DREXEL UNIVERSITY
Department of Mechanical Engineering & Mechanics

MEM 633

Robust Control Systems I

Final Exam

12/08/2021 Wednesday

NAME: ____________________________

Attention:

1. Open books and notes.
2. **Absolutely no discussions about the exam before, during, or even after the test until all the students turn in their exams.**
3. If there is any question about the exam, please contact Dr. Chang (changbc@drexel.edu).
4. **Show detailed procedure!** A solution without detailed procedure will receive no credit even the answer is correct.
5. **No hard copy will be accepted this time!!** The exam report should be saved as **ONE** LEGIBLE pdf file with Pencil Tool enabled and size less than 5Mb. The report should be **self contained, clearly written and well organized according to the order of the problems**. Be sure to include your name on the front page of the report. **In addition to the exam report, you also need to submit the executable computer programs (like m-files, mdl, and slx-files) used to obtain the solutions for the exam.**
6. The exam report and all computer program files are required to be **zipped** and **uploaded to Drexel Bb Learn MEM633 Web Page before 11:59PM, 12/08/2021, Wednesday.**
7. The name of the zipped file should include MEM633_FinalExam_YourName.
8. **Read the problems carefully** before answering the questions.
9. **Use discreet judgment to determine if it is appropriate to use MATLAB (or other software) commands to obtain the solutions. Do NOT use a black-box command to answer the questions that may defeat the purpose of the test.**
10. **Do NOT use MATLAB or any software to automatically generate a report for the exam.**

Problem #1: (30%)

Consider the following transfer function matrix

$$H(s) = \frac{\begin{bmatrix} -s & s \\ s+1 & 1 \end{bmatrix}}{(s-1)(s+2)^2}$$

(a) Find a state-space realization of $H(s)$ in block controller form. (10%)

(b) Use PBH Test to check the controllability and observability of the realization. (5%)

(c) Is it a minimal realization? If not, find a similarity transform to transform the realization into either a noncontrollable or a nonobservable canonical form. Then find a minimal realization by eliminating the uncontrollable and/or unobservable parts of the canonical form. (10%)

(d) Determine the poles and zeros by using the (A, B, C) of the minimal realization in 1(c). (5%)

Note that the zeros of (A, B, C) are the frequencies at which the rank of

$$\begin{bmatrix} sI - A & B \\ -C & 0 \end{bmatrix}$$

drops below its normal rank.

Hint: Computation of zeros (from [Harry and Kwatny 2021, page 378])

Theorem 10.17 (Computation of the Zeros of State-Space Model (A, B, C, D))

The zeros of the state-space model (A,B,C,D) are the generalized eigenvalues λ_i so that

$$\begin{bmatrix} \lambda_i I - A & B \\ -C & D \end{bmatrix} v_i = \left[\lambda_i \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \right] v_i = [\lambda_i E - Z] v_i \qquad (10.33)$$

where v_i are the corresponding generalized eigenvectors. ■

The MATLAB command

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Dz=eig(Z,E)   or   [Vz,Dz]=eig(Z,E)
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can be employed to obtain the generalized eigenvalues and the generalized eigenvectors in Dz and Vz, respectively. This state-space approach was employed to find the zero of a simple first-order SISO system in Example 10.3. In Example 10.13, we obtained the poles and zeros of a fourth-order F/A–18 SISO system by converting the state-space model to a transfer function. We will verify the zeros of the fourth-order F/A–18 SISO system in the next example by the Theorem 10.17 approach.

Problem #2: (37%)

For the transfer function matrix $H(s)$ shown in Problem #1, which can be represented as

$$H(s) = N(s)D(s)^{-1}$$

where

$$N(s) = \begin{bmatrix} -s & s \\ s+1 & 1 \end{bmatrix}, \quad D(s) = (s-1)(s+2)^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(a) Find a greatest common right divisor (gcrd) of $N(s)$ and $D(s)$. (10%)

(b) Find an irreducible right MFD for $H(s)$ by extracting the gcrd of $N(s)$ and $D(s)$. (4%)

(c) Determine the poles and zeros of the system based on the irreducible MFD in (b). (3%)

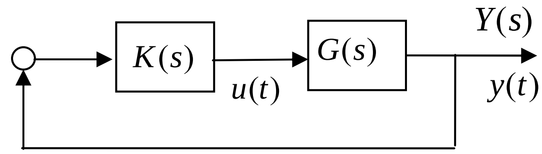
(d) Find a state-space realization for the right MFD in (c). (10%)

(e) Is the state-space realization controllable? observable? Is it a minimal realization? (5%)

(f) Can you find a similarity transformation which relates the realizations in Problem #1(c) and Problem #2(d)? If yes, show the results and procedure. If not, explain. (5%)

Problem #3: (33%)

Consider the following system,



where

$$G(s) = \frac{s-2}{s^2 - 4s}.$$

In the following you will design an observer-based controller $K(s)$ so that the closed-loop system is stable.

(a) Find a state-space representation of $G(s)$. (3%)

(b) Use the Riccati-equation approach to determine an observer-based controller $K_1(s)$ so that the closed-loop system is stable. (5%)

(c) Find a state-space representation of the closed-loop system with $y(t)$ as the output. (3%)

(d) Verify that the closed-loop system poles are the regulator poles together with the observer poles. (3%)

(e) Plot state response for $x(t)$ and $u(t)$ due to initial conditions y(0), amd $\dot{y}(0)$. (2%)

(f) Repeat (b), (c), (d) and (e) with a different observer-based controller $K_2(s)$, which is obtained by choosing different weighting matrices in the Riccati equations. (13%)

(g) Comment on how the weighting matrices in the Riccati equation and the pole location of the closed-loop system affect the closed-loop system performance. (4%)