 1]$$

a. Use the **discrete controllability Gramian** to test controllability.
b. Use the **discrete observability Gramian** to test observability.