Final
MEM 633

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Problem 1

Consider the following transfer function matrix

$$H(s) = \frac{\begin{bmatrix} -s & s \\ s+1 & 1 \end{bmatrix}}{(s-1)(s+2)^2}$$

(a) Find a state-space realization of $H(s)$ in block controller form.

(b) Use the PBH Test to check the controllability and observability of the realization.

(c) Is it a minimal realization? If not, find a similarity transformation to transform the realization into either controllablility or observability decompostion. Then find a minimal realization by eliminating the uncontrollable and/or unobservable parts of the system.

(d) Determine the poles and zeros by using the (A, B, C) matrices of the minimal realization in 1(c). Note that the zeros of (A, B, C) are the frequencies at which the rank of

$$\begin{bmatrix} sI - A & B \\ -C & D \end{bmatrix}$$

drops below its normal rank.

Problem 2

For the transfer matrix $H(s)$ shown in Problem #1, which can be represented as

$$H(s) = N(s)D(s)^{-1}$$

Where

$$N(s) = \begin{bmatrix} -s & s \\ s+1 & 1 \end{bmatrix}, \quad D(s) = (s-1)(s+2)^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(a) Find a greatest common right divisor (gcrd) of $N(s)$ and $D(s)$.

(b) Find an irreducible right MFD for $H(s)$ by extracting the gcrd of $N(s)$ and $D(s)$.

(c) Determine the poles and zeros of the system based on the irreducible MFD in (b).

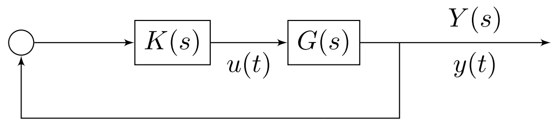
(d) Find a state-space realization for the MFD in (c)

(e) Is the state-space realization controllable and observable? Is it a minimal realization?

(f) Can you find a similarity transformation which relates the realization in Problem #1(c) and problem #2(d)? If yes, show the results and procedure. If not explain.

Problem 3

Consider the following system, where



$$G(s) = \frac{s-2}{s^2-4s}$$

In the following you wil ldesign an observer-based controller $K(s)$ so that the closed-loop system is stable.

(a) Find a state space representation of $G(s)$.

(b) Use the Riccati-equation approach to determine an observer based controller $K_1(s)$ such that the closed loop system is stable.

(c) Find a state space representation of the closed loop system with $y(t)$ as the output.

(d) Verify that the closed loop system poles are the regulator poles together with the observer poles.

(e) Plot the state response for $x(t)$ and $u(t)$ due to the initial conditions $y(0)$, and $\dot{y}(0)$.

(f) Repeat (b), (c), (d), and (e) with a different observer-based controller $K_2(s)$, which is obtained by choosing different weighting matrices in the Riccati equations.

(g) Comment on how the weighting matrices in the Riccati equation and the pole location of the closed-loop system affect the closed loop system performance.