MEM633 Robust Control Systems I

HW#1

Problem 1:

Given: A system is governed by

$$\ddot{r} = r\dot{\theta}^2 - \frac{k}{r^2} + u_1$$
$$\ddot{\theta} = \frac{-2\dot{\theta}\dot{r}}{r} + \frac{1}{r}u_2$$

If $u_1 = u_2 = 0$, the equations admit the solution

$r = \sigma$ (σ is a constant)

$\theta = \omega t$ (ω is a constant)

where $\sigma^3\omega^2 = k$. Define $x_1 = r - \sigma$, $x_2 = \dot{r}$, $x_3 = \sigma(\theta - \omega t)$, $x_4 = \sigma(\dot{\theta} - \omega)$, and write a set of linearized differential equations which describe the resulting motion for small deviations from a given circular orbit.

Problem 2:

Given: A system is governed by

$$\dot{v}_1 + 5v_1 = 2u$$
$$\dot{v}_2 + v_2 - 2v_1 = 0$$

$$y = 50v_2$$

where $v_1(t)$ and $v_2(t)$ are state variables and $y(t)$ and $u(t)$ are the output and the input of the system respectively.

(a) Compute the state transition matrix, the impulse response function, and the step response function of the system. Plot the impulse and the step responses based on the obtained functions. Is the system stable? Why?

(b) Determine the transfer function of the system. Draw the Bode plots of the frequency response function of the system, and explain the physical meaning of the plots.