Problem 1: Given the state matrices $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$ $C = [1\ 0]$, state $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and input u with the objective function:

$$J = \int_0^{\infty} {x_1}^2 + \beta^2 u \, dt$$

where m, β are some unknow constants. And $\beta > 0$.

a. Based on the given information, indicate the matrix Q and R in terms of m and β.
b. Solve the ARE using the Q and R. Find matrix P in terms of m and β.
c. Find the gain vector K in terms of m and β.
d. How does the solution and cost change with β values?
e. Verify that the optimal input $F = -x_1 - 4x_2$ when $m = 8$ and $\beta = 1$.
f. Write out the time domain solution for both open loop and closed loop system. ($u = unit\ step$)
g. Simulate both systems and plot the states using Matlab.

Problem 2: Given the state matrices $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.4 & -4.2 & -2.1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

a. Find the open loop eigenvalues.
b. If $Q = I_3$, find P, K and closed loop eigenvalues as $R = 0.01, 0.1$ and 1 respectively.
C. Use Matlab to plot the states and optimal input with the Q and different R values given in part b.