MEM 633 Robust Control I HW #3

1. $A = T\Lambda T^{-1}$ and $\Lambda = diag\left[\lambda_1, \lambda_2, ...\lambda_n\right]$ where $\lambda_1, \lambda_2, ...\lambda_n$ are distinct.

   Show that

   (a) $e^{At} = Te^{\Lambda t}T^{-1}$

   (b) $e^{\Lambda t} = diag\left[e^{\lambda_1 t}, e^{\lambda_2 t}, ...e^{\lambda_n t}\right]$

2. Consider the time-invariant system $\dot{x}(t) = Ax(t)$ where the nxn matrix A has distinct eigenvalues $\lambda_1, \lambda_2, ...\lambda_n$. The corresponding eigenvectors are $e_1, e_2, ...e_n$.

   Let $T = \left[e_1\ e_2\ .....\ e_n\right]$ and $v_1', v_2', ...v_n'$ be row vectors of T^{-1}. Show that the solution of $\dot{x}(t) = Ax(t)$ can be written as

$$x(t) = \sum_{i=1}^{n} v_i' \cdot x(0) \cdot e^{\lambda_i t} \cdot e_i$$

3. The A matrix in Problem 2 is given as

$$A = \begin{bmatrix} 0 & 1 \\ 6 & -5 \end{bmatrix}$$

   Write down the solution of $\dot{x}(t) = Ax(t)$ by using the result of Problem 2. You will see the system is unstable. However, $x(t)$ will be bounded if the initial state vector is in the stable subspace. Describe the stable subspace of the system.

4. Find realizations in controller and observability forms of the transfer function,

$$H(s) = \frac{2s^3 + 13s^2 + 31s + 32}{s^3 + 6s^2 + 11s + 6}$$

   Give both block diagrams and state-space equations.