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ADOPTION OF ELECTRIC VEHICLES - AN AGENT-BASED MODEL APPROACH

Dariusz Pałatyński

keywords:
electric vehicles, diffusion of innovation, opinion dynamics, agent-based modeling, multi-layer networks, Monte Carlo simulations

short summary:
In this work, an agent-based model of adoption of electric vehicles in Poland is presented. The agents are put on a multi-layer network representing different communication channels between them. Their opinion about electric cars evolves according to the q-voter model with independence in the presence of an external field. The model is simulated by means of Monte Carlo methods. The role of different parameters is discussed.

Supervisor	dr hab. Janusz Szwabiński	....	....
	Title/degree/name and surname	grade	signature

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Chapter 1

Introduction

An electric vehicle is a vehicle that has at least one electric motor inside [32]. What is essential, this motor does not need to be used every day. There are cars with both combustion and electric motors and still counts as electric vehicles. The group of electric vehicles [21] (EVs) consists of BEVs (Battery electric vehicle) - also known as fully electric vehicles, which have only electric motors - and need to be charged using electric vehicle charger, and PHEVs (Plug-in hybrid electric vehicle), which are combination of electric and standard combustion motors. This type of vehicles can be charged. The third group of EVs are HEVs (Hybrid electric vehicle) - they have small electric motors, which are charged during usage of the combustion engine. It is impossible to charge them using standard charging infrastructure.

The manufacturing of electric vehicles is constantly increasing as well as people's interests in having such a car. This high demand for electric vehicles can cause considerable changes in public and private transport. People are interested in having electric vehicles due to reduced production of pollution in comparison to standard fuel vehicles. Electric vehicles are the most wanted choice of alternative fuel vehicles. It seems like this type of cars could be a suitable replacement for public transport, which uses fuel engines. However, for many citizens, it can be only a deliberation due to the relatively high price for an average employee. Another reason is the restricted maximum range, which is stated as one of the most debatable topics about electric vehicles in their development in the next years.

In 2017 the Ministry of Energy in Poland signed an act Electromobility Development plan to increase the popularization of electric vehicles [30]. In the contents of this article, it is stated that in 2025 the number of electric vehicles will exceed one million. The corresponding forecasts are presented in table 1.1. It seems that the development of these alternative fuel vehicles should rapidly increase and reach enormous values.

Year	Number of electric vehicles	New registrations
2016	2 397	1 389
2018	13 576	7 871
2019	32 310	18 734
2020	76 898	44 587
2022	366 034	183 016
2024	823 576	274 525
2025	1 029 470	205 894

Table 1.1: Forecast for the number of electric vehicles and new registrations based on the Ministry of Energy in Poland [30] [34].

Values presented in the table 1.1 have been verified by NIK (Supreme Audit Office) [6] in 2020. The predictions turned out to be too optimistic. If we consider the predicted value for 2020 with a number of electric vehicles in the middle of 2020, which is presented in the figure 1.1, there is more than 60 thousand difference. Values in the table 1.1 are heavily overestimated. NIK in its report states that electromobility in Poland is still in its initial stage. What is more, the number of electric charging infrastructures is still not properly adjusted to allow travel using electric vehicles without worries about electric chargers.

Despite that failure in planning the development of electromobility in Poland for future years, the number of electric vehicles is still increasing yearly. In order to speed up the adoption process, there are additional subsidies to buy an electric vehicle if part of assumptions about potential consumers and cars are satisfied. Based on monthly reports from ACEA (*fr. Association des Constructeurs Européens d'Automobiles*) in 2020, only in three months, sale of EVs were lower than one million.

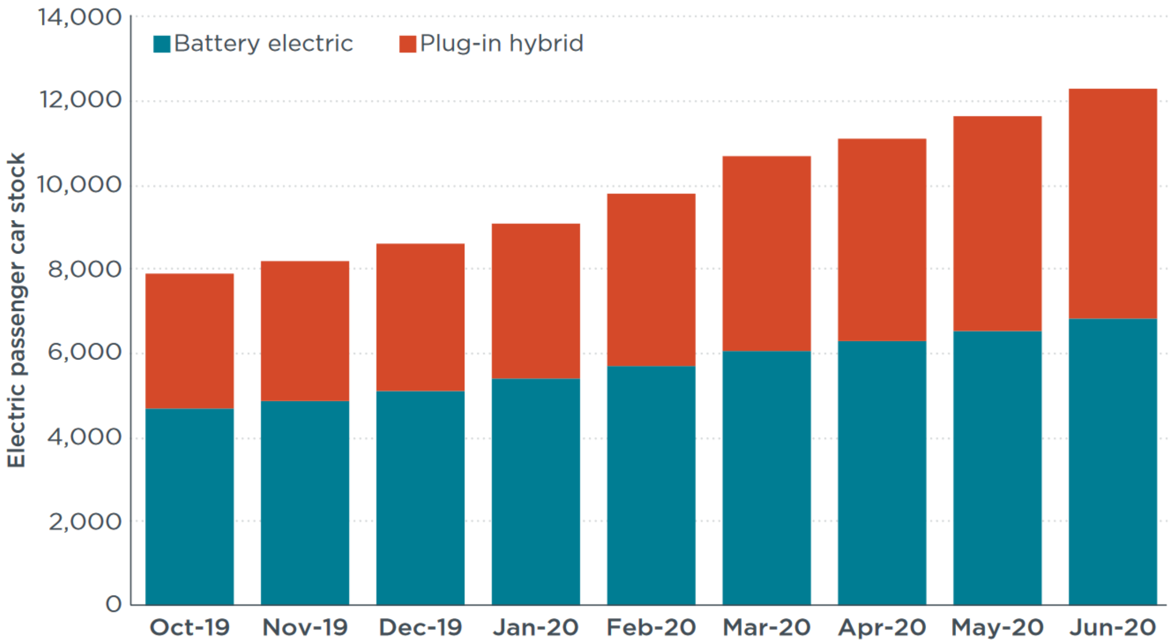

Figure 1.1: Development of electric passenger cars in Poland [33].

In the figure 1.1 we can see changes in the number of electric vehicles in 9 months from October 2019 to June 2020. In this period, the number of electric alternative fuel vehicles increased by 33% in comparison to the beginning of the considered period. Interestingly, the fractions of BEVs and PHEVs are quite similar over the entire period, with the number of BEVs increasing a little bit faster.

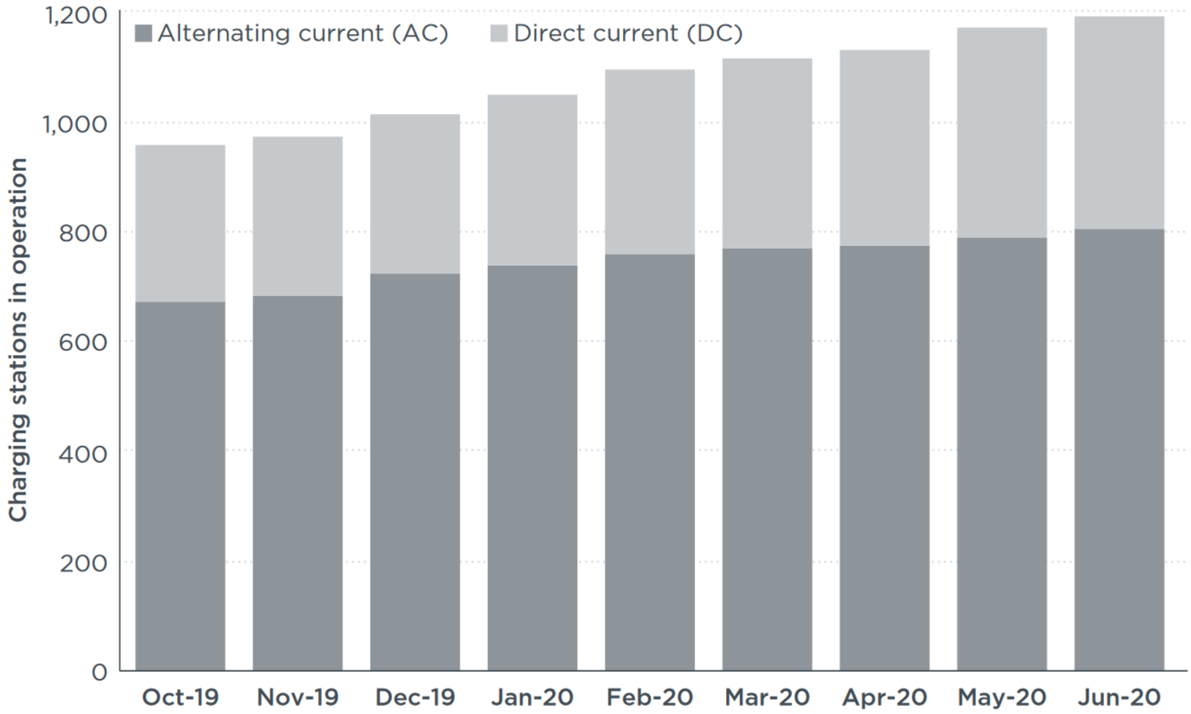

Figure 1.2: Development of charging stations in Poland [33].

With an increasing number of electric vehicles in Poland for the same period from October 2019 to June 2020, we can see changes in a number of electric charging stations (Figure 1.2). It shows that government constantly plans and develops new charging infrastructure, which promotes electric vehicles. In October 2019, there were 950 charging stations, but in June 2020, this number increased to almost 1200.

At the same time, when we have observed 4 000 new electric vehicles on Polish roads, 250 new charging stations were built. The ratio of newly established electric chargers to newly registered electric vehicles in the same period is equal to 1 : 16, which means one new charger for 16 electric cars. Due to the continually growth of electric vehicles, the development of infrastructure needs to stay on a similar or higher level to encourage people to this type of alternative fuel vehicles.

Motivation

Due to recent changes in the number of electric vehicles and the constant growth of electric chargers infrastructure, the adoption of electric vehicles could be a debatable topic to discuss. Without a doubt, there are a lot of allies and enemies in the development of electric vehicles. Their influence on the environment is also controversial. EVs are considered as zero-emission vehicles, however, in a lot of countries, this energy is produced in coal-based power plants, which produce a lot of carbon dioxide, which is finally harmful for the environment. The misunderstandings in the maximum range of cars also have a bad impact on the opinion about electric vehicles. Nevertheless, people are interested in this type of transport, and they adjust slowly to this innovation.

The goal of this work will be to model the adoption process of the electric cars in a population. The agent-based approach [13] approach will be used, as it allows for a detailed analysis of complex systems at the microscopic level.

Chapter 2

Theory

In this chapter, the building blocks of our model will be introduced and briefly discussed.

2.1 Graphs

Graphs are abstract data structures which are commonly used in various mathematical models and real-world analyses. They are quite useful in describing systems in which single components can interact with each other. There are many applications of graphs in different fields like physics, chemistry, computer technology, genetics, and a lot more. Especially in this paper, we will use graphs to model opinion diffusion.

Definition 2.1. [20] Graph G is a data structure, represented by the tuple $G = (V(G), E(G))$, where $V(G)$ is not an empty set of all vertices inside graph G. $E(G)$ is considered as a set of edges between vertices $V(G)$ - one edge between two vertices, with assumptions that in graph G it is possible to have an empty set $E(G)$. It can be said that the graph is labeled if it is possible to distinguish nodes using labels.

Definition 2.2. [31] A subgraph H of a graph G is a graph that is created by removing some vertices and edges which were connected to the removed vertex. A subgraph H can be represented as a tuple $(V(H), E(H))$, where $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

Definition 2.3. [31] The degree of a node $d(v)$ in graph G is represented by the number of connections between i and other nodes, which belongs to graph G. All vertices which are connected to v are called neighbors of v. The average degree of a graph G is a sum of degrees of all nodes divided by a number of vertices. It can be represented as the following formula:

$$d(G) = \frac{1}{|V|} \sum_{i=1}^{n} d(i),$$

where $|V|$ is the number of all nodes in graph G.

Example 2.4. Consider graph $G = (V(G), E(G))$, in which $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$. Representation of this graph is shown in the figure 2.1.

Example 2.5. A subgraph of graph G is created by removing the following vertices: v_5, v_6, which removes edges: e_6, e_7. A subgraph of graph G is represented in the figure 2.1.

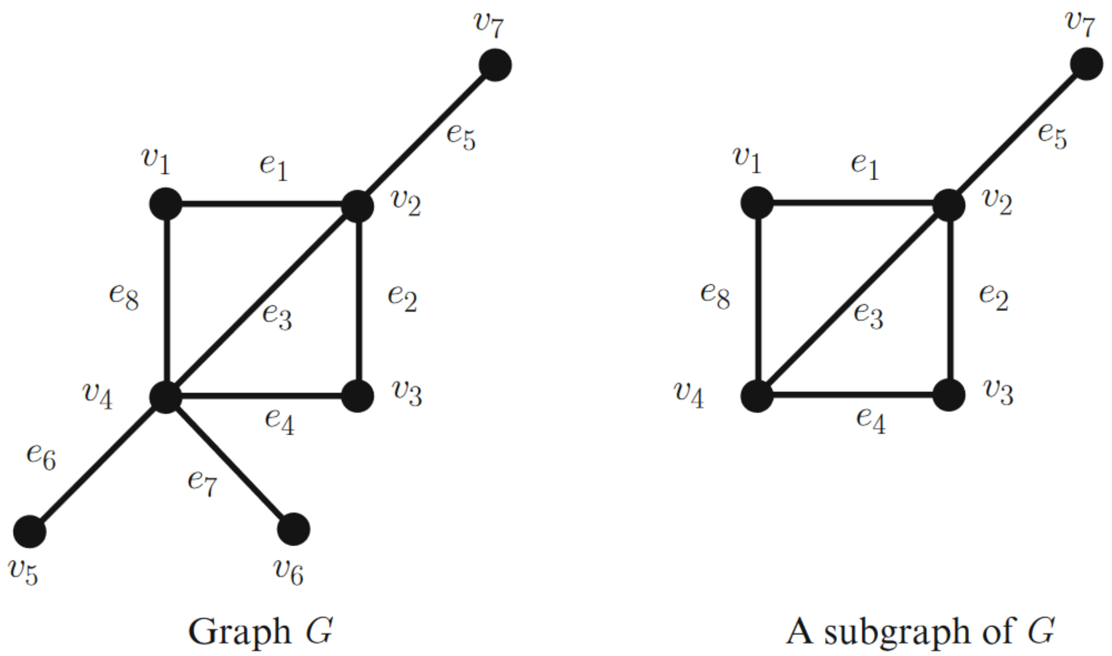

Figure 2.1: An example of graph G and its subgraph [31].

Vertex (node) v_i is a point in the defined set of nodes on which the creation of the graph is based. Nodes v_i, v_j can be connected by an edge e_i, but it is not obligatory. In the considered example we have 7 nodes and 8 edges. This graph is labeled because it is possible to recognize all vertices by labels. The subgraph of G consists of 5 nodes and 6 edges. The degree of a node can differ in graph and subgraph, for graph G degree of a node v_4 is equal to 5, because it has 5 possible neighbors, but for the same vertex in the subgraph of G, the degree $v_4 = 3$, because two vertices and two edges were removed, which was connected to v_4.

2.2 Multi-layer networks

A lot of interactions in real life are described by many complicated processes [12]. Interaction between objects represented by vertices in a graph can be more sophisticated than standard relationships based on edges. To reflect real-life problems, we can create a system consisting of numerous layers, where each layer can describe different dependencies. One layer in such a system is called the single-layer network, and it can be expressed as a graph, where nodes are e.g., humans and edges - some kind of interactions between them.

Definition 2.6. [20] A multi-layer network M consists of several graphs (single layer networks), each of which represents a different kind of interactions. It can be represented as $M = (G_1, G_2, \ldots, G_n)$, where a single layer G_i, $i = 1, 2, \ldots, n$ is described as $G_i = (V(G_i), E(G_i))$.

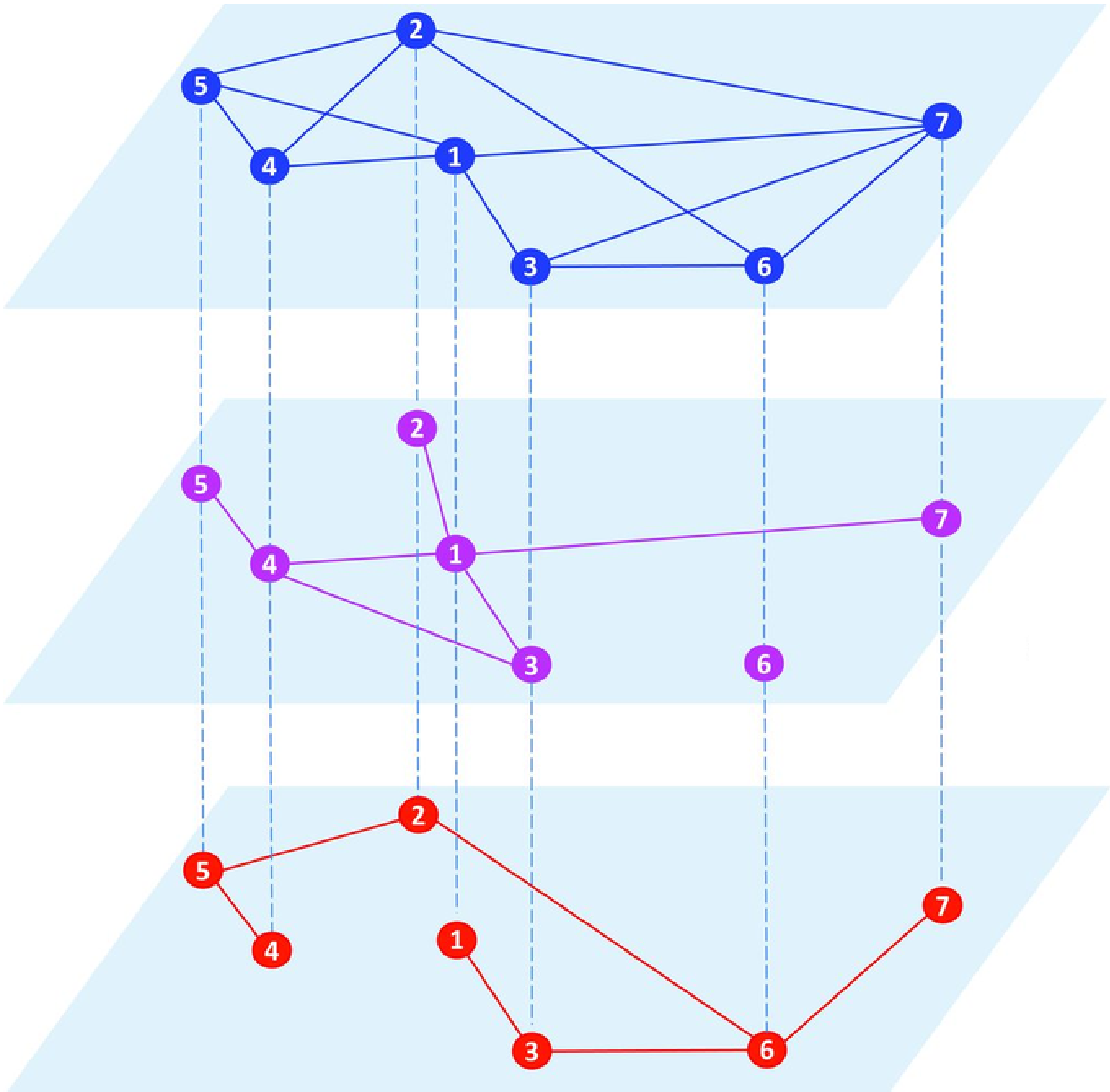

Figure 2.2: A representation of a multi-layer network [22].

In this paper, we assume that in all layers of a multi-layer network, there is the same set of nodes as presented in the figure 2.2. The only difference between the layers is a number of edges. At every level of these networks, vertices can have different neighbors. The nodes and edges represent different interactions between people like social interaction or media influence. Dotted lines stand for vertical connection between nodes. In this paper we will assume that the layers are connected only by the shared nodes. Moreover, each of the nodes will have only one state variable, visible in all of the layers.

2.3 Agent Based Modelling

Agent based models [13] are dynamic models which are widely used to simulate actions and interactions of autonomous agents. These models contain agents, connections between them usually in form of a graph, and the environment in which actions are made. As agents, we can consider people as well as inanimate objects. Agents have a set of attributes and behaviors, and results of modification in time are analyzed after simulations are done. The connection between agents describes a relationship between them, which states the number of links between agents and the type of connection. An environment for agents is a place where all interaction between them is performed.

Each agent examines its situation independently and decides about its opinion or state based on rules. Agents can perform a lot of various activities implemented in the model in which they were created. Their actions lead to modification of their properties, and these changes can be influential for other agents in the environment. An agent based model could be used to work with the complicated system of interactions and produce valuable information about the dynamics of the real-world system.

Chapter 3

Model

In this chapter, we will introduce our model. To simulate the number of people, who are for or against electric vehicles, we will use a multi-layer network to accommodate different types of relationships between people.

3.1 Assumptions

To simulate the diffusion of electric vehicles in Poland, we need to estimate the number of agents in our network. Due to computational restrictions, we limit the area of Poland to Lower Silesian Voivodeship because it's a highly industrial area of a country, so it can be a good region to consider. We take a number of people in Lower Silesian Voivodeship in the 2018 year [2], which is equal to 2 901 225 people in different ages. Additionally, we take into account only people who are living in a city. Degree of urbanization in 2018 is equal 0.606 [3]. It means that more than 60% of society live in a city and have broad access to many electric vehicles charging stations because most of them are located in highly urbanized areas [27]. The last assumption for restriction of agent's number is based on salaries. We assume that only agents who earn more than 200% of an average salary can consider buying and maintaining an electric vehicle. An average salary in the 2018 year was 5003,78zł [5], so finally, we estimate the minimal wage of 10 000zł. In this year, salary at at least this level was reached by 6.2% people in Poland [5]. Finally, we calculate the final number of agents for our model:

$$N = 2\ 901\ 225 \cdot 0.606 \cdot 0.062 \approx 109\ 004.$$

We need to consider the starting value of electric vehicles for simulation. At the end of 2018, on Polish roads, there were 2021 electric cars [7]. This value consists of BEVs and PHEVs. Since we are interested only in Lower Silesian Voivodeship, we take a fraction of EVs based on a population of Lower Silesian Voivodeship and an overall population for Poland [8]. Finally, we calculate the final number of electric vehicles:

$$N_{EV} = 2\ 021 \cdot \frac{2\ 901\ 225}{37\ 921\ 592} \approx 154.$$

Due to the very long computing times of a single simulation, we restrict the number of agents to a value ten times lower. It significantly shortens the time of a single iteration to such an amount, which let us investigate a lot of different scenarios. On the other hand, it is still big enough to mimic the behavior of a real community. As a result, in this model, and for calculation, we use the number of agents $N = 10\ 900$ and the initial number of electric vehicles $N_{EV} = 15$.

3.2 Networks

The multi-layer network consists of four networks, which describe dependencies between agents. It reflects the behavior of people in real life. All relationships between people have different properties. For this model, we analyze following networks:

- neighbourhood network,
- social interaction network,
- temporary social interaction network,
- social media network.

For simulation and visualization purposes, all networks were generated using NetworkX[18] library from Python.

3.2.1 Neighbourhood network

The basic network, which will be used in the created model, is connected with interaction between neighbors. Based on assumptions that Lower Silesian Voivodeship is a highly developed part of Poland and can be represented by a rectangular area, we are able to equally distribute all agents on a two-dimensional plane. To arrange all agents in a correct way, we consider the geographic limits of this region.

borders	decimal degrees
North	51.804° N
East	$14.817°E$
South	$50.096°N$
West	$17.795°E$

Table 3.1: The table presents topography of Lower Silesian Voivodeship containing borders of this region.

Using longitude and latitude values from the table 3.1, it is possible to calculate height and width in decimal degrees. Based on this value and the number of agents N, we get to know how they should be distributed on an area of 2.97 x 1.71 decimal degrees. In a rectangular area, we need to determine the number of agents for length n_x and width n_y sizes. To find the estimated number, we need to solve the following system of equations:

$$\begin{cases} N = n_x \cdot n_y, \\ \frac{2.97}{1.71} = \frac{n_x}{n_y}. \end{cases} \tag{3.1}$$

Finally, we round our results to integers, and we have got $n_x = 137$ and $n_y = 79$, which can be used to create rectangular networks. Finally, the number of agents in our network is $N = 10823$. This value will also be used to generate all further networks.

Thus, we created a rectangular neighborhood network. Each node in the bulk of the lattice has four nearest neighbors. Nodes on edges have three connections except the ones in the corners, which are connected to only two other nodes. The network was created with the function `grid_2d_graph` [14] from the NetworkX library. An exemplary neighborhood network is presented in the figure 3.1.

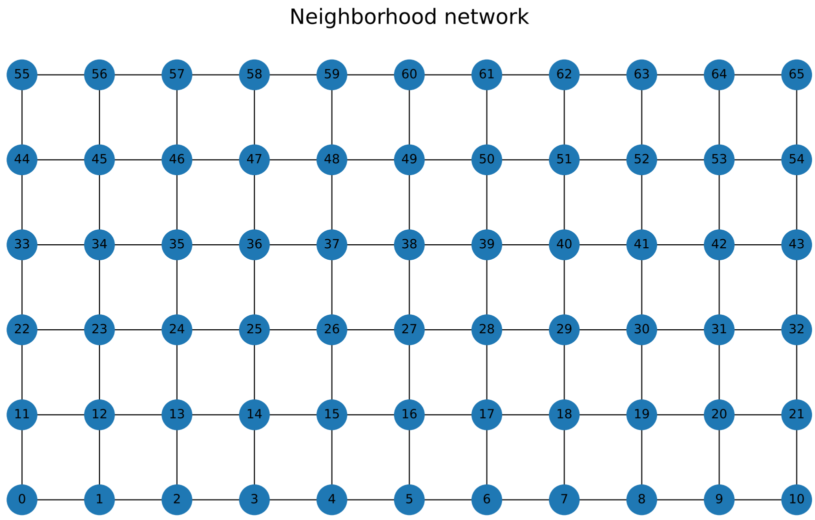

Figure 3.1: An example of a 2d lattice network.

3.2.2 Social interaction network

The second network in our multi-layer model of diffusion of electric vehicles is the social interaction network. To create this layer, we use Barabasi-Albert graph[11]. Its construction starts with a group of nodes, which are connected with each other. Then, all new nodes are added to the network with a predefined number of connections to the already existing nodes with probability [29]:

$$p_i = \frac{d_i}{\sum_j d_j},$$

where d_i is a degree of a node i. This value of probability means that a higher degree of a node (higher number of neighbors) corresponds to the higher probability p_i of making a connection between a new node and an already existing one. This property is called preferential attachment because nodes prefer to link to highly-connected nodes. Barabasi-Albert model reproduces some real networks with a few hubs with the highest number of connections with other vertices. It imitates real interaction between people, where the most famous people have the most friends, and the newcomers - want to create a link with those popular ones. In our model, we use the Barabasi-Albert method with one node as the starting vertex and one preferential attachment link for new nodes. An exemplary network is presented in the figure 3.2. The graph we created with the `barabasi_albert_graph` function from NetworkX library.

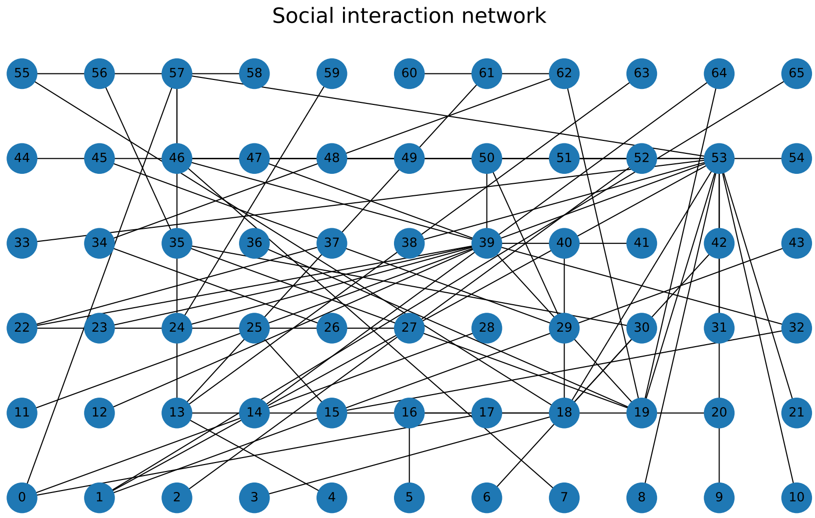

Figure 3.2: An example of Barabasi Albert Graph with single preferential attachment.

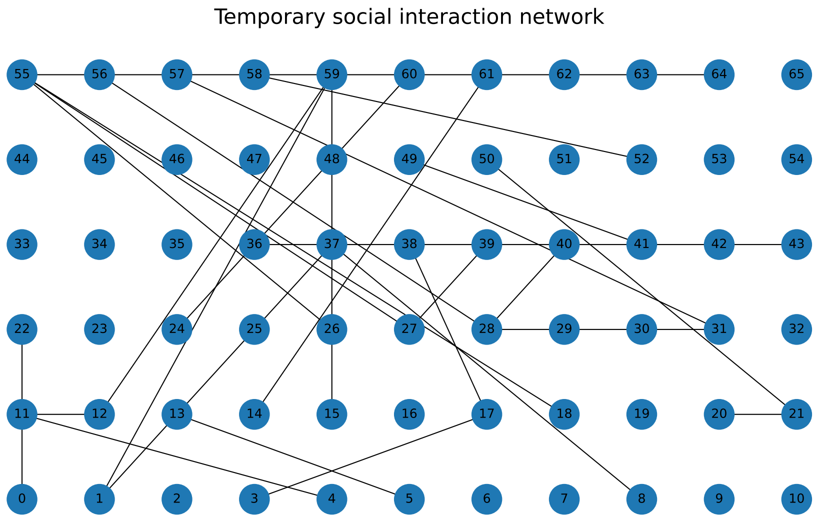

Figure 3.3: An example of Erdős-Rényi graph.

3.2.3 Temporary social interaction network

The next layer is also connected with social interaction within the community. This time, it is connected with temporary and unexpected meetings. This graph reflects situations like small-talk in public transport or meeting in work with random person and discussing about electric vehicles. One important property of this graph is that it is generated in every step. It means that for simulation with 120 time steps, for every time step it looks different. This approach makes this graph more reliable as a reflection of standard's people behavior. To generate this network, we use Erdős-Rényi graph [17], also known as the random graph or binomial graph. It has one parameter r, $r \in \mathbb{R}$, which is the probability of the existence of an edge between two nodes. For simulation, we choose r as a random uniform variable from $r \in (0.0001, 0.001)$. The expected number of connections should be equal o the number of nodes multiplied by the probability. This network is presented in the figure 3.3 and it was created using the `fast_gnp_random_graph` function [16] from NetworkX library.

3.2.4 Social media network

The last network, which is used in our model, is connected to social media influence and other external factors (e.g. regulations). It is represented by a star graph, which means that there is one central node having edges to all remaining nodes. Outer vertices are connected only to that one node in the middle. The idea of this network is shown in the figure 3.4. It was created using the `star_graph` function [25] from NetworkX library. The final message from this node has only two states: being for EV with probability h, which means that the central node of social media has opinion -1 and otherwise being against EV with probability $1 - h$ and opinion 1. A star node interacts and sends its opinion about innovation to the remaining vertices.

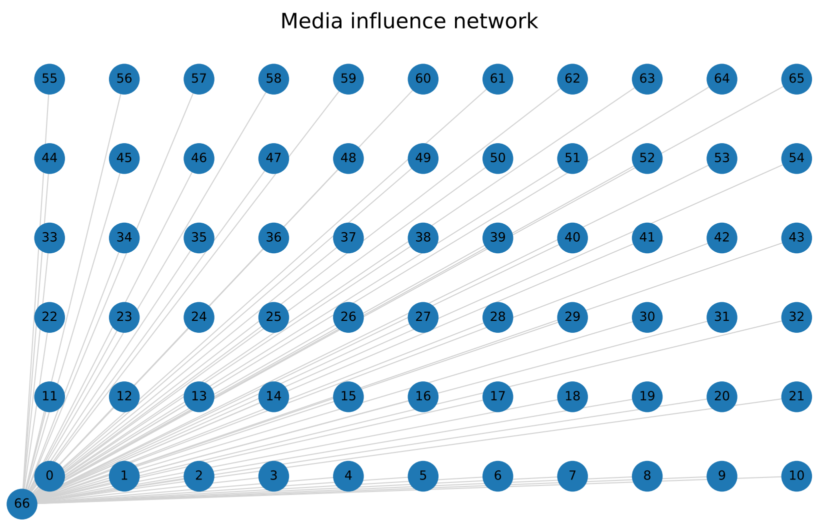

Figure 3.4: An example of social media influence network.

3.3 Algorithm

Considering all assumptions and properties of networks from this chapter, each time step within the simulation consists of choosing N agents independently and changing their opinion if some conditions are satisfied. The algorithm is as follows:

1. Generate temporary social interaction network as random graph with probability $r \in (0.0001, 0.001)$.
2. Determine an opinion of social media with probability h.
3. Repeat N times
   (a) Choose randomly one agent from network.
   (b) Decide with probability p, if the agent will act independently.
   (c) In case of independence, an agent changes state with probability 1/2.
   (d) In other case (conformity), choose q neighbors from all networks, sum their opinions and multiply by weight of considered graph:
      i. If weighted opinion > 0 - agent is against EV.
      ii. otherwise - agent is for EV.

In case of conformity, we need to take q neighbors from every network, it means that we consider q neighbors from the neighborhood, social interaction, temporary social interaction, and social media network. It creates a situation where from every level, we take a subgraph of the corresponding graph. Then, we are able to calculate the sum of opinions for each network and multiply by an adequate weight coefficient and then sum all these values to a variable weighted opinion. Based on this value, the agent decides if it adopts to innovation or declines this. However, this decision is made in every iteration, it means that in the next iteration or time step, it can adapt to this.

The algorithm presented above is an extension of the q-voter model [19]. It consists of two different types of behavior: independence and conformity. The first attitude lets the agent to randomly decide with probability p what does it think about this innovation. Conformity depends on a number of q neighbors and weighted opinion value, which is calculated using opinions in a current state multiplied by coefficients of networks. For this model, we consider $q = 4$ for the q-voter model as a good value to investigate sociophysical behaviors in opinion dynamics [24]. The model presented in this chapter is called the majority weighted model. The majority stands for a situation where a threshold to change opinion is located in the middle of possible opinions and weighted due to coefficients from considered networks.

3.4 HPC cluster BEM

To generate one Monte Carlo simulation with time $t = 120$, this model requires about 200 seconds using a medium-quality CPU. It is caused by a number of agents in created society - we consider more than 10 thousand agents. What is more, in every single time step, we consider as many MC events as the number of defined agents. In one iteration, we randomly choose one node from neighborhood network. It leads to the situation that in every time opinion of a single agent could be changed 10 thousand times as well as in a single time step every agent can change their opinion once. Finally, obtaining one trajectory of an average opinion (an average of 50 Monte Carlo simulations), it requires about 3 hours of computation for only one set of parameters.

Due to the long computational time of a single simulation, it would be almost impossible to investigate such an enormous set of parameters for this model using only a personal computer. To be able to perform many simulations, we used high-performance computing hardware, which the Wroclaw Centre for Networking and Supercomputing[10] at the University of Science and Technology in Wroclaw gave access to us. We used cluster BEM[1], which has 22 thousand computing cores with a total computing power of 860 TFLOPS.

To be able to effectively use its power and equipment, it is required to understand the concept and possibilities of parallel programming. It is a reasonable application of all computational power to optimize time and CPU usage by dividing simulations into smaller tasks, which can be run independently [28]. For those Monte Carlo simulations it is not difficult to separate tasks into numerous ones, so this problem could be described as an embarrassingly parallel problem[23]. This type of computation does not need any connection between partitioned tasks.

The majority weighted model is considered as an embarrassingly parallel problem if we divide one full-time simulation with N Monte Carlo steps into 50 independent runs. Due to this separation, we are able to obtain a lot of results in a short period of time. Finally, we obtain a list of many single trajectories, which are arrays of results from simulations, but we are able to add them into one array and divide by a number of all parallel Monte Carlo simulations [26]. As a result, we obtain the same average value as running all simulations in one approach.

Chapter 4

Results

Based on the model described in the previous chapter, we can run simulations for a specific set of parameters, which come from assumptions of this model. In this part of the paper, we focus on investigating following parameters:

- p - the probability of being a conformist by an agent,
- a - the weight of neighbourhood network,
- b - the weight of social interaction network,
- c - the weight of temporary social interaction network,
- d - the weight of social media network,
- h - the probability of positive influence by social media.

We will assume that sum of all weights of networks should be equal to 1, so $a+b+c+d=1$. This condition makes iterations through all variables harder than standard loops. To investigate as many parameters as possible, we choose deterministic weights of networks studying different parameter scenarios, which means cases with a higher impact of particular variables, equally distributed weights, or different configurations based on already seen results. Taking into consideration chosen weights of parameters, we iterate through different values of probability of independence p and influence by social media h. The exemplary set of parameters is shown in the Table 4.1.

p	a	b	c	d	h
0.1	0.25	0.25	0.25	0.25	0.5
0.37	0.6	0.3	0.05	0.05	0.25
0.45	0.1	0.7	0.1	0.1	0.5
0.25	0.15	0.5	0.3	0.05	0.75
0.6	0.25	0.25	0.4	0.1	0.25
0.3	0.25	0.25	0.1	0.4	0.75

Table 4.1: Exemplary sets of parameters used to simulate diffusion opinion about electric vehicles within our model.

4.1 Results

The basic characteristic to analyze the majority weighted model results is the magnetization, which is defined as a sum of opinions $S_1, S_2, \dots S_N$ of every agent divided by the number of nodes N in a single time step [15]:

$$m(t) = \frac{1}{N}\sum_{i=1}^{N} S_i(t).$$

Except for standard magnetization, we can consider final magnetization for a defined set of parameters. It is an average value of magnetization in the last time step of the simulation. This characteristic can describe the final state of the system, and based on it, and we can evaluate a number of electric vehicles when the simulation is finished. The greatest possible value of magnetization for every iteration is 1, which means that all agents in considered networks are against EV. On the other side, the smallest value could be -1, which stands for the complete acceptance of electric vehicles in society. A balanced situation is when the magnetization is equal to 0, which denotes that agents are not unanimous.

We simulated our model for more than 1000 sets of parameters. Every set of parameters consists of the probability of independence of agents, four coefficients of influence for every network, and the probability of social media opinion. Each trajectory is averaged over 50 independent Monte Carlo simulations. We used Monte Carlo simulation with trajectories divided into 120 time steps, where every step is deliberated as a single month. Such assumptions let these simulations predict adoption of opinions about EVs for the next ten years. In the figure 4.1 we can see examples of independent runs of one magnetization trajectory and an average value of magnetization for each time step.

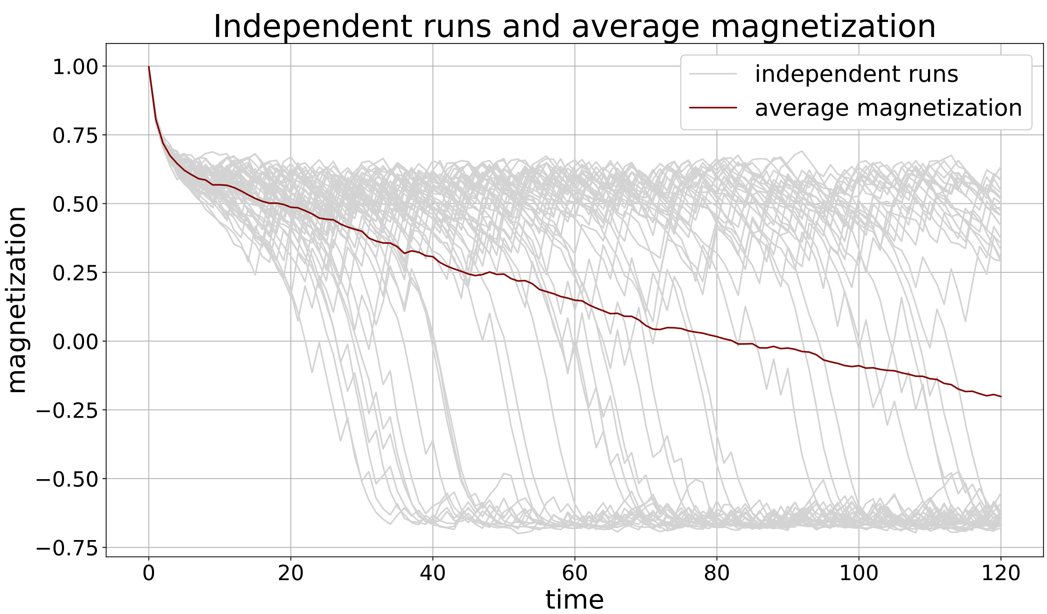

Figure 4.1: Simulation for 120 time steps with following parameters: $p = 0.3$, $q = 4$, $a = 0.6$, $b = 0.3$, $c = 0.05$, $d = 0.05$, $h = 0.75$. In the figure we can see independent runs for these parameters and an average values of magnetization in each time step.

In Fig. 4.2 we can see ten trajectories of average magnetization for randomly chosen sets of parameters from more than thousands of simulations. Investigating many trajectories like those presented in the graph lets us notice some dependencies between particular curves. Therefore in the next part of this paper, we present the classification of magnetization curves.

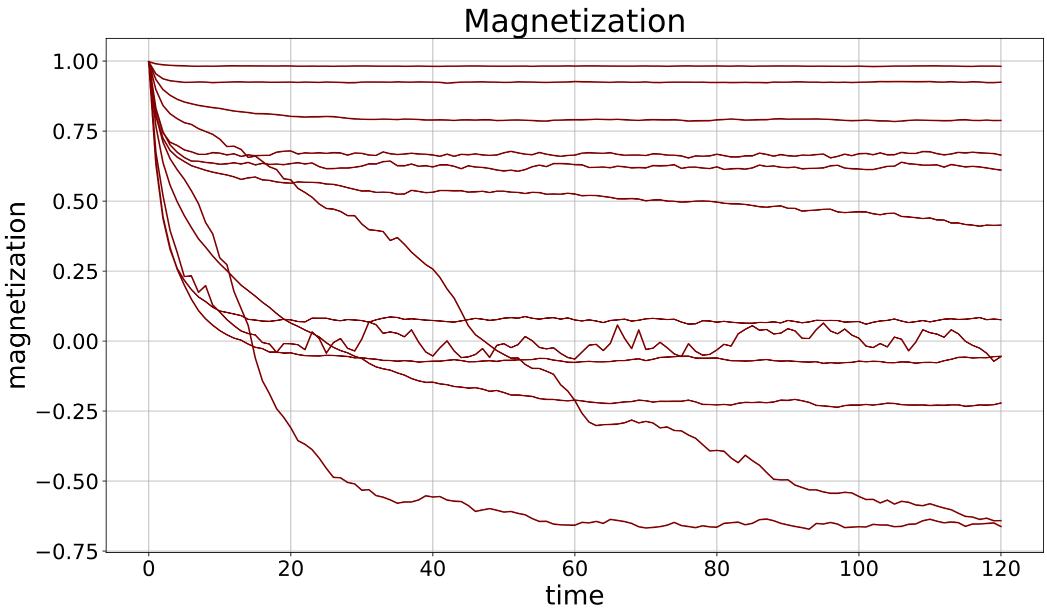

Figure 4.2: Exemplary magnetization trajectories of simulation for 120 time steps with different combinations of parameters.

4.2 Similarity classes

Analyzing more than 1000 sets of parameters, which part of them are presented in the figure 4.2, we were able to find some similarities in their behavior during the simulation time. We were able to identify the following similarity classes:

1. monotonic decrease of magnetization with p, slow convergence of some trajectories to steady states,
2. quick convergence to steady states,
3. irregular fluctuations,
4. intersections of some trajectories.

4.2.1 Slow convergence of some trajectories

In the first group, the parameters cause the behavior of magnetization as presented in the figure 4.3. We can see that for lower values of probabilities p, the system stabilizes almost immediately at a given level, and it is difficult for the society to make at least little changes in the opinion about EVs. On the other hand, if values of probability of independence are big enough ($p > 0.25$), we can see that the system becomes disordered and for all parameters, final magnetization fluctuates around 0. However, there are two trajectories that seem different than the remaining ones. For two values of p, $p = 0.15$ and $p = 0.2$, we can see that system tries to fight against the adoption of electric vehicles, but finally, the slope of the curve is negative, hence when we consider a longer period of time, it should approach to the zero as well. Both trajectories of magnetization could be good representatives of the kind of threshold values for which the system behaves in a defined way. Values below that threshold blocks diffusion of innovation however those above support innovation. A closer look at the behavior of the system upon the threshold is shown in Fig 4.4. In this figure we can see final magnetization as a function of probability. For small values of p ($p < 0.13$), most of the agents stay against the EVs. Then the magnetization drops quickly with p to fluctuate around zero for $p > 0.21$. This behavior can be interpreted as an order-disorder phase transition.

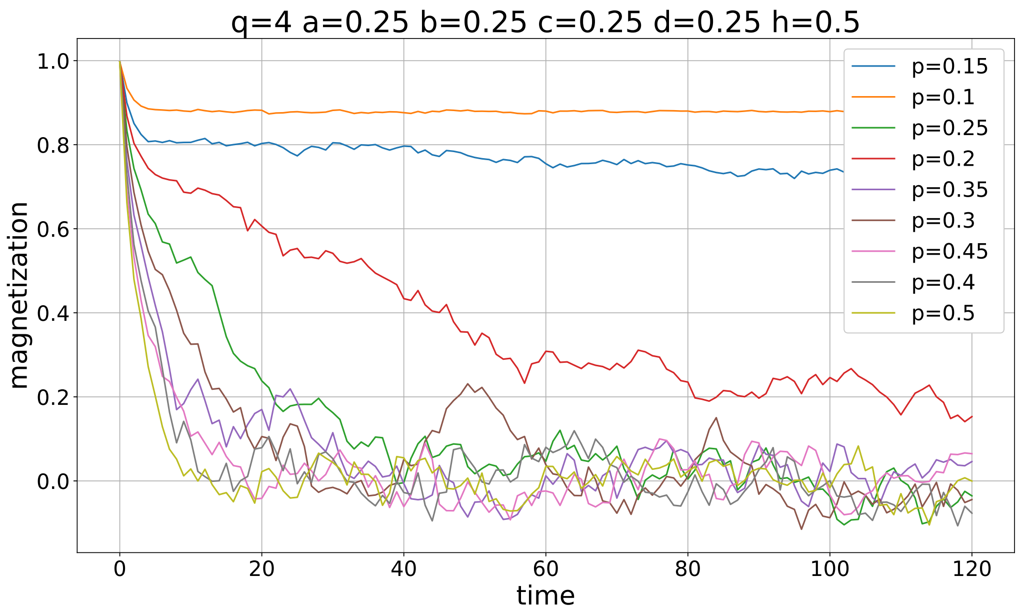

Figure 4.3: Trajectories of magnetization for slight slope trajectories similarity class - simulation for 120 time steps made for set of parameters: $q = 4$, $a = 0.25$, $b = 0.25$, $c = 0.25$, $d = 0.25$, $h = 0.5$ and different values of p.

In the table 4.2 we described more sets of parameters, which produce similar output to the one in the figure 4.3. Looking at values of this table, we can notice that this class of trajectories is present mostly for relatively high values of coefficients a and b of weighted opinion model. The higher value of a or b means the opinion taken from these networks is considered more influential than from others. From table 4.2 we can see that one more characteristic, which is connected with the value of the probability of a social media opinion h, is important. In this similarity class, we have $h = 0.5$. It could be the reason why all trajectories above the threshold $p = 0.2$ finish nearby 0.

This similarity class could represent society, where a is strongly dependent on the probability of being a conformist by an agent with also big influence of social media opinion. Its threshold value of $p = 0.18$ for one specific set of parameters can describe slow adoption to innovation based on neighborhood opinion and social interaction connections. It shows that these networks are the crucial ones and the most influential for the weighted model. However, the final value of magnetization is controlled by a parameter h, which in most cases is equal to a half. Due to such a value of a parameter, sooner or later the system is forced to reach a disorder phase and fluctuate around zero.

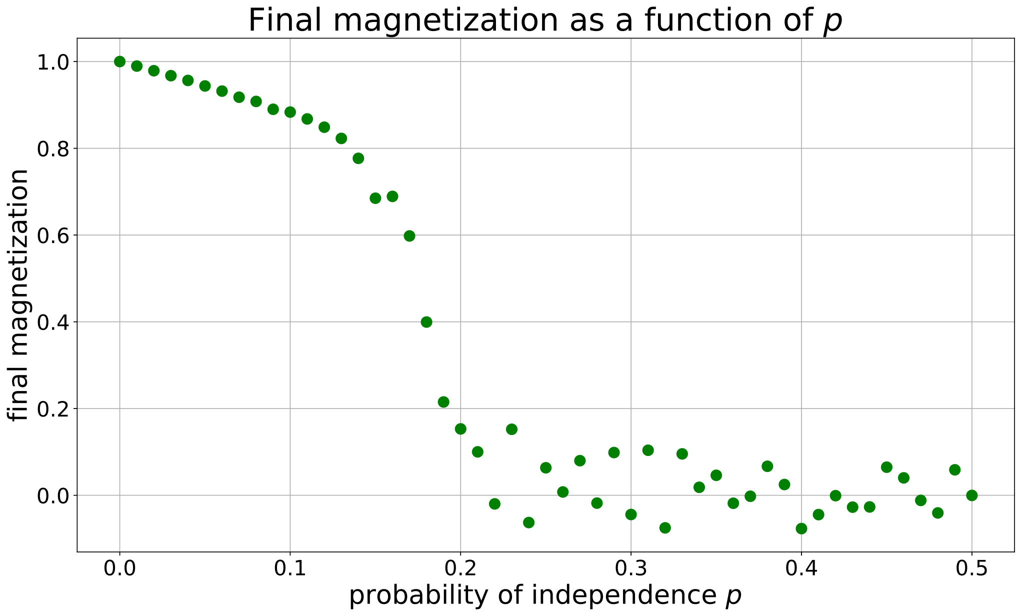

Figure 4.4: Final magnetization as a function of p for set of parameters: $q = 4$, $a = 0.25$, $b = 0.25$, $c = 0.25$, $d = 0.25$, $h = 0.5$.

a	b	c	d	h
0.15	0.5	0.3	0.05	0.5
0.3	0.5	0.1	0.1	0.5
0.5	0.3	0.1	0.1	0.5
0.7	0.2	0.05	0.05	0.5
0.2	0.7	0.05	0.05	0.5

Table 4.2: The similarity class of slowly convergent trajectories.

4.2.2 Quick convergence to steady states

This group is characterized by magnetization being almost constant - except a short initial phase. Trajectories from this group approach the steady state almost immediately. An example of this similarity class is presented in the figure 4.5. From this plot, we can observe that all trajectories are set in order from the low probabilities p to the higher values. Hence, it is the most influential parameter for this type of graph. Based on the probability of being a conformist, we can estimate the level of adoption of EVs in society.

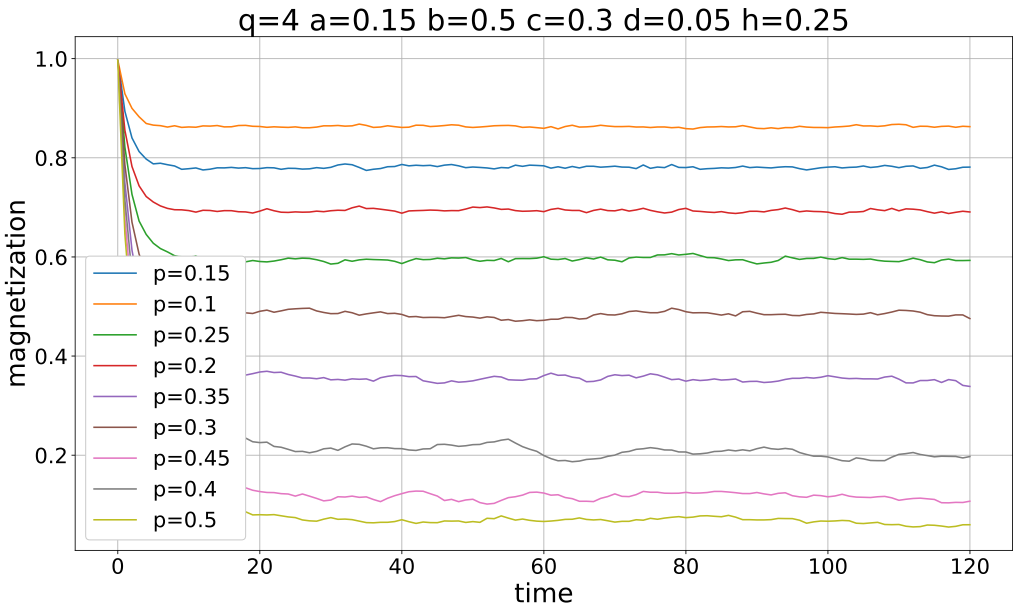

Figure 4.5: Trajectories of magnetization for quick convergence similarity class - simulation for 120 time steps made for set of parameters: $q = 4$, $a = 0.15$, $b = 0.5$, $c = 0.3$, $d = 0.05$, $h = 0.25$ and different values of p.

This situation states that only through the independence of agents adoption of EV can immediately explode, and then the adoption level stays constant. For the next time steps, magnetization could not be changed because there are not enough fluctuations in the system to spread an opinion. The final opinion is presented in the figure 4.6 as a function of the probability p. Other parameters do not seem to have a high impact on the outcome, but they may change it just a bit. We can analyze the influence of all parameters to observe what happens if any parameter is modified. To illustrate that, we choose coefficient b, which stands for the weight of social interaction network. In the figure 4.7 we can see a final magnetization as a function of b. Due to assumptions described in the previous chapter, all weights of networks a, b, c and d sum to one, hence we take constant values of c and d, such that $c = 0.3$ and $d = 0.05$, and modified values of a based on value of coefficient b such that $b + a = 0.65$. For this situation, we investigate final magnetization as a function of $p(b)$ being aware that changing b involves modification for a parameter.

Figures 4.6 and 4.7 show final magnetization as a function of probability of independence p and the weight of social interaction network b. From the first graph, we can see that the higher value of probability p, the lower value of final magnetization. The dependence between p and final magnetization is almost linear, hence it makes this model highly dependent on a parameter p. We can conclude that society, which is not independent, has some limitations in spreading positive opinions for electric vehicles. The figure 4.7 represents the relation between the coefficient of the social interaction graph and final magnetization. It is visible that difference for two extreme values of b, $b = 0$ and $b = 0.5$ is not as big. Hence the parameter b has not much impact of the final magnetization especially for $b < 0.35$. For $b \geq 0.35$ we can observe a small decrease of magnetization with b, which enlarges the amount of people who are for electric vehicles.

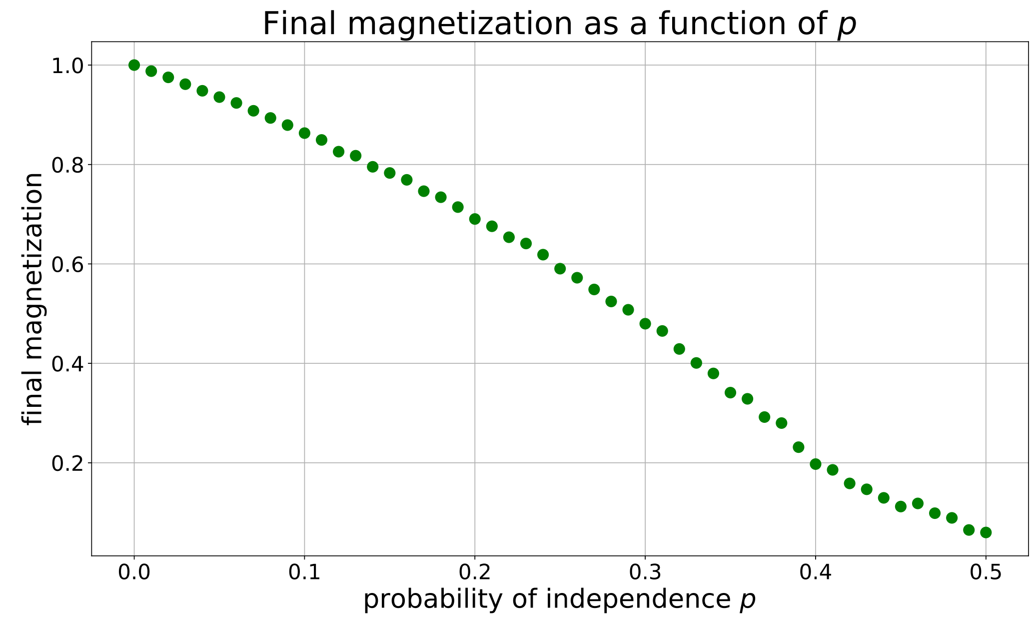

Figure 4.6: Final magnetization as a function of p for set of parameters: $q = 4$, $a = 0.15$, $b = 0.5$, $c = 0.3$, $d = 0.05$, $h = 0.25$.

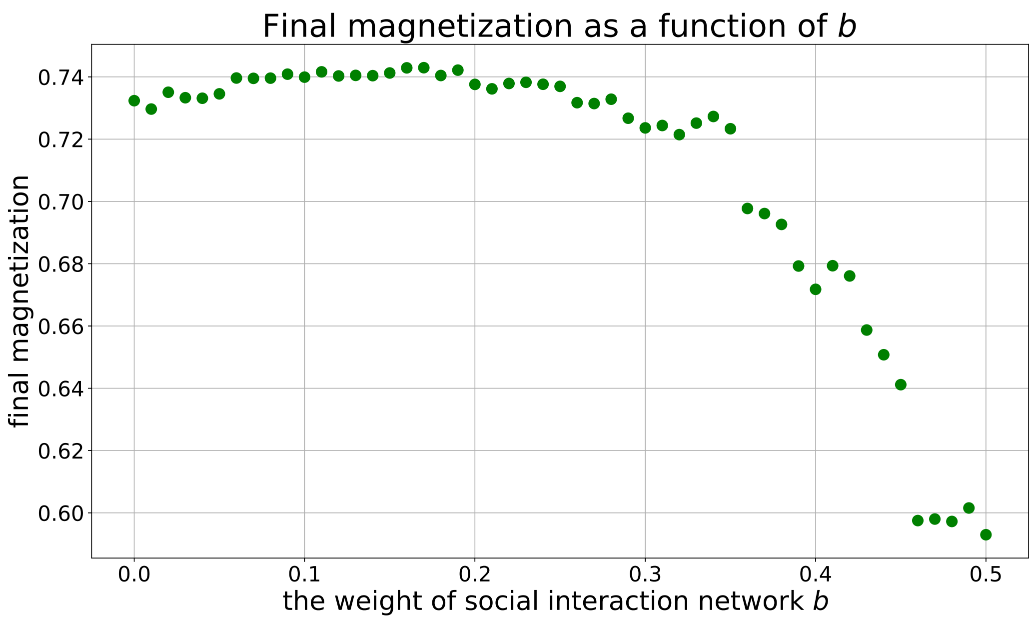

Figure 4.7: Final magnetization as a function of b for set of parameters: $q = 4$, $p = 0.25$, $c = 0.3$, $d = 0.05$, $h = 0.25$.

Example parameter values typical for this class are presented in the table 4.3. Analyzing previous figures, we concluded that the most important parameter for this class of similarity is probability p, but in this table, we can notice one important feature. Mostly, to obtain almost horizontal lines, we have $a + b > c + d$, and usually, it goes with a value of

$h = 0.25$. It means that the lower impact of social media node in this model restricts further fluctuations, and as a result, it blocks the diffusion of innovation.

a	b	c	d	h
0.15	0.5	0.3	0.05	0.25
0.3	0.5	0.1	0.1	0.25
0.5	0.3	0.1	0.1	0.25
0.25	0.25	0.4	0.1	0.5
0.35	0.3	0.2	0.15	0.25
0.1	0.7	0.1	0.1	0.25

Table 4.3: The table presents exemplary configurations of sets of parameters, which belongs to the horizontal lines similarity class.

4.2.3 Irregular fluctuations

The next class of similarity is characterized by quick drops of magnetization to a given value and its large irregular fluctuations around this value. This group is highly dependent on social media influence because the corresponding parameter d is high. Exemplary trajectories of this similarity class are presented in the figure 4.8.

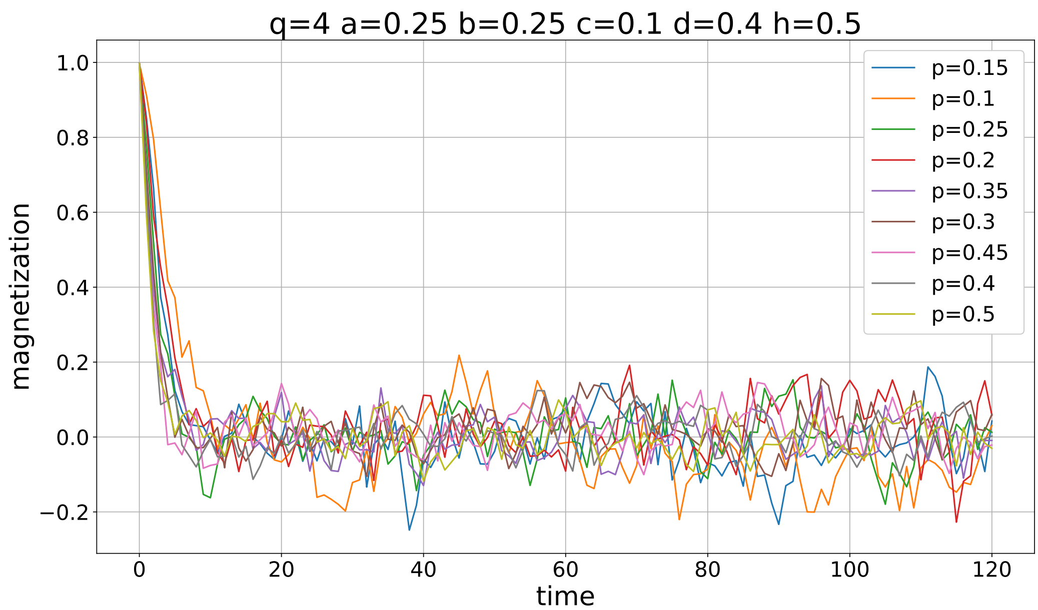

Figure 4.8: Trajectories of magnetization for irregular fluctuations similarity class - simulation for 120 time steps made for set of parameters: $q = 4$, $a = 0.25$, $b = 0.25$, $c = 0.1$, $d = 0.4$, $h = 0.5$ and different values of p.

The weight of the social media network describes how much this graph influences the agents' opinions. If d is the highest parameter from all weights of networks, it has the biggest impact on the opinion of an agent. If social media network is considered as the most influential, we should expect the parameter h to be important as well. As a reminder - based on the value of h, the social media node in the star graph decides if it is for or against EVs. A higher value of h means that social media promotes this innovation. It can be visible in the figure 4.9. There are three different types of magnetization for chosen value of $h = 0.25$, $h = 0.5$ and $h = 0.75$. The middle value of h means that social media is for/against EVs half a time value of social media vertex is generated. Value $h = 0.25$ stands for a situation when an opinion of social media node is rather negative, and finally $h = 0.75$ for a positive impact of diffusion of EV.

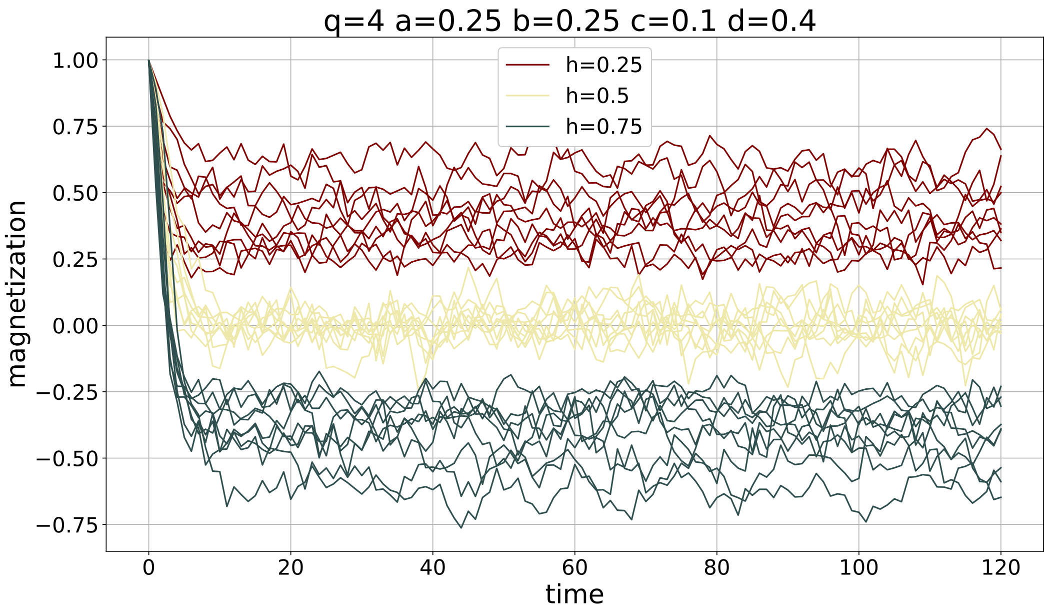

Figure 4.9: Trajectories of magnetization for irregular fluctuations similarity class - simulation for 120 time steps made for set of parameters: $q = 4$, $a = 0.15$, $b = 0.5$, $c = 0.3$, $d = 0.05$ and different values of p and $h = 0.25, h = 0.5$, $h = 0.75$.

Due to the high impact of the coefficient d in this class of similarity, we can measure final magnetization as a function of d. Being aware of the assumption that $a + b + c + d$ needs to be equal one, we take parameters in a way that coefficient d would be the highest. This situation is presented in the figure 4.10. Again, we consider three different values of h to visualize how final magnetization depends both on coefficient d and parameter h.

From graph 4.10 we can see that different values of d do not make significant changes in final magnetization. All values stabilize at similar point as in the figure 4.9 for chosen h. What we can notice from these points is a tendency for $h = 0.25$ and $h = 0.75$, which makes final magnetization closer to the equilibrium point for a higher value of the coefficient of social media network d. Meantime, when we consider $h = 0.5$ for larger values of d we can observe higher fluctuations of final magnetization, which makes this model more unpredictable, but finally, all values end nearby point $m = 0$.

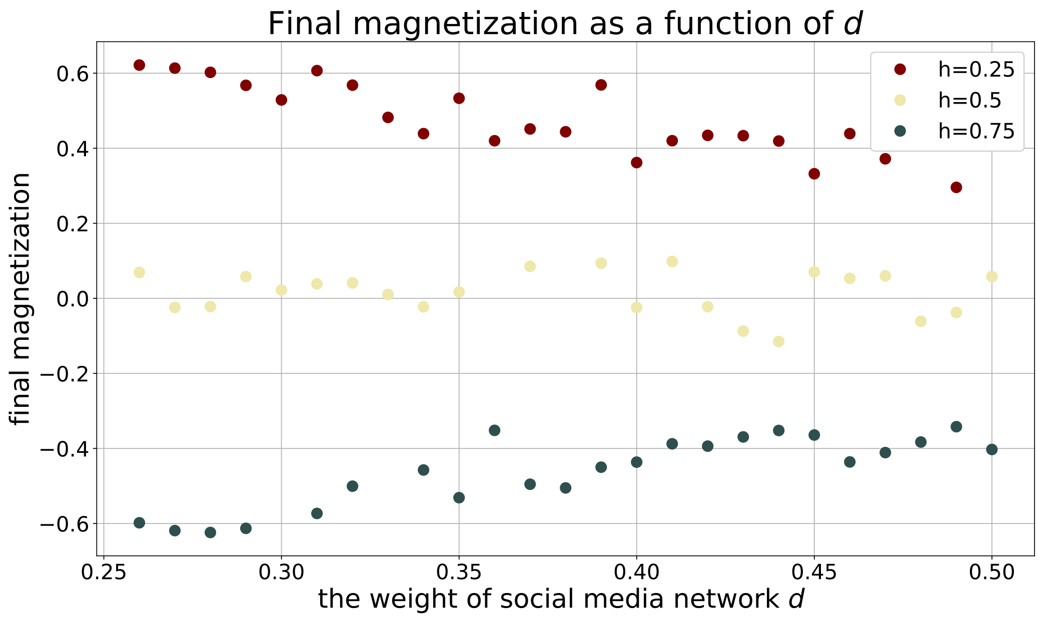

Figure 4.10: Final magnetization as a function of d for set of parameters: $q = 4$, $p = 0.25$, $c = 0.3$, $d = 0.05$ and $h = 0.25, h = 0.5$, $h = 0.75$.

This class of similarity is extremely dependent on h. Almost at the beginning of the simulation, curves rapidly drop, stabilize at a specific level and do not change their magnetization till the end of the simulation. Irregular fluctuations describe situations when opinion about electric vehicles constantly changes due to some funding by the government, news read in magazines or social websites or the necessity of changing standard vehicles for those that used alternative fuel vehicle. Converging to the defined level appears for all values of probabilities, which means that in this group of trajectories, independence does not impact people as much as social media. We could imagine a situation when all famous people publish posts about the advantages of having an electric vehicle.

4.2.4 Intersections of some trajectories

Among all sets of parameters, there are some values for which we can observe behavior not covered by the previous classes. These trajectories are shown in the figure 4.11.

Firstly, considering probabilities p we can see that for lower values of this parameter $p \in (0.1, 0.25)$ the magnetization behaves like in the quick convergence group. It immediately drops to a value around $1 - p$ with an additional influence of other parameters, which raise or lower a trajectory by a small amount. Then, as for the slow converging, there are some values of p for which we can see a smooth decrease in time to a specific level based on other parameters in the considered set. What is more, for $p > 0.35$ we can see some unexpected behavior in this model. The trajectory for $p = 0.35$ crosses other higher values of probability of independence. Finally, all trajectories stabilize at some point, however their order is not anticipated here. To better understand what has been done in these simulations, let us consider a figure 4.12, in which we can see the same simulations but for a higher value of p.

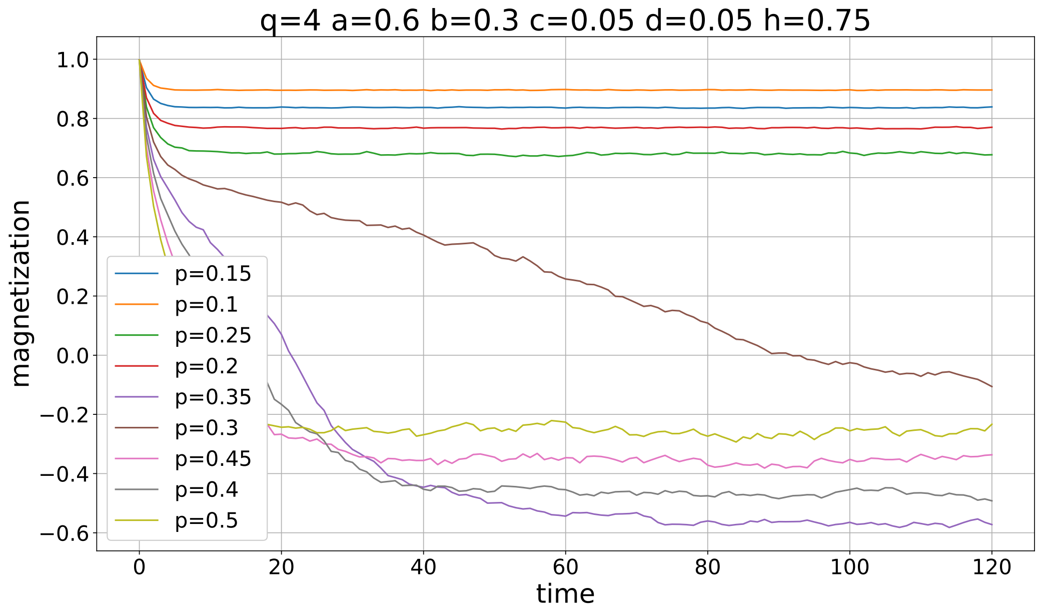

Figure 4.11: Trajectories of magnetization for horizontal lines similarity class - simulation for 120 time steps made for set of parameters: $q = 4$, $a = 0.6$, $b = 0.3$, $c = 0.05$, $d = 0.05$, $h = 0.75$ and different values of p.

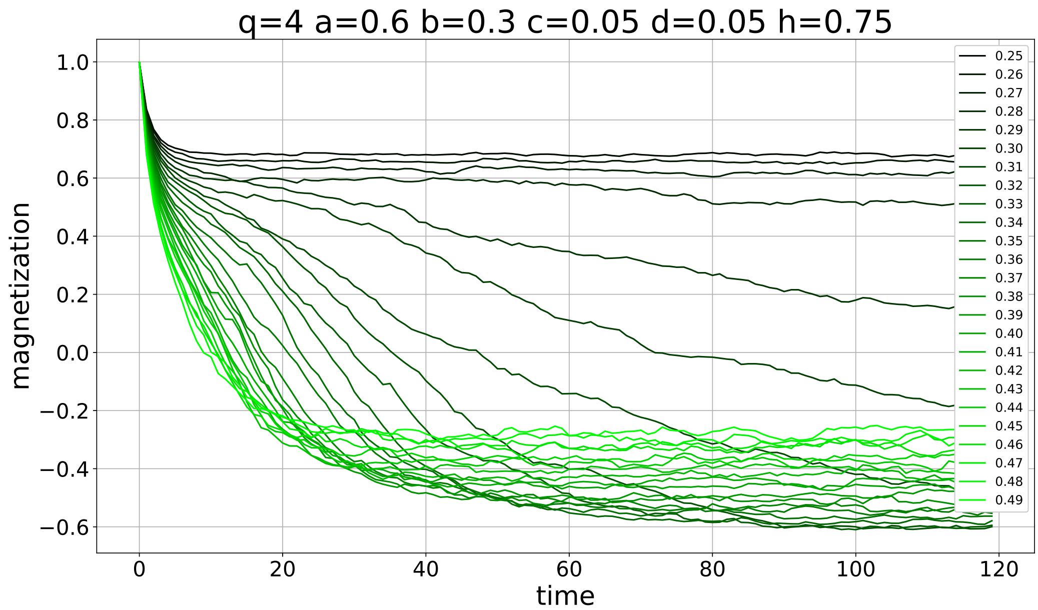

Figure 4.12: Trajectories of magnetization for horizontal lines similarity class - simulation for 120 time steps made for set of parameters: $q = 4$, $a = 0.6$, $b = 0.3$, $c = 0.05$, $d = 0.05$, $h = 0.75$ and different values of p.

From this figure, we can see that for lower probability $p < 0.29$, magnetization stabilizes at a high level, which reflects the situation when the final number of electric vehicles has been reached in the society and then it is impossible to enlarge a number of them. This situation blocks the diffusion of innovation on a specific level. However, increasing the value of p such that $p \in (0.29, 0.35)$ makes few trajectories crawl to their stable point. For all these values of the probability p there are intersections with trajectories in which the probability of independence is higher. Due to the restricted time of simulation and consideration of how middle trajectories will develop in the future time steps, we made simulations for the same set of parameters, which last three times longer. Magnetization of the longer case is shown in the figure 4.13. We can notice from this figure that for some parameters, it is still not enough time for trajectories to arrive at a steady state. What is more, trajectories which did not stabilize for $t = 120$, in the figure 4.13 of longer simulation for time $t = 360$ reach a stable level below every previously balanced ones. Lastly, all values of p which are in the interval $p \in (0.4, 0.5)$ finish their slope nearby value of magnetization $m \in (-0.25, -0.35)$. This situation creates a higher value of electric vehicles, but their amount after stabilization does not change to time $t = 360$.

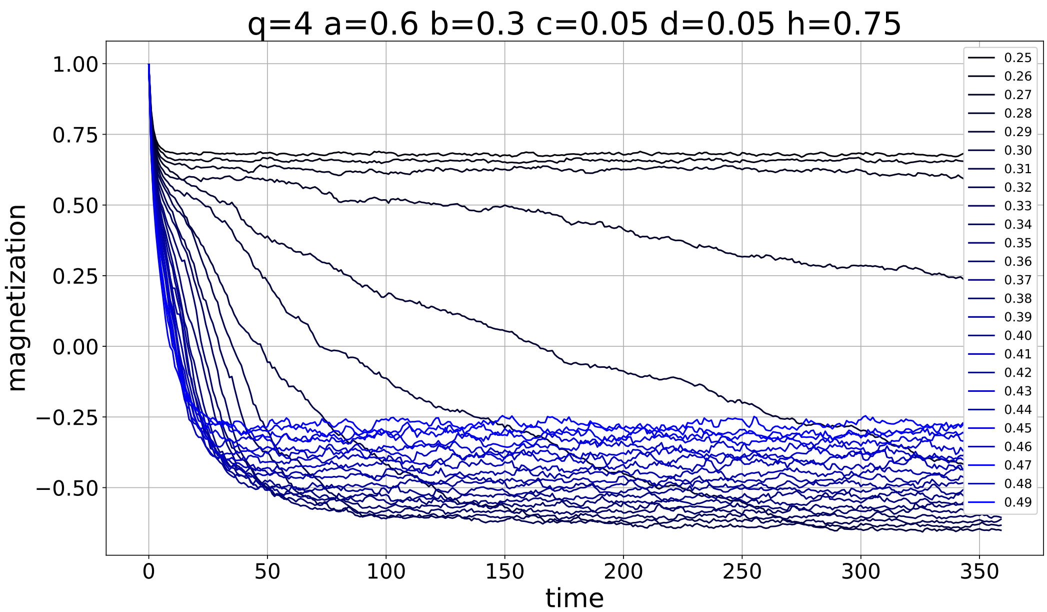

Figure 4.13: Trajectories of magnetization for horizontal lines similarity class - simulation for 360 time steps made for set of parameters: $q = 4$, $a = 0.6$, $b = 0.3$, $c = 0.05$, $d = 0.05$, $h = 0.75$ and different values of p.

For this case, it would be interesting to look at the final magnetization as well. It is presented in the figure 4.14 both for standard simulation $t = 120$ and longer ones $t = 360$. We can see that for the first values of probabilities p, it behaves exactly the same no matter of length of simulations. The significant changes are present for $p \in (0.27, 0.31)$ - it stands for trajectories that need more time to reach their stable level. All higher values of the probability of independence p behave the same no matter of the number of time steps. It is worth to mention that if we consider absolute values of final magnetization, which is the last value of every trajectory of magnetization, we obtain a similar order of trajectories as probability p. The absolute value of magnetization is shown in the figure 4.15.

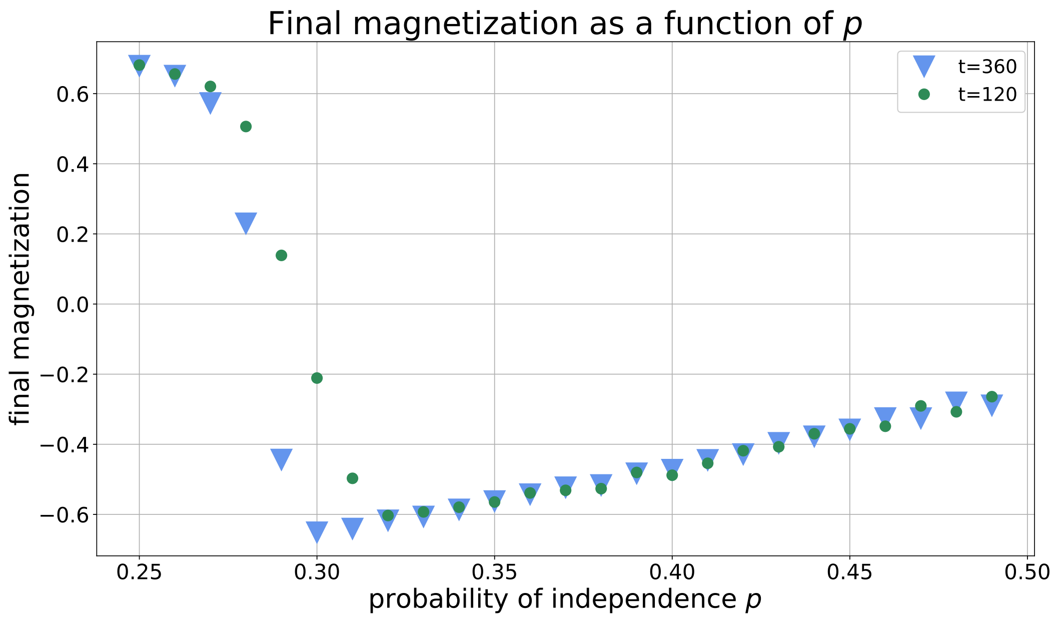

Figure 4.14: Final magnetization as a function of p for set of parameters: $q = 4$, $a = 0.6$, $b = 0.3$, $c = 0.05$, $d = 0.05$, $h = 0.75$ for $t = 120$ and $t = 360$.

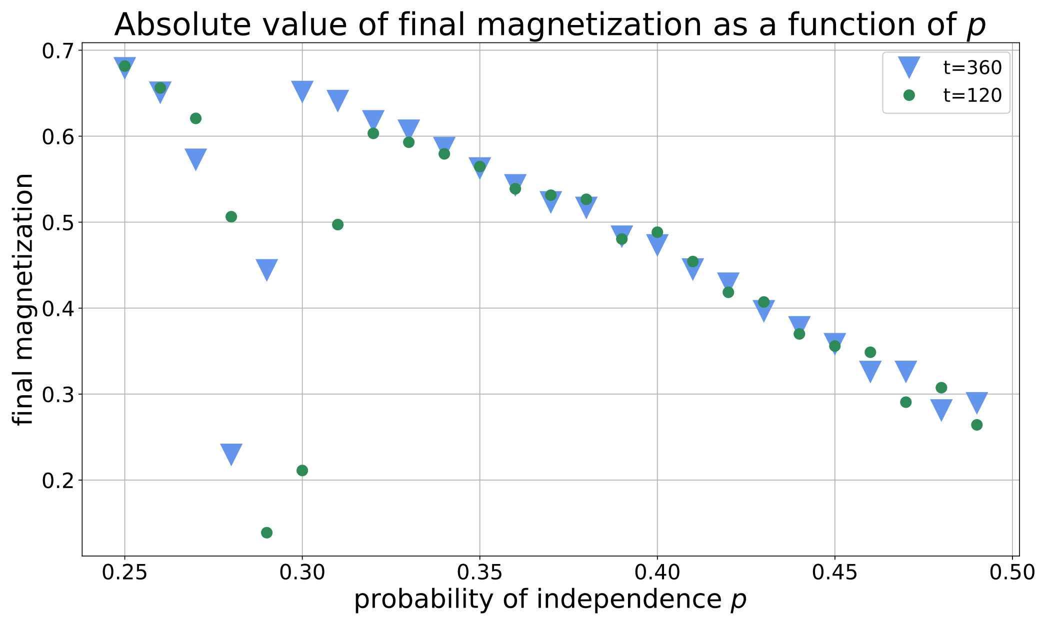

Figure 4.15: Absolute value of final magnetization as a function of p for set of parameters: $q = 4$, $a = 0.6$, $b = 0.3$, $c = 0.05$, $d = 0.05$, $h = 0.75$. for $t = 120$ and $t = 360$.

Based on the plot in the figure 4.15, we can see that a v-shape valley for simulation $t = 120$ is wider than for $t = 360$. It is caused by the length of the simulation, for the longer one, the trajectory has enough time to reach its steady state, e.g., as for probability $p = 0.3$, 120 time steps were not enough for this variable to stabilize, but 240 time steps more solve this problem. This v-shape valley for both values of t lead to the conclusion that in the even longer simulation such that for $t = 360$, it is possible to obtain a standard linear function of the absolute value of final magnetization as a function of p. Nevertheless, such long simulations require more computational power and can be a good candidate for future research.

a	b	c	d	h
0.6	0.3	0.05	0.05	0.75
0.5	0.3	0.1	0.1	0.75
0.15	0.5	0.3	0.05	0.75
0.8	0.1	0.05	0.05	0.75
0.25	025	0.4	0.1	0.75
0.35	0.3	0.2	0.15	0.75

Table 4.4: Exemplary sets of parameters, which belongs to the intersections similarity class.

In the table 4.4, we can see more sets of parameters, which have similar intersections for part of trajectories as in the figure 4.11. Based on all simulations, which create comparable shapes, we can notice a common high value of $h = 0.75$. Other parameters do not impact magnetization significantly, however the probability of independence p decides if intersections appear or not. This set of parameters and their behavior should be investigated in a more accurate way, and it could be an interesting starting point for future research and development of this model.

Chapter 5

Summary

5.1 Conclusions

In this paper, we investigate the adoption of innovation, i.e. is the diffusion of electric vehicles. In the beginning, it was required to create a mathematical and sociophysical model, which has a lot of assumptions based on real-world values like the initial number of electric vehicles or the degree of urbanization in Poland. These assumptions are necessary points of every model, but we need to remember that due to these limitations, some proposed solutions of the majority weighted model could be incorrect or unequally weighted compared to real life. When the final model to investigate the diffusion of adoption was designed, thousands of simulations were made to analyze obtained curves of magnetization, find similarities between them and classify them. Finally, we were able to divide all magnetization into similarity classes and detect similarities between magnetization, then, by manipulating some parameters, it was possible to understand more how this model behaves and how much sensitive it is. Based on similarity classes defined in this thesis, we can divide our conclusions into three different scenarios of the adoption of electric vehicles in Poland.

First scenario

All the slowly convergent trajectories , the quick convergent ones and those with irregular fluctuations have a similar shape for values of $p < 0.2$. All trajectories stabilize almost immediately, and due to little fluctuations, it is impossible for magnetization to significantly change its values. It reflects a situation when the number of electric vehicles reaches a specific level and then, no other people want to adapt to this innovation. Final magnetization is reaching to value around $1 - p$ with some changes due to values of the other parameters.

This scenario is highly dependent on the probability p, which signifies an opportunity to change the opinion of innovation. Stabilization at a given level means difficulties in convincing society to use electric vehicles. Numerous drawbacks of electric vehicles could cause it. It could represent a situation when different alternative fuel vehicles become popular, and society decides to adapt to a new alternative instead of electric vehicles. However, there are still people who like the solution with electric motors and firmly think EVs are a great choice. Even when some people get discouraged, there are some new customers who want to try this innovation. Nevertheless, electric vehicles will not become the most common type of vehicle, and their sale and usage will not increase.

Second scenario

The second scenario is the most controversial because it reflects a situation when people in a short period of time adjust to the same level of distribution of electric vehicles no matter of their independence. This scenario relates to the irregular fluctuations similarity class. In this case it is important that the probability of being a conformist does not influence the shape of curves. It corresponds to the situation when people are forced to change their standard vehicles to different alternative fuel vehicles immediately - for instance, due to some acts or laws, which restrict usage of standard motor engines. Hence, no matter of probability of independence, people would be obligated to make a change. Here, the opinion of social media h is important - the higher value of h, which means that social media are for electric vehicles stands for bigger interests of this type of motor as an alternative fuel. As in the figure 4.9, the value h decides about the final magnetization, which is responsible for the level of adoption of electric vehicles.

We can consider if this scenario is really possible. In last year Europen Commission published “Fit for 55” article[9], which regulates production of greenhouse gases of at least 55% by 2030. These regulations are a continuation of a European Green Deal published in 2019, which also focuses on reducing greenhouse gases. Due to these limitations, every country in the European Union should try to meet the imposed standards. What is more, in recent weeks, the European Parliament has voted to ban combustion engine cars from 2035 as a part of “Fit for 55” package. In Brussels, 9 June 2022, members of the European Parliament decided to restrict targets for reducing CO2 for cars and vehicles [4]. Such restrictions imposed by governments are a good example of this scenario, especially if people do not have enough time to adjust to the situation and need to make changes immediately. Luckily, these laws will take effect in more than 10 years, so society has enough time to adapt to new solutions.

Third scenario

The last scenario is connected to the slow adoption of electric vehicles in time. In the figure 5.1 we collect exemplary sets of parameters, which show an unhurried increase of electric vehicles. This scenario is a mix of two similarity classes: slowly convergent for middle values of probability of independence p and trajectories with intersections, which also adjust to this scenario.

The first visible feature of this scenario is the steep slope at the beginning, which is required for the future slow adoption of electric vehicles. The large changes in the first steps of simulation create possibilities for evolution in the next part of trajectories. Then, if in the system there are enough people with a positive opinion about electric vehicles, it can be widely spread in following steps. However, since time $t = 5$ we can notice a slow adjusting to innovation. In the last part of simulation, we can see a slowdown and stabilization to a specific level. At the end for $t = 120$, magnetization has a value nearby $m = -0.3$, which means that majority of vehicles used are electric ones, which stands for about 65% of all vehicles in Poland.

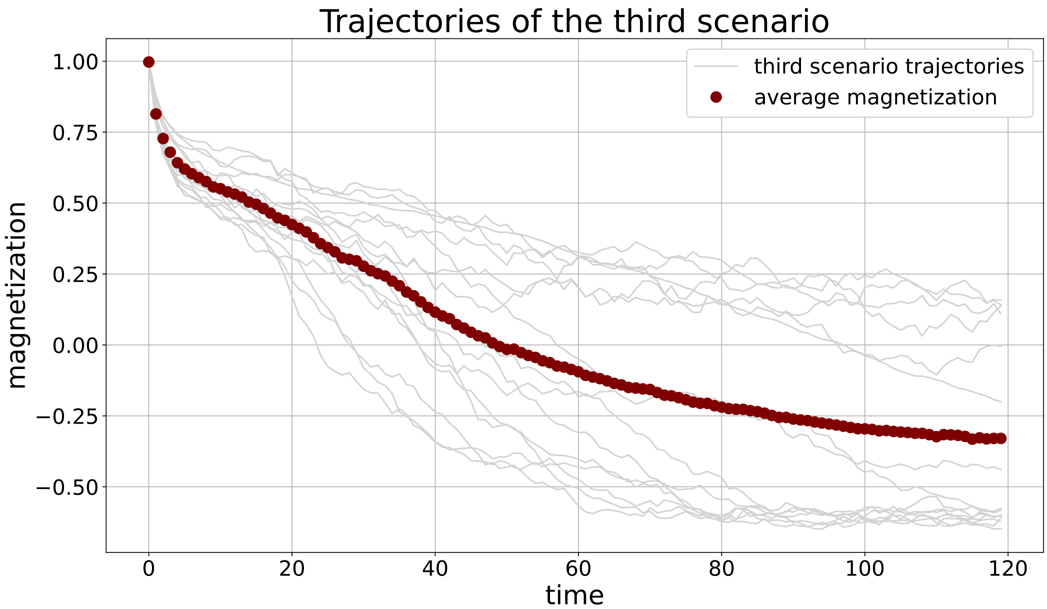

Figure 5.1: Few trajectories, which satisfy conditions of the third scenario and an average value of magnetization.

This percentage of electric vehicles is possible only when some additional conditions are satisfied. As in the second scenario, it could be reasonable if the regulation from the European Union will be slowly applied to the community. It reflects situations in which people are aware of future restrictions and they want to prepare for such limitations from governments. The second condition is the fact, that in the environment, there will not appear any different and better substitute to combustion motors than electric ones. Therefore, people will require to buy electric cars and use them for everyday life. It would be also valuable, when all disadvantages of electric vehicles are solved, their maximum range is the longer, the service life of battery relatively high, and well-placed infrastructure of electric chargers.

5.2 Future works

Parameters

In our model, we consider the influence of 6 parameters, which is the probability of independence p, weights of all proposed networks a, b, c and d and probability of positive influence by social media h. However, this model could have be improved or modified in various ways. Firstly, the size q of the influence group in the q-voter model was set to 4. However, it would be interesting to investigate the impact of other values for q. Except of the conformity value, it is possible to add more layers in the multi-layer network model, which means that in future research, we can find more relationships between agents and describe those within a network. The number of considered graphs could be greater than 4 with no limitation to the maximum value. The only remark is that it enlarges the time of a single simulation.

Unexpected trajectories

The next topic to investigate is the last presented similarity class - unexpected intersections. The shape of this group of trajectories and their crossings are a good starting point for further investigation. We have found that these curves appear when the probability of social media opinion is equal to $h = 0.75$, however it is not always a necessary condition. It would be good to investigate and decide what parameters have an additional impact on those trajectories.

It would be worth to calculate the final magnetization, but for significantly longer simulation times. In the figure 4.15, we can see that enlarging time in simulation, lowers the width of the valley between $p \in (0.25, 0.30)$. Hence, it is probably that for these types of trajectories, more time is needed to stabilize the system, but it is difficult to define how much time they need and what would be the final magnetization value. When we consider standard final magnetization figure 4.14 it would be good to focus on that steep slope for probability of independence $p \in (0.25, 0.30)$. Additional analysis would cover the slope of that straight line when this final magnetization immediately drops and determine the slope in comparison to the time of simulations.

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