Midterm
MEM 633

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Problem 1

For the nonlinear cart-inverted pendulum system shown in page 11 of Chapter 1 notes, a linearized model at the upright stick equilibrium can be obtained as follows if both of the rotational and translational friction coefficients are negligible.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -1.4 \\ 0 \\ 0.6 \end{bmatrix} u(t)$$

Assume the initial condition is

$$x(0) = \begin{bmatrix} -0.3\text{rad} \\ 0 \\ 0.2m \\ 0 \end{bmatrix}$$

(a) Is the system stable? Controllable? Explain.

(b) Let $u(t) = 0$. Solve for $x(t)$ and then plot it and comment on the initial state response.

(c) Now, use the state feedback $u(t) = Fx(t)$ with $F = \begin{bmatrix} 139 & 24 & 30 & 31 \end{bmatrix}$ to stabilize the system. Find the closed-loop pole locations of the closed-loop system to verify that all these poles are in the left half of the complex plane.

(d) Plot the displacement of the state variables $x_1(t)$ and $x_3(t)$ on the first graph, the velocity state variables $x_2(t)$ and $x_4(t)$ on the second graph, and the control input $u(t)$ on the third graph for the closed-loop system.

(e) Comment on the physical meaning of the three time response graphs in (d).

Problem 2

Consider the following block diagram where $G(s)$ is given as

$$\xrightarrow[u(t)]{U(s)} \boxed{G(s)} \xrightarrow[y(t)]{Y(s)}$$

$$G(s) = \frac{s-2}{s(s^2+1)}$$

(a) Explain why the system is not BIBO stable based on the **definition** of BIBO stability, and find two bounded inputs that would cause the output to be unbounded. Plot the output due to these two inputs.

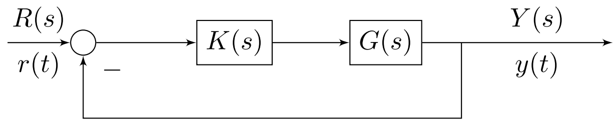
(b) Find a state-space representation in the controller canonical for for the system $G(s)$.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

(c) Explain why the system is not internally stable based on the **definition** of internal stability. Plot the state response, $x(t)$, of the system with initial state $x(0) = \begin{bmatrix} 2 & 0 & -2 \end{bmatrix}^T$ and zero input.

Problem 3

Consider the following feedback control system. Given



$$G(s) = \frac{s-1}{s^2 - 2s}$$

(a) Find a controller $K(s)$ with the least order, which is either strictly proper or proper, so that the closed-loop system is BIBO stable. Hint: You can start from a zero-order controller $K_0(s) = b_0$. If it does not work, try a 1^{st}-order controller $K_1(s) = \frac{b_1 s + b_0}{s + a_0}$ or a 2^{nd}-order controller $K_2(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$.

(b) With the controller $K(s)$ you have just designed, determine the closed-loop transfer function $\frac{Y(s)}{R(s)}$.

(c) Assume zero initial conditions and the reference input $r(t)$ is a unit step function. Plot the output response $y(t)$ of the feedback control system you have just designed.

Problem 4

Consider the system described by the following state equation,

$$\dot{x}(t) = \begin{bmatrix} -21 & -22 & -20 \\ 26 & 27 & 23 \\ -9 & -9 & -7 \end{bmatrix} x(t) + \begin{bmatrix} -3 \\ 5 \\ -2 \end{bmatrix} u(t)$$

(a) Use PBH Test to check if the system is controllable.

(b) Characterize the controllable subspace.

(c) If possible, design a state feedback controller so that the closed-loop system is internally stable. If not, explain the reason.