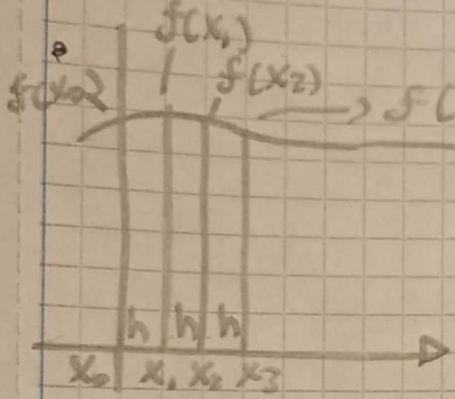
Deducción Polinomio, funciones Cardinales regla de Simpson 3/8



• $h = \frac{x_3 - x_0}{3}$

$$A_1 = f(x_0)\int_{x_0}^{x_3} L_0(x)\,dx + f(x_1)\int_{x_0}^{x_3} L_1(x)\,dx + f(x_2)\int_{x_0}^{x_3} L_2(x)\,dx + f(x_3)\int_{x_0}^{x_3} L_3(x)\,dx$$

En donde:

$$L_0 = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \qquad L_1 = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$L_2 = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \qquad L_3 = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$\therefore$

$$L_0 = \frac{}{(-h)(-2h)(-3h)} \qquad L_1 = \frac{}{(h)(-h)(-2h)}$$

$$L_2 = \frac{}{(2h)(h)(-h)} \qquad L_3 = \frac{}{(3h)(2h)(h)}$$

y partiendo de:

$x = x_0 + th \rightarrow x_0 = x_0 + 0h$
$x_1 = x_0 + 1h$
$x_2 = x_0 + 2h$
$x_3 = x_0 + 3h$

$x - x_0 = th$
; $x - x_1 = x - x_0 - h = th - h = h(t-1)$
$x - x_2 = x - x_0 - 2h = th - 2h = h(t-2)$
$x - x_3 = x - x_0 - 3h = th - 3h = h(t-3)$

Entonces

$$L_0 = \frac{h(t-1)\,h(t-2)\,h(t-3)}{-6h^3}$$

$$L_1 = \frac{(th)\,h(t-2)\,h(t-3)}{2h^3}$$

$$L_2 = \frac{th(h)(t-1)\,h(t-3)}{-2h^3}$$

$$L_3 = \frac{(th)\,h(t-1)\,h(t-2)}{6h^3}$$

$P:\ X = X_0 + th$

- $X_0 = X_0 + th \to t = 0$
- $X_3 = X_0 + th$
  $X_3 - X_0 = th$
  $3h = th$
  $\to t = 3$

$\to t \in [0,3]$
límites de integración

$$\to \frac{dx}{dt} = h\ ;\ dx = h\,dt$$

$$\to A_1 = \frac{f(X_0)}{6}h\overset{i}{\int_0^3}(t-1)(t-2)(t-3)\,dt + \frac{f(X_2)}{2}h\overset{ii}{\int_0^3}t(t-2)(t-3)\,dt - \frac{f(X_2)}{2}h\overset{iii}{\int_0^3}t(t-1)(t-3$$

$$+ \frac{f(X_3)}{6}h\overset{iv}{\int_0^3}t(t-1)(t-2)\,dt$$

$$\to \overset{i}{\int_0^3}(t-1)(t-2)(t-3)\,dt = \int_0^3 t^3-6t^2+11t-6 = \frac{t^4}{4} - \frac{6t^3}{3} + \frac{11t^2}{2} - 6t\,\Big|_0^3 = -\frac{9}{4}$$

$$\to A_1 = -\frac{f(X_0)}{6}h\overset{i}{\left(\frac{-9}{4}\right)} + \frac{f(X_1)}{2}h\overset{ii}{\left(\frac{9}{4}\right)} -$$

$$\to \overset{ii}{\int_0^5} t(t-2)(t-3)\,dt = \frac{t^4}{4} - \frac{5t^3}{3} + \frac{6t^2}{2}\,\Big|_0^5 = \frac{9}{4}$$

$\to \int_0^3 t(t-1)(t-3)\,dt = \frac{t^4}{4} - \frac{4t^3}{3} + \frac{3t^2}{2}\Big|_0^3 = \frac{9}{4}$

$\to A_1 = -\frac{f(x_0)}{6} h\left(-\frac{9}{4}\right) + \frac{f(x_1)}{2} h\left(\frac{9}{4}\right) - \frac{f(x_2)}{2} h\left(-\frac{9}{4}\right)$

$\to \int_0^3 t(t-1)(t-2)\,dt = \frac{t^4}{4} - 3\frac{t^3}{3} + \frac{2t^2}{2}\Big|_0^3 = \frac{9}{4}$

$\to A_1 = -\frac{f(x_0)}{6} h\left(-\frac{9}{4}\right) + \frac{f(x_1)}{2} h\left(\frac{9}{4}\right) - \frac{f(x_2)}{2} h\left(-\frac{9}{4}\right) + \frac{f(x_3)}{6} h\left(\frac{9}{4}\right)$

$\to \int_{x_0}^{x_3} f(x)\,dx = f(x_0)\left(\frac{3h}{8}\right) + f(x_1)\left(\frac{9h}{8}\right) + f(x_2)\left(\frac{9h}{2}\right)$

$+ f(x_3)\left(\frac{3h}{8}\right)$

$= \frac{3h}{8}\left(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right)$

b) $h = \frac{x_3 - x_0}{3}$ $\quad x_1 = x_0 + h = a + \frac{(b-a)}{3}$

$x_1 = \frac{2a+b}{3}$

$x_2 = x_0 + 2h = a + \left(\frac{2b-2a}{3}\right)$

$x_2 = \frac{a+2b}{3}$

Dem.