MEM 633 Robust Control I HW #4

1. Consider the system described by the following state equation,

$$\dot{x}(t) = Ax(t) + Bu(t) := \begin{bmatrix} -18 & -19 & -15 \\ 20 & 21 & 16 \\ -5 & -5 & -4 \end{bmatrix} x(t) + \begin{bmatrix} -3 \\ 5 \\ -2 \end{bmatrix} u(t)$$

(a) Use PBH Test to check if the system is controllable.

(b) Characterize the controllable subspace using the controllability decomposition approach.

(c) Is the system stabilizable? Explain.

(d) Design a state feedback controller using the controllability decomposition approach so that the closed-loop system is internally stable.

(e) Assume the initial state of the system is $x(0) = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}^T$, plot the state response $x(t)$ of the closed-loop system.

2. Consider the same system described by the state equation shown in Problem 1.

(a) Find a similarity transformation to transform the state equation to the one with diagonal "A" matrix.

(b) Characterize the controllable subspace using the eigenvectors obtained in Problem 2(a).

(c) Check the stabilizability of the system based on the result of Problem 2(b)..

(d) Design a state feedback controller using the eigen structure obtained in Problem 2(b) so that the closed-loop system is internally stable.

(e) Assume the initial state of the system is $x(0) = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}^T$, plot the state response $x(t)$ of the closed-loop system.

3.

(a) If $\{A, b, c, d\}$, $d \neq 0$, is a realization with $H(s) = c(sI - A)^{-1}b + d$, show that

$$\left\{ A - \frac{bc}{d},\ \frac{b}{d},\ \frac{-c}{d},\ \frac{1}{d} \right\}$$

is a realization for a system with transfer function 1/H(s).

(b) If we are given $\{A, b, c, d\}$, $d \neq 0$, show that the zeros of $c(sI - A)^{-1}b + d$ are the eigenvalues of the matrix $A - \frac{bc}{d}$.