Abbotsleigh 1999 MX2 Trial Q4(a)

Q4 (a)(i)

Show that tangent to the ellipse $\dfrac{x^2}{12} + \dfrac{y^2}{4} = 1$ at the point $P(3,1)$ has the equation $x + y = 4$

Q4 (a)(ii)

If this tangent cuts the directrix at the point T and S is the corresponding focus, show that SP and ST are at right angles to each other.

Worked Solutions:

Q4 (a)(i)

$$\frac{x^2}{12} + \frac{y^2}{4} = 1$$
$$\frac{8x}{12} + \frac{2y}{4} \cdot \frac{dy}{dx} = 0$$
$$\therefore \frac{dy}{dx} = -\frac{x}{3y}$$

At $P(3,1)$: $\dfrac{dy}{dx} = 1$

$\therefore y - 1 = -1(x - 3)$
$\Rightarrow x + y = 4$

Q4 (a)(ii)

$$e = \sqrt{1 - \frac{4}{12}} = \frac{\sqrt{2}}{\sqrt{3}}$$

So the focus is $(2\sqrt{2}, 0)$ and the directrix is $x = 3\sqrt{2}$

T lies on the tangent and directrix, so solving the simultaneous equations, $x + y = 4$ and $x = 3\sqrt{2}$ gives $y = 4 - 3\sqrt{2}$.
$\Rightarrow P(3,1),\ S(2\sqrt{2}, 0),\ T(3\sqrt{2},\ 4 - 3\sqrt{2})$

To show that $SP \perp ST$, calculate the product of the gradients

gradient SP $\times$ gradient ST

$$= \frac{1}{3 - 2\sqrt{2}} \cdot \frac{4 - 3\sqrt{2}}{\sqrt{2}}$$
$$= \frac{4 - 3\sqrt{2}}{3\sqrt{2} - 4}$$
$$= -1$$

$\therefore$ SP and ST are at right angles to each other.