1999 Abbotsleigh Maths Ext 2 Trial - Q4 (a) (Worked Solutions)

[i] Show that tangent to the ellipse $\dfrac{x^2}{12}+\dfrac{y^2}{4}=1$ at the point $P\,(3,1)$ has the equation $x+y=4$

[ii] If this tangent cuts the directrix at the point T and S is the corresponding focus, show that SP and ST are at right angles to each other.

Worked Solutions:

[i]
$\dfrac{x^2}{12}+\dfrac{y^2}{4}=1$
$\dfrac{8x}{12}+\dfrac{2y}{4}\cdot\dfrac{dy}{dx}=0$
$\therefore \dfrac{dy}{dx}=-\dfrac{x}{3y}$

At $P\,(3,1)$, $\dfrac{dy}{dx}=1$
$\therefore y-1=-1\,(x-3)$
$\Rightarrow x+y=4$

[ii]
$e=\sqrt{1-\dfrac{4}{12}}=\frac{\sqrt{2}}{\sqrt{3}}$
So the focus is $\left(2\sqrt{3}\cdot\dfrac{\sqrt{2}}{\sqrt{3}},0\right)=\left(2\sqrt{2},\ 0\right)$ and the directrix is $x=\dfrac{2\sqrt{3}}{\left(\dfrac{\sqrt{2}}{\sqrt{3}}\right)}=3\sqrt{2}$

At T, $x+y=4$ and $x=3\sqrt{2}$
$\Rightarrow y=4-3\sqrt{2}$
$\therefore P\,(3,1)\,,\ S\left(2\sqrt{2},0\right),\ T\left(3\sqrt{2},\ 4-3\sqrt{2}\right)$

$\therefore$ gradient SP $\times$ gradient ST
$=\dfrac{1}{3-2\sqrt{2}}\cdot\dfrac{4-3\sqrt{2}}{\sqrt{2}}=\dfrac{4-3\sqrt{2}}{3\sqrt{2}-4}$
$=-1$

$\therefore SP$ and ST are at 90° to each other.