DREXEL UNIVERSITY
Department of Mechanical Engineering & Mechanics

MEM 633

Robust Control Systems I

Mid-Term Exam

11/03/2021 Wednesday

NAME: ______________________________

Attention:

1. Open books and notes.
2. **Absolutely no discussions about the exam before, during, or even after the test until all the students turn in their exams.**
3. If there is any question about the exam, please contact Dr. Chang (changbc@drexel.edu).
4. **No hard copy will be accepted this time!!** The exam report should be saved as **ONE** LEGIBLE pdf file with Pencil Tool enabled and size less than 5Mb. The report should be self contained, clearly written and well organized according to the order of the problems. Be sure to include your name on the front page of the report. **In addition to the exam report, you also need to submit the executable computer programs (like m-files, mdl, and slx-files) used to obtain the solutions for the exam.**
5. The exam report and all computer program files are required to be **zipped** and **uploaded to Drexel Bb Learn MEM633 Web Page before 11:59PM, 11/03/2021, Wednesday.**
6. The name of the zipped file should include MEM633_MidTerm_YourName.
7. **Read the problems carefully** before answering the questions.
8. **Use discreet judgment to determine if it is appropriate to use MATLAB (or other software) commands to obtain the solutions. Do NOT use a black-box command to answer the questions that may defeat the purpose of the test.**

Problem #1: (34%)

For the nonlinear cart-inverted pendulum system shown in page 11 of Chapter 1 notes, a linearized model at the upright stick equilibrium can be obtained as follows if both of the rotational and translational friction coefficients are negligible.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -1.4 \\ 0 \\ 0.6 \end{bmatrix} u(t)$$

Assume the initial condition is

$$x(0) = \begin{bmatrix} -0.3\text{ rad} \\ 0 \\ 0.2\text{ m} \\ 0 \end{bmatrix}.$$

(a) Is the system stable? Controllable? Explain. (6%)

(b) Let $u(t) = 0$. Solve for $x(t)$ and then plot it and comment on the initial state response. (7%)

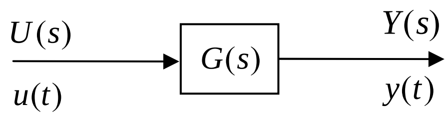
(c) Now, use the state feedback $u(t) = Fx(t)$ with $F = \begin{bmatrix} 139 & 24 & 30 & 31 \end{bmatrix}$ to stabilize the system. Find the closed-loop pole locations of the closed-loop system to verify that all these poles are in the left half of the complex plane. (10%)

(d) Plot the displament state variables $x_1(t)$ and $x_3(t)$ on the first graph, the velocity state variables $x_2(t)$ and $x_4(t)$ on the second graph, and the control input $u(t)$ on the third graph for the closed-loop system. (6%)

(e) Comment on the physical meaning of the three time response graphs in (d). (5%)

Problem #2: (25%)

Consider the following block diagram



where $G(s)$ is given as

$$G(s) = \frac{s-2}{s(s^2+1)}$$

(a) Explain why the system is not BIBO stable based on the **definition** of BIBO stability, and find two bounded inputs that would cause the output to be unbounded. Plot the output response due to these two inputs. (10%)

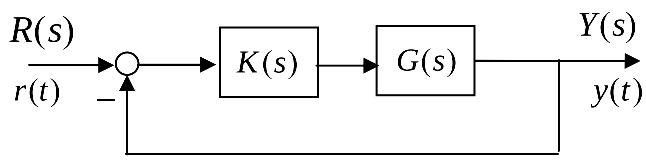
(b) Find a state-space representation in the controller canonical form for the system $G(s)$. (5%)

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

(c) Explain why the system is not internally stable based on the **definition** of internal stability. Plot the state response, $x(t)$, of the system with initial state $x(0) = \begin{bmatrix} 2 & 0 & -2 \end{bmatrix}^T$ and zero input. (10%)

Problem #3: (25%)

Consider the following feedback control system.



Given

$$G(s) = \frac{s-1}{s^2 - 2s}$$

(a) Find a controller $K(s)$ with the least order, which is either strictly proper or proper, so that the closed-loop system is BIBO stable. (15%)

Hint: You can start from a zero-order controller $K_0(s) = b_0$. If it does not work, try a 1^{st}-order controller $K_1(s) = \frac{b_1 s + b_0}{s + a_0}$ or a 2^{nd}-order controller $K_2(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$.

(b) With the controller $K(s)$ you have just designed, determine the closed-loop transfer function $Y(s)/R(s)$. (5%)

(c) Assume zero initial conditions and the reference input $r(t)$ is a unit step function. Plot the output response $y(t)$ of the feedback control system you have just designed. (5%)

Problem #4: (16%)

Consider the system described by the following state equation,

$$\dot{x}(t) = \begin{bmatrix} -21 & -22 & -20 \\ 26 & 27 & 23 \\ -9 & -9 & -7 \end{bmatrix} x(t) + \begin{bmatrix} -3 \\ 5 \\ -2 \end{bmatrix} u(t)$$

(a) Use PBH Test to check if the system is controllable. (6%)

(b) Characterize the controllable subspace. (4%)

(c) If possible, design a state feedback controller so that the closed-loop system is internally stable. If not, explain the reason. (6%)