Taller 4

1. (Theoretical) Hacer pasos intermedios para regla del trapecio Simple.

$$P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$f(x) \approx P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b), \forall x \in [a,b]$$

$$I = \int_a^b f(x)\,dx \cong \int_a^b P_1(x)\,dx = \frac{b-a}{2}(f(a)+f(b)) \quad (1,74)$$

$$\int_a^b P_1(x)\,dx = \int_a^b \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)\,dx =$$

$$\int_a^b \frac{x-b}{a-b} f(a)\,dx + \int_a^b \frac{x-a}{b-a} f(b)\,dx = \frac{f(a)}{a-b}\int_a^b x-b\,dx + \frac{f(b)}{b-a}\int_a^b x-a\,dx$$

$$\frac{f(a)}{a-b}\Big|_a^b \left(\frac{x^2}{2} - bx\right) + \frac{f(b)}{b-a}\Big|_a^b \left(\frac{x^2}{2} - ax\right) =$$

$$\frac{f(a)}{a-b}\left(\left(\frac{b^2}{2} - \frac{b^2}{1}\right) - \left(\frac{a^2}{2} - \frac{ba}{1}\right)\right) + \frac{f(b)}{b-a}\left(\left(\frac{b^2}{2} - \frac{ab}{1}\right) - \left(\frac{a^2}{2} - a^2\right)\right)$$

$$\frac{f(a)}{a-b}\left(\left(-\frac{b^2}{2}\right) - \left(\frac{a^2-2ba}{2}\right)\right) + \frac{f(b)}{b-a}\left(\left(\frac{b^2-2ab}{2}\right) - \left(-\frac{a^2}{2}\right)\right) =$$

$$\frac{f(a)}{a-b}\left(-\left(\frac{a^2-2ab+b^2}{2}\right)\right)+\frac{f(b)}{b-a}\left(\frac{b^2-2ab+a^2}{2}\right)=$$

$$\frac{f(a)}{a-b}\left(-\frac{(a-b)^2}{2}\right)+\frac{f(b)}{b-a}\left(\frac{(b-a)^2}{2}\right)=$$

$$f(a)\left(-\frac{a-b}{2}\right)+f(b)\cdot\frac{b-a}{2}=$$

$$f(a)\,\frac{b-a}{2}+f(b)\cdot\frac{b-a}{2}=$$

$$\frac{b-a}{2}\left(f(a)+f(b)\right)$$

Se llegó. ☺

2 (Theoretical) Encontrar el error para regla de trapecio simple. Ecuacion (1,77)

$$\epsilon(x) = \frac{f''(\varepsilon)}{2}(x-a)(x-b), \quad a \leq \varepsilon \leq b$$

$$E_1 = \int_a^b \epsilon(x)\,dx = -\frac{h^3}{12} f''(\varepsilon) \quad (1,77)$$

$h = b - a$ $\quad b = h + a$ $\quad a = b - h$

$$\int_a^b \frac{f''(\varepsilon)}{2}(x-a)(x-b)\,dx = \frac{f''(\varepsilon)}{2} \cdot \int_a^b (x-a)(x-b)\,dx$$

$$\int_a^b x^2 - bx - ax + ab\,dx = \frac{x^3}{3} - \frac{bx^2}{2} - \frac{ax^2}{2} + abx \Big|_a^b =$$

$$\left(\frac{b^3}{3} - \frac{b^3}{2} - \frac{ab^2}{2} + \frac{ab^2}{1}\right) - \left(\frac{a^3}{3} - \frac{ba^2}{2} - \frac{a^3}{2} + \frac{a^2 b}{1}\right) =$$

$$-\frac{b^3}{6} + \frac{ab^2}{2} + \frac{a^3}{6} - \frac{a^2 b}{2} = b^2\left(\frac{a}{2} - \frac{b}{6}\right) + a^2\left(\frac{a}{6} - \frac{b}{2}\right) =$$

$$= b^2\left(\frac{3a-b}{6}\right) + a^2\left(\frac{a-3b}{6}\right) = b^2\left(\frac{3(b-h)-b}{6}\right) + (b-h)^2\left(\frac{(b-h)-3b}{6}\right)$$

$$= b^2\left(\frac{2b-3h}{6}\right) + (b-h)^2\,\frac{-2b-h}{6} = \frac{2b^3 - 3b^2h}{6} + (b^2 - 2bh + h^2)\left(\frac{-h-2b}{6}\right)$$

$$\frac{\overbrace{2b^3}^{///} - \overbrace{3b^2h}^{//} - \overbrace{b^2h}^{//} - \overbrace{2b^3}^{///} + \overbrace{2bh^2}^{/} + \overbrace{4b^2h}^{//} - h^3 - \overbrace{2bh^2}^{/}}{6}$$

$$= \frac{-h^3}{6} = -\frac{h^3}{6} = \int_a^b (x-a)(x-b)\,dx$$

$$E = \frac{f''(\varepsilon)}{2} \cdot \int_a^b (x-a)(x-b)\,dx =$$

$$E = \frac{f''(\varepsilon)}{2} \cdot \left(-\frac{h^3}{6}\right) = -\frac{h^3}{12} f''(\varepsilon)$$

3. Codigo.

3.

$$x_m = \frac{a+b}{2}$$

$$\int_a^b f(x)\,dx \cong \int_a^b P_2(x)\,dx = \int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)\,dx$$

$$= \int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a)\,dx + \int_a^b \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m)\,dx + \int_a^b \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)\,dx =$$

$$f(a)\int_a^b \frac{x^2 - x x_m - bx + b x_m}{a^2 - a x_m - ba + b x_m}\,dx + f(x_m)\int_a^b \frac{x^2 - bx - ax + ab}{x_m^2 - b x_m - a x_m + ab}\,dx + f(b)\int_a^b \frac{x^2 - x x_m - ax + a x_m}{b^2 - b x_m - ab + a x_m}\,dx =$$

$$\frac{f(a)}{a^2 - a x_m - ba + b x_m}\int_a^b x^2 - x x_m - bx + b x_m\,dx + \frac{f(b)}{b^2 - b x_m - ab + a x_m}\int_a^b x^2 - x x_m - ax + a x_m\,dx$$

$$+ \frac{f(x_m)}{x_m^2 - b x_m - a x_m + ab}\int_a^b x^2 - bx - ax + ab\,dx$$

$$=$$

$$\frac{f(a)}{a^2 - a x_m - ba + b x_m}\left.\left(\frac{x^3}{3} - \frac{x^2}{2}x_m - \frac{bx^2}{2} + b x_m x\right)\right|_a^b + \frac{f(x_m)}{x_m^2 - b x_m - a x_m + ab}\left.\left(\frac{x^3}{3} - \frac{bx^2}{2} - \frac{ax^2}{2} + abx\right)\right|_a^b + \frac{f(b)}{b^2 - b x_m - ab + a x_m}\left.\left(\frac{x^3}{3} - \frac{x^2}{2}x_m - \frac{ax^2}{2} + a x_m x\right)\right|_a^b$$

4. (Theoretical) Verificar el resultado presentado en la Ecuación (1.89).

$$E = \int_a^b \epsilon(x)\,dx = \int_a^b \frac{f''''(\varepsilon)}{4!}(x-a)(x-b)(x-(a+b)/2)\,dx = 0$$

$$= \frac{f'''(\varepsilon)}{4!}\int_a^b (x^2 - bx - ax + ab)\left(x - \frac{a+b}{2}\right)dx$$

$$\frac{f'''(\varepsilon)}{4!}\int_a^b x^3 - bx^2 - ax^2 + abx - \frac{a+b}{2}x^2 + \frac{(a+b)bx}{2} + \frac{(ax)(a+b)}{2} - \frac{ab(a+b)}{2}\,dx$$

$$\frac{f'''(\varepsilon)}{4!}\int_a^b x^3 - bx^2 - ax^2 + abx - \frac{a+b}{2}x^2 + \frac{ab+b^2}{2}x + \frac{a^2+ab}{2}x - \frac{a^2b+ab^2}{2}\,dx$$

$$\frac{f'''(\varepsilon)}{4!}\Bigg|_a^b\left(\frac{x^4}{4} - \frac{bx^3}{3} - \frac{ax^3}{3} + \frac{abx^2}{2} - \frac{(a+b)x^3}{2\cdot 3} + \frac{(ab+b^2)x^2}{2\cdot 2} + \frac{(a^2+ab)x^2}{2\cdot 2} - \frac{a^2b+ab^2}{2}x\right)$$

$$\frac{f'''(\varepsilon)}{4!}\left(\left(\frac{b^4}{4} - \frac{b^4}{3} - \frac{ab^3}{3} + \frac{ab^3}{2} - \frac{ab^3+b^4}{6} + \frac{ab^3+b^4}{4} + \frac{a^2b^2+ab^3}{4} - \frac{a^2b^2+ab^3}{2}\right)\right.$$
$$\left.-\left(\frac{a^4}{4} - \frac{ba^3}{3} - \frac{a^4}{3} + \frac{a^3b}{2} - \frac{a^4+a^3b}{6} + \frac{a^3b+a^2b^2}{4} + \frac{a^4+a^3b}{4} - \frac{a^3b+a^2b^2}{2}\right)\right)$$

$$\frac{f'''(\varepsilon)}{4!}\left(\left(-\frac{b^4}{12} + \frac{ab^3}{6} + \frac{ab^3+b^4}{12} - \frac{a^2b^2+ab^3}{4}\right) - \left(-\frac{a^4}{12} + \frac{a^3b}{6} + \frac{a^4+a^3b}{12} - \frac{a^3b+a^2b^2}{4}\right)\right)$$

$$\frac{f'''(\varepsilon)}{4!}\left(\left(-\frac{b^4}{12} + \frac{ab^3}{6} + \frac{ab^3}{12} + \frac{b^4}{12} - \frac{a^2b^2}{4} - \frac{ab^3}{4}\right) - \left(-\frac{a^4}{12} + \frac{a^3b}{6} + \frac{a^4}{12} + \frac{a^3b}{12} - \frac{a^3b}{4} - \frac{a^2b^2}{4}\right)\right)$$

$$\frac{f'''(\varepsilon)}{4!}\left(\left(-\frac{a^2b^2}{4}\right) - \left(-\frac{a^2b^2}{4}\right)\right) = \frac{f'''(\varepsilon)}{4!}\cdot(0) = 0$$