



Generative Adversarial Networks (GANs)

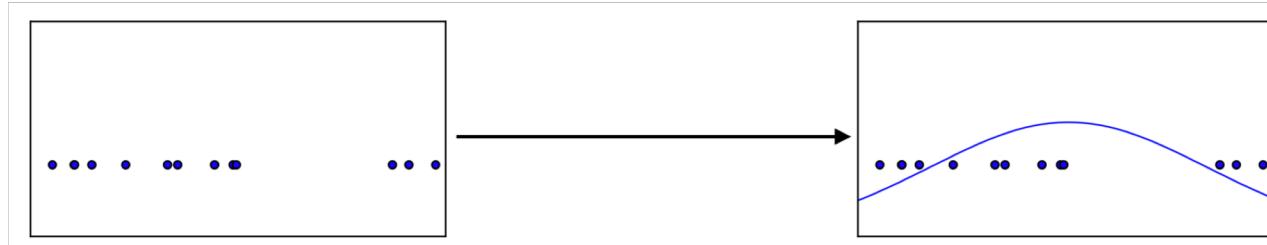
Javier Montoya (montoya@ethz.ch)

Arequipa, 9th January 2019

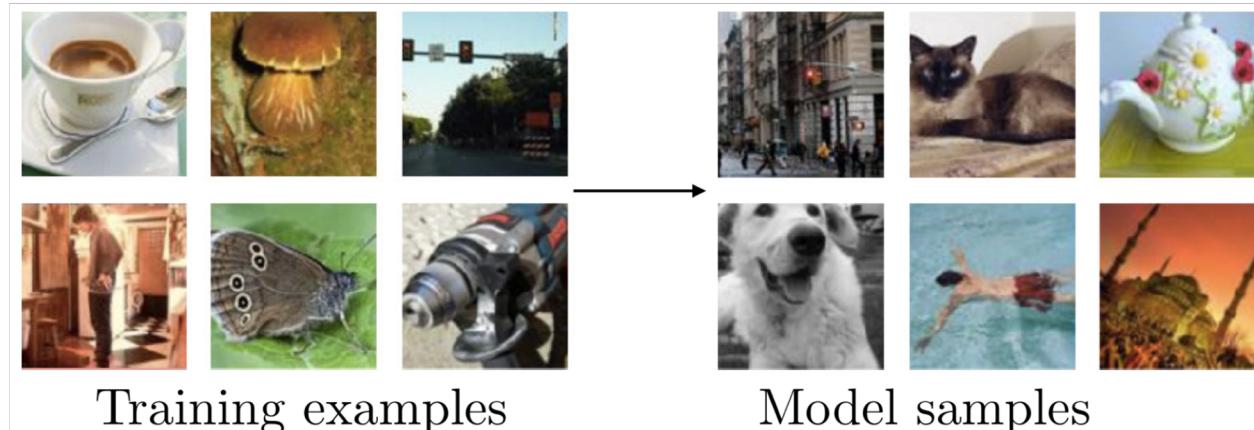
What is a generative model?

- **Key Idea:**

- It cares about what distribution $P_{\text{data}}(X)$ generated the data X



- It generates new samples from the learned distribution $P_{\text{model}}(X)$.



See NIPS 2016 Tutorial: Generative Adversarial Networks (I. Goodfellow)

Generative vs. Discriminative models

- **Discriminative Models:**

- Given an image \mathbf{X} , predict its label \mathbf{Y} .
- Estimates $\mathbf{P}(\mathbf{Y}|\mathbf{X})$.

- **Limitations of discriminative models:**

- Can't model $\mathbf{P}(\mathbf{X})$, i.e. the prob. of seeing a certain image \mathbf{X} .
- They can't sample from $\mathbf{P}(\mathbf{X})$, i.e. can't generate new images.

Why generative models?

What are some recent and potentially upcoming breakthroughs in deep learning?

 Answer

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Yann LeCun, Director of AI Research at Facebook and Professor at NYU
Answered Jul 28, 2016 · Upvoted by Joaqin Quiñonero Candela, studied Machine Learning and Gokul Krishnan, M.Sc Computer Science & Machine Learning, ETH Zurich (2018)

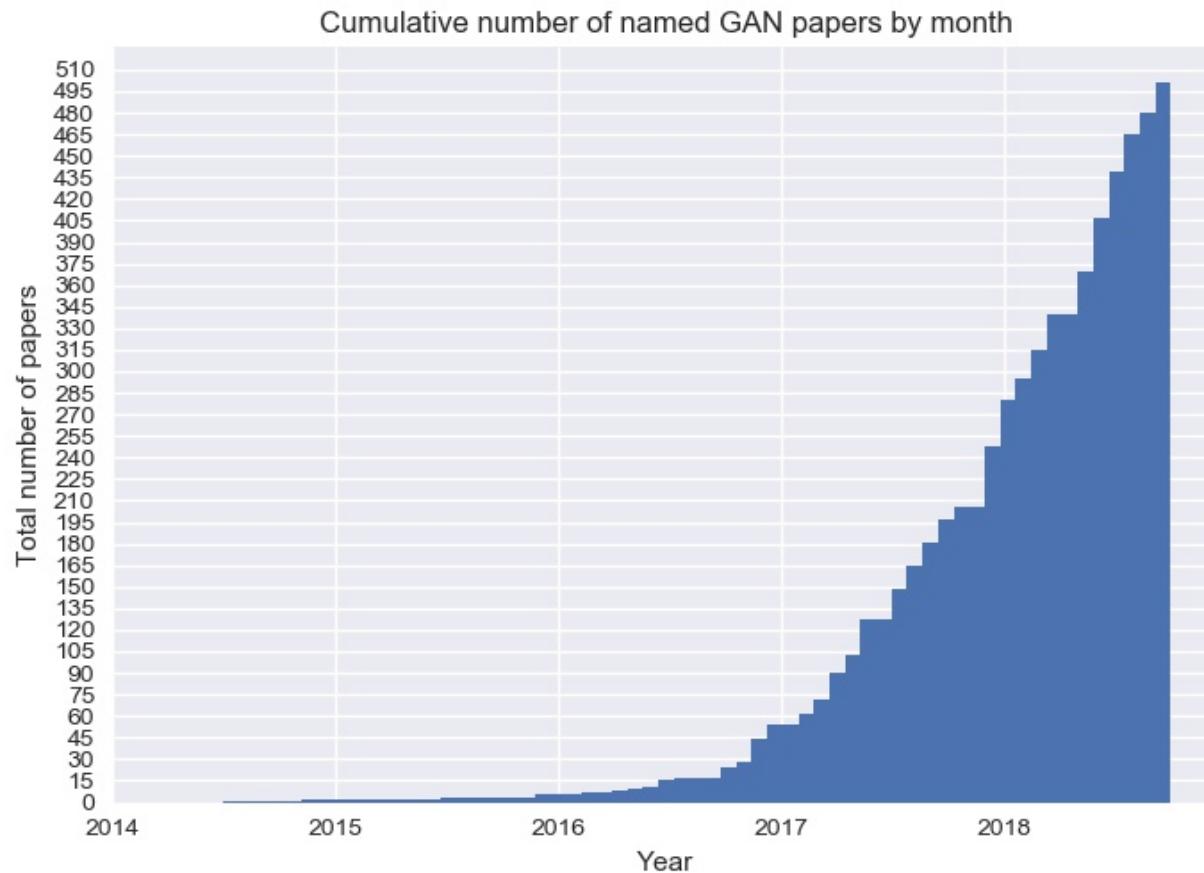


There are many interesting recent development in deep learning, probably too many for me to describe them all here. But there are a few ideas that caught my attention enough for me to get personally involved in research projects.

The most important one, in my opinion, is adversarial training (also called GAN for Generative Adversarial Networks). This is an idea that was originally proposed by Ian Goodfellow when he was a student with Yoshua Bengio at the University of Montreal (he since moved to Google Brain and recently to OpenAI).

This, and the variations that are now being proposed is the most interesting idea in the last 10 years in ML, in my opinion.

Why generative models?



<https://github.com/hindupuravinash/the-gan-zoo>

Why generative models?



2014



2015



2016



2017

Brundage et al, 2018

Why generative models?

Odena et al
2016



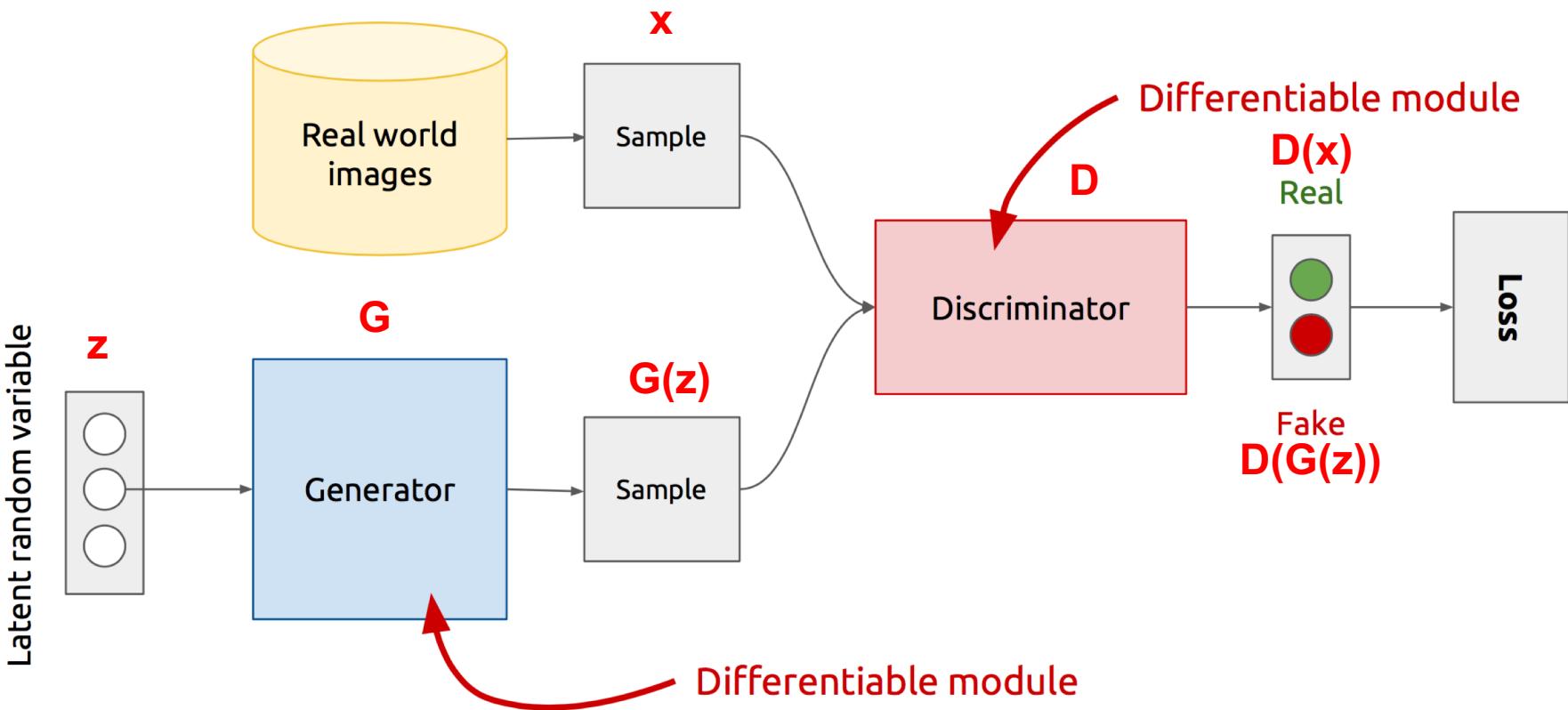
Miyato et al
2017



Zhang et al
2018

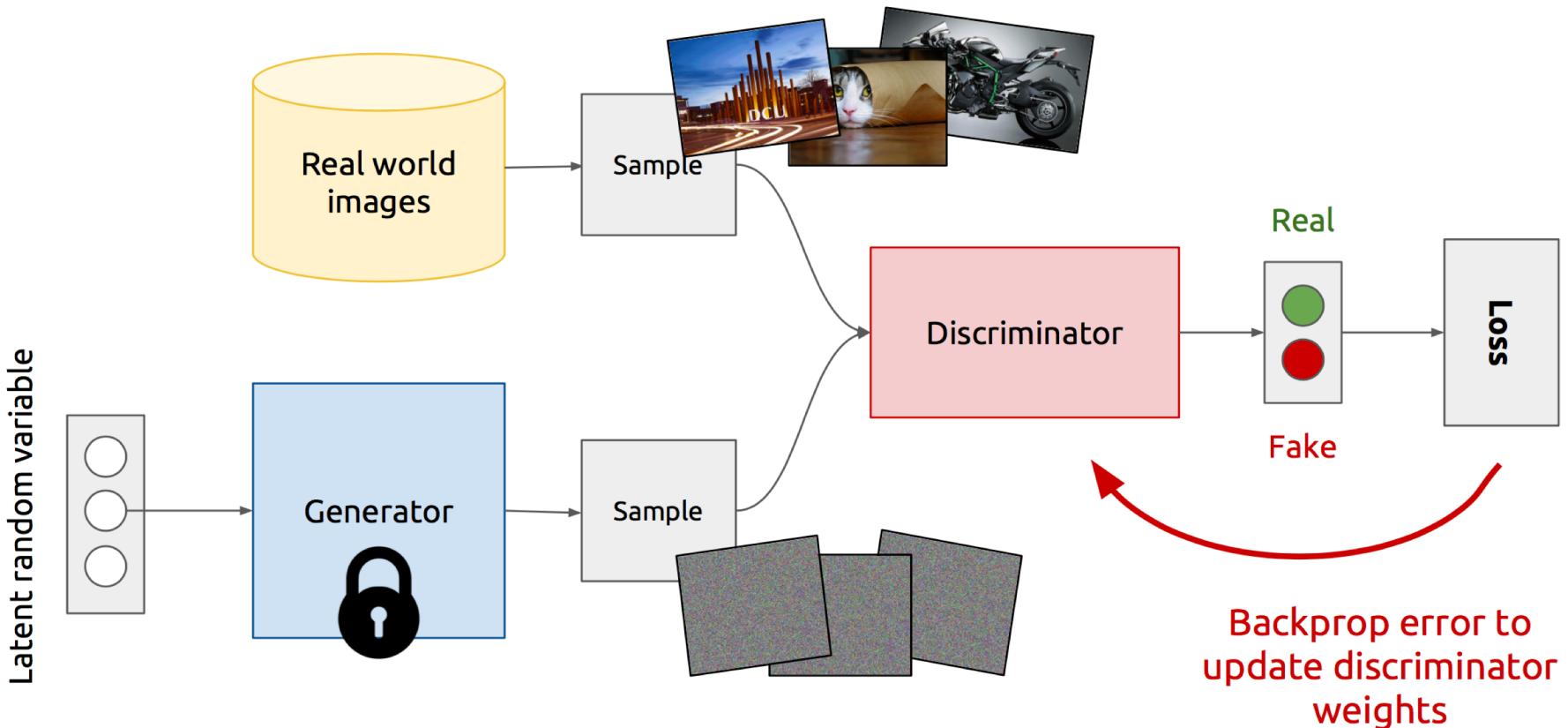


GAN's Architecture

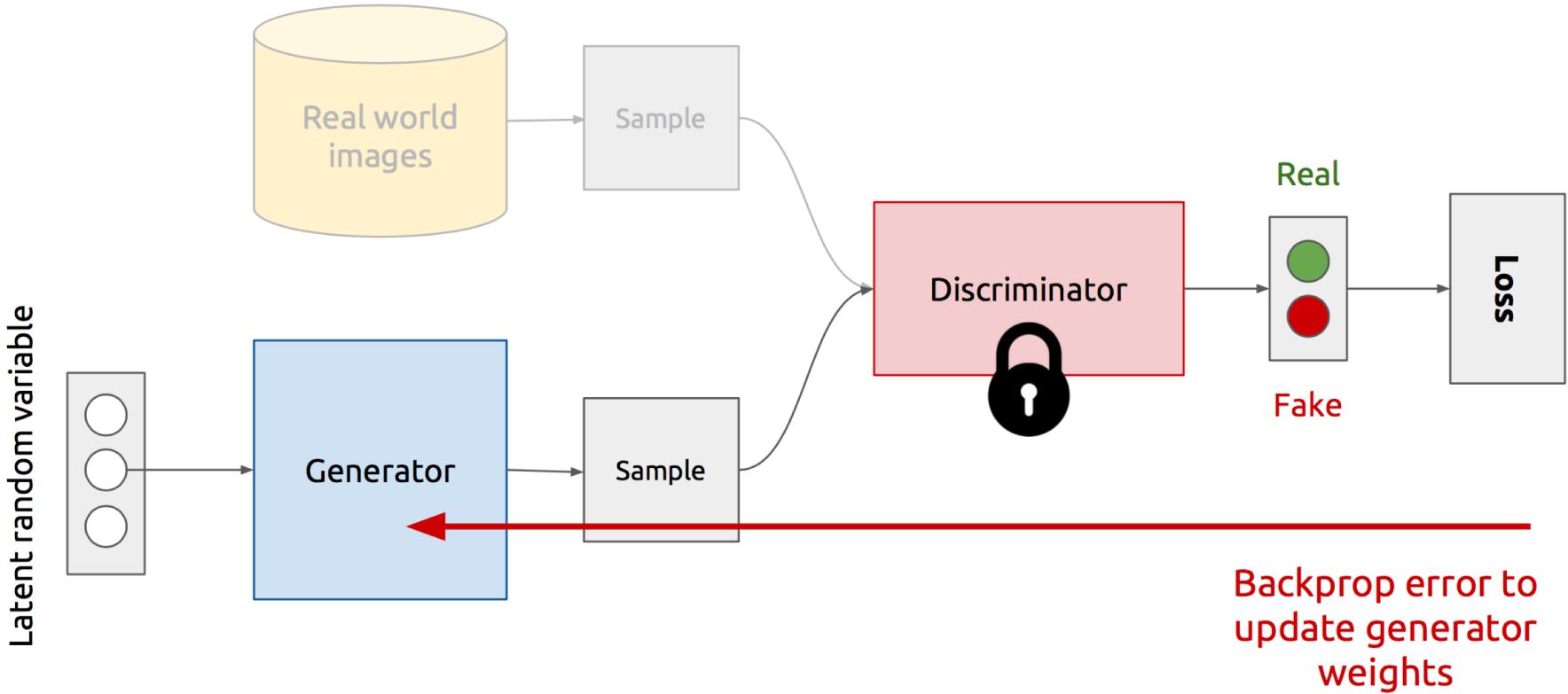


- Z is some random noise (Gaussian/Uniform).
- Z can be thought as the latent representation of the image.

GAN's Architecture

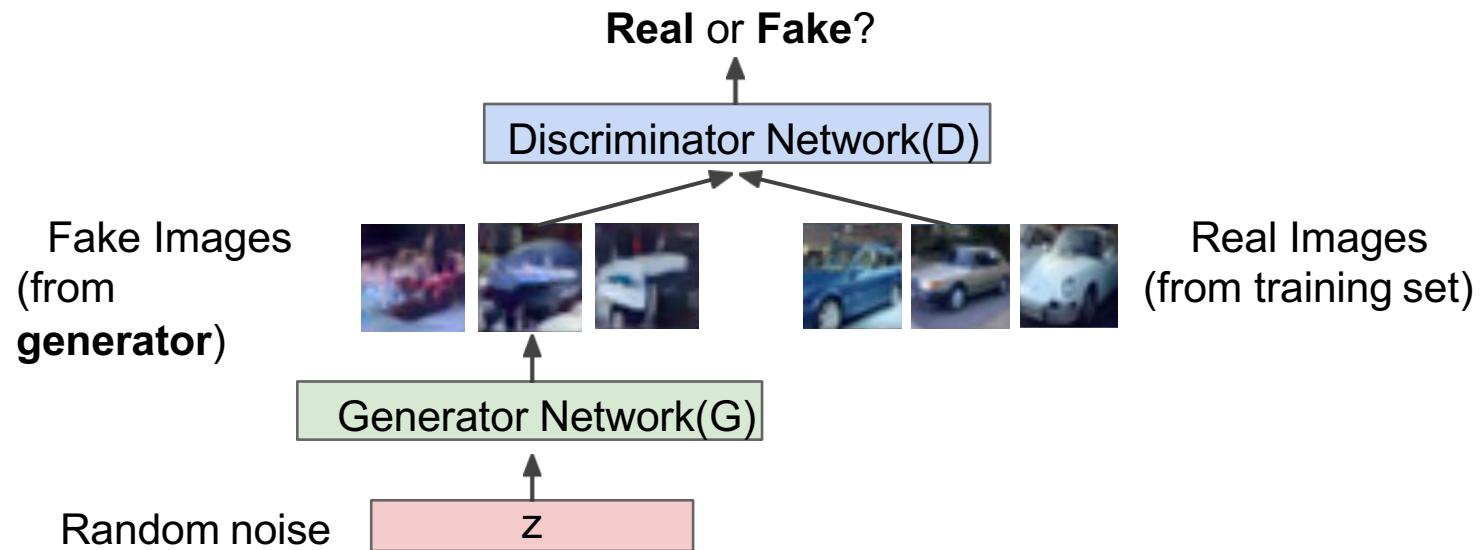


GAN's Architecture



Training GANs: Two-player game

- **Generator network (G)**: tries to **fool** the **discriminator** by generating real-looking images.
- **Discriminative network (D)**: tries to distinguish between **real** and **fake** images.



Training GANs: Two-player game

- **Generator network (G)**: tries to **fool** the **discriminator** by generating real-looking images.
- **Discriminative network (D)**: tries to distinguish between **real** and **fake** images.
- Train jointly in **minimax game**

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

Training GANs: Two-player game

- **Generator network (G)**: tries to **fool** the **discriminator** by generating real-looking images.
- **Discriminative network (D)**: tries to distinguish between **real** and **fake** images.
- Train jointly in **minimax game**

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log \underbrace{D_{\theta_d}(x)}_{\text{Discriminator output for real data } x} + \mathbb{E}_{z \sim p(z)} \log(1 - \underbrace{D_{\theta_d}(G_{\theta_g}(z))}_{\text{Discriminator output for generated fake data } G(z)}) \right]$$

Training GANs: Two-player game

- **Generator network (G)**: tries to **fool** the **discriminator** by generating real-looking images.
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Minimax objective function:

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- **Discriminator (θ_d)** wants to **maximize objective** such that $D(x)$ is close to 1 (**real**) and $D(G(z))$ is close to 0 (**fake**)
- **Generator (θ_g)** wants to **minimize objective** such that $D(G(z))$ is close to 1 (discriminator is fooled into thinking generated $G(z)$ is real)

Training GANs: Two-player game

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

end for

Training GANs: Two-player game

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by ascending its stochastic gradient:

Some find $k=1$ more stable, other use $k>1$, no best rule.

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

end for

Training GANs: Two-player game

Discriminator (D)

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascend** its stochastic gradient:

$$\nabla_{\theta_d} \left[\frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right] \right].$$

Loss function to
maximize for the
Discriminator

end for

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descend** its stochastic gradient:

$$\nabla_{\theta_g} \left[\frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))) \right].$$

Loss function to
minimize for the
Generator

Generator (G)

end for

Training GANs: Two-player game

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascend** its stochastic gradient:

Gradient w.r.t the parameters of
the **Discriminator**

$$\nabla_{\theta_d}$$

$$\frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

Loss function to
maximize for the
Discriminator

end for

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descend** its stochastic gradient:

$$\text{minimize}$$

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

Loss function to
minimize for the
Generator

end for

Gradient w.r.t the parameters of
the **Generator**

Training GANs: Two-player game

for number of training i

for k steps **do**

- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the discriminator by **maximize** its stochastic gradient:

Gradient w.r.t the parameters of the **Discriminator**

$$\nabla_{\theta_d}$$

$$\frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

Loss function to maximize for the **Discriminator**

end for

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descending** its stochastic gradient:

minimize

$$\nabla_{\theta_g}$$

$$\frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

Loss function to minimize for the **Generator**

end for

Gradient w.r.t the parameters of the **Generator**

Training GANs: Two-player game

```
for number of training iterations do
```

```
  for k steps do
```

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution

Uniform noise vector (random numbers)

Real images

```
end for
```

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Uniform noise vector (random numbers)

Real images

Discriminator: (input=generated/real image,
output=prediction of real image)

end for

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Uniform noise vector (random numbers)

Real images

Generator:
 input=random numbers,
 output=synthetic image

Discriminator: (input=generated/real image,
 output=prediction of real image)

end for

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

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- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(\mathbf{x}^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

end for

Uniform noise vector (random numbers)

Real images

Generator:
 input=random numbers,
 output=synthetic image

Real image



Generated image



Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

 for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by ascending its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

Discriminator: (input=generated/real image,
output=prediction of real image)

end for

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by maximizing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

Discriminator: (input=generated/real image,
output=prediction of real image)

Let's do an example

end for

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

 for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **maximize** its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

$$D(x) = 0.8$$

$$\log(0.8) = -0.2$$

Imagine for a real image $D(x)$ scores 0.8 that it is a real image (good)

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

end for

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by maximizing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

Discriminator: (input=generated/real image,
output=prediction of real image)

$$D(x) = 0.8$$

$$\log(0.8) = -0.2$$

$$D(G(z)) = 0.2$$

$$\log(1-0.2) = \log(0.8) = -0.2$$

end for

Then for a generated image, $D(x)$ scores 0.2
that it is a real image (good)

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascending** its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

$$D(x) = 0.8$$

$$\log(0.8) = -0.2$$

$$D(G(z)) = 0.2$$

$$\log(1-0.2) = \log(0.8) = -0.2$$

end for

$$-0.2 + -0.2 = -0.4$$

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

Discriminator: (input=generated/real image,
output=prediction of real image)

We add them together and this gives us a fairly high (-0.4) loss (note we ascend so we want to maximize this).

Also note that we are adding two negative numbers so 0 is the upper bound

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by maximizing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

$$D(x) = 0.8$$

$$\log(0.8) = -0.2$$

$$D(G(z)) = 0.2$$

$$\log(1-0.2) = \log(0.8) = -0.2$$

end for

$$-0.2 + -0.2 = -0.4$$

Let's do another example

Generator:
input=random numbers,
output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by ascending its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

$D(x)$ scores 0.2 that a real image is a real image (bad)
 $\log(0.2) = -1.39$

Discriminator: (input=generated/real image, output=prediction of real image)

$D(x) = 0.2$
 $\log(0.2) = -1.39$

Generator:
input=random numbers,
output=synthetic image

$D(G(z)) = 0.2$

$$\log(1-0.2) = \log(0.8) = -0.2$$

end for

$$-0.2 + -0.2 = -0.4$$

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by maximizing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

$$\begin{aligned} D(x) &= 0.8 \\ \log(0.8) &= -0.2 \end{aligned}$$

$$\begin{aligned} D(x) &= 0.2 \\ \log(0.2) &= -1.6 \end{aligned}$$

$D(x)$ scores 0.8 that the generated image is a real image (bad)

$$\begin{aligned} D(G(z)) &= 0.8 \\ \log(1-0.8) &= \log(0.2) = -1.6 \end{aligned}$$

end for $-0.2 + -0.2 = -0.4$

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by maximizing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \log D(x^{(i)}) + \log (1 - D(G(z^{(i)})))$$

end for

Average over m samples

$$D(x) = 0.8$$

$$\log(0.8) = -0.2$$

$$D(G(z)) = 0.2$$

$$\log(1 - 0.2) = \log(0.8) = -0.2$$

end for

These "bad" predictions combined gives a loss of -3.2

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

Discriminator: (input=generated/real image,
output=prediction of real image)

$$D(x) = 0.2$$

$$\log(0.2) = -1.39$$

$$D(G(z)) = 0.8$$

$$\log(1 - 0.8) = \log(0.2) = -1.39$$

$$-1.39 + -1.39 = -3.2$$

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by maximizing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

$$D(x) = 0.8$$

$$\log(0.8) = -0.2$$

$$D(x) = 0.2$$

$$\log(0.2) = -1.6$$

$$D(G(z)) = 0.2$$

$$\log(1-0.2) = \log(0.8) = -0.2$$

$$D(G(z)) = 0.8$$

$$\log(1-0.8) = \log(0.2) = -1.6$$

end for

$$-0.2 + -0.2 = -0.4$$

$$-1.6 + -1.6 = -3.2$$

Compare the loss to when we had "good" predictions (remember we are maximizing)

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by maximizing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by descending its stochastic gradient:

Uniform noise vector (random numbers)

Real images

Generator:

input=random numbers,
output=synthetic image

Discriminator: (input=generated/real image,
output=prediction of real image)

end for

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by maximizing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

end for

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascending** its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by descending its stochastic gradient:

$$D(G(z)) = 0.2$$

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

end for

$D(G(z))$ scores 0.2 that a generated image is a real image = bad :(We didn't fool D(.)

Uniform noise vector (random numbers)

Real images

Generator:

input=random numbers,
output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **maximize** its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descending** its stochastic gradient:

$$D(G(z)) = 0.2$$

$$\log(1-0.2) = \log(0.8) = -0.2$$

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

end for

Gets assigned a loss of -0.2

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascend**ing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descend**ing its stochastic gradient:

$$D(G(z)) = 0.2$$

$$\log(1-0.2) = \log(0.8) = -0.2$$

minimize

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

end for

Notice here we want to minimize this loss function (not maximize like before)

Uniform noise vector (random numbers)

Real images

Generator:

input=random numbers,
output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascend**ing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descend**ing its stochastic gradient:

$$D(G(z)) = 0.2$$

$$\log(1-0.2) = \log(0.8) = -0.2$$

minimize

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

Generator:

input=random numbers, output=synthetic image

end for

One more example ...

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascending** its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descending** its stochastic gradient:

$$D(G(z)) = 0.2$$

$$\log(1-0.2) = \log(0.8) = -0.2$$

$$D(G(z)) = 0.8$$

minimize

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

$D(G(z))$ scores 0.8 that a generated image is a real image = good :) We fooled $D(\cdot)$!

end for

Uniform noise vector (random numbers)

Real images

Generator:

input=random numbers,
output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascending** its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descending** its stochastic gradient:

$$D(G(z)) = 0.2$$

minimize

$$\log(1-0.2) = \log(0.8) = -0.2$$

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

$$D(G(z)) = 0.8$$

end for

$$\log(1-0.8) = \log(0.2) = -1.6$$

This gives loss of -1.6

Generator:

input=random numbers,
output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascend**ing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descend**ing its stochastic gradient:

$$D(G(z)) = 0.2$$

minimize

$$\log(1-0.2) = \log(0.8) = -0.2$$

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

$$D(G(z)) = 0.8$$

end for

$$\log(1-0.8) = \log(0.2) = -1.6$$

So minimizing this loss means to generate images that fool $D(\cdot)$

Generator:

input=random numbers, output=synthetic image

Training GANs: Two-player game

$D(x)$ = probability that x is a real image
 0 = generated image; 1 = real image

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by **ascend**ing its stochastic gradient:

Gradient w.r.t the parameters of the Discriminator

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log (1 - D(G(z^{(i)}))) \right].$$

end for

Average over m samples

Discriminator: (input=generated/real image, output=prediction of real image)

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by **descend**ing its stochastic gradient:

minimize

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^{(i)}))).$$

end for

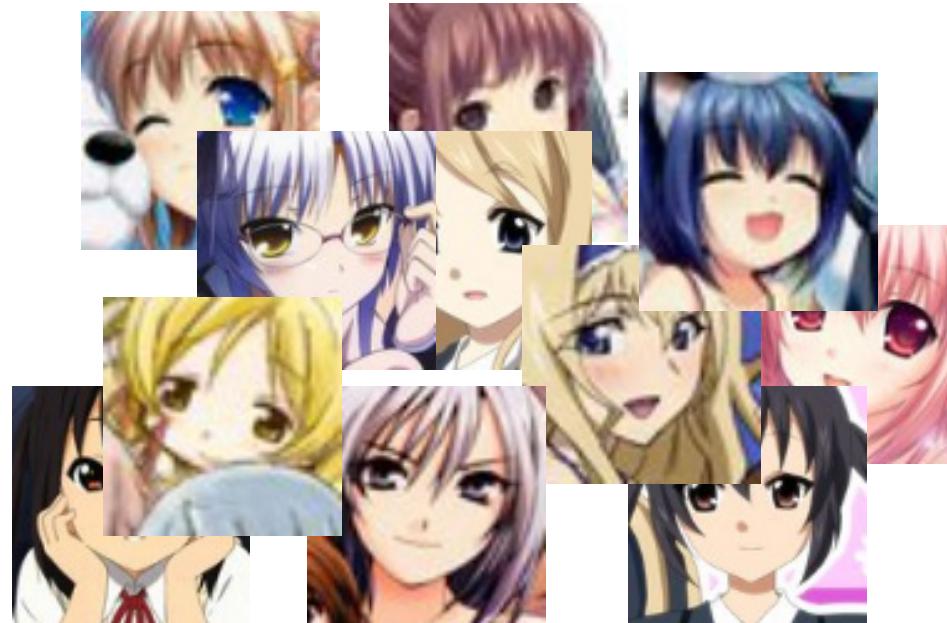
Gradient w.r.t the parameters of the Generator

Uniform noise vector (random numbers)

Real images

Generator:
input=random numbers,
output=synthetic image

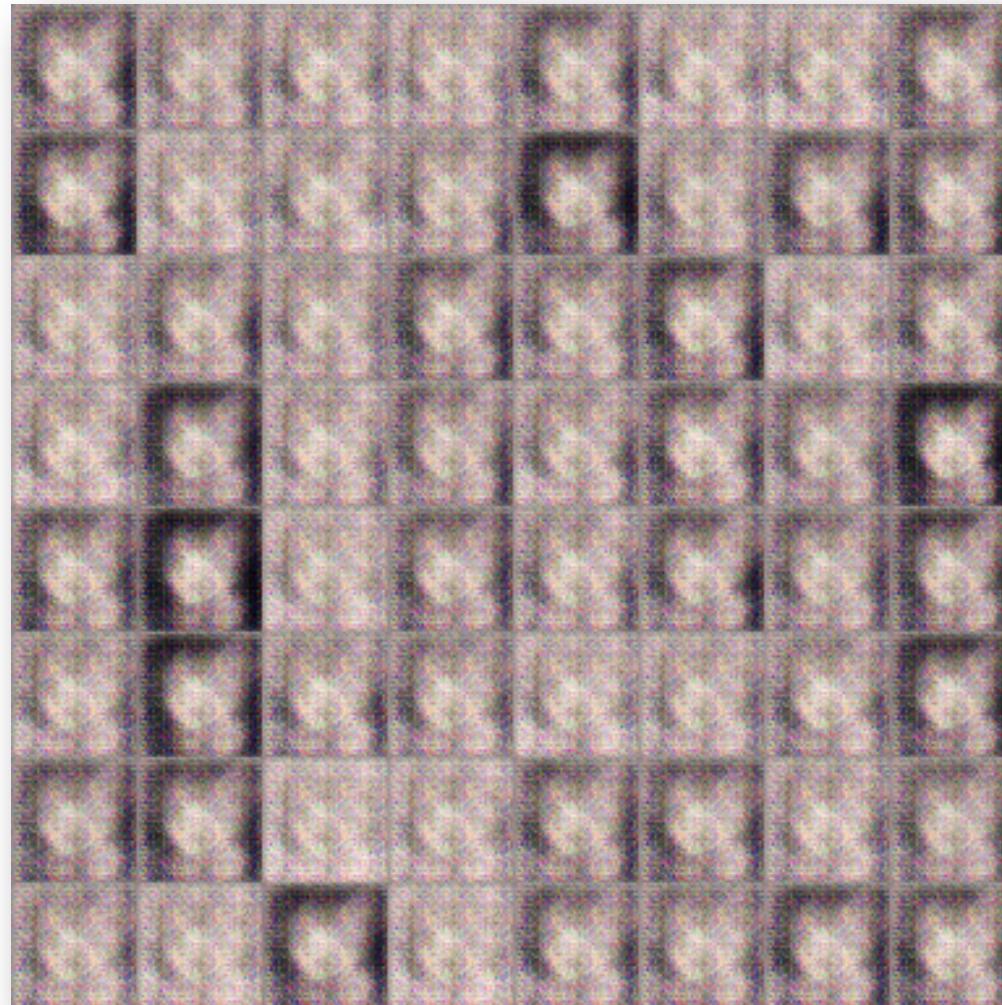
Training GANs: Iterations



Generating Anime Figures

Training GANs: Iterations

100 rounds



Training GANs: Iterations

1000 rounds



Training GANs: Iterations

2000 rounds



Training GANs: Iterations

5000 rounds



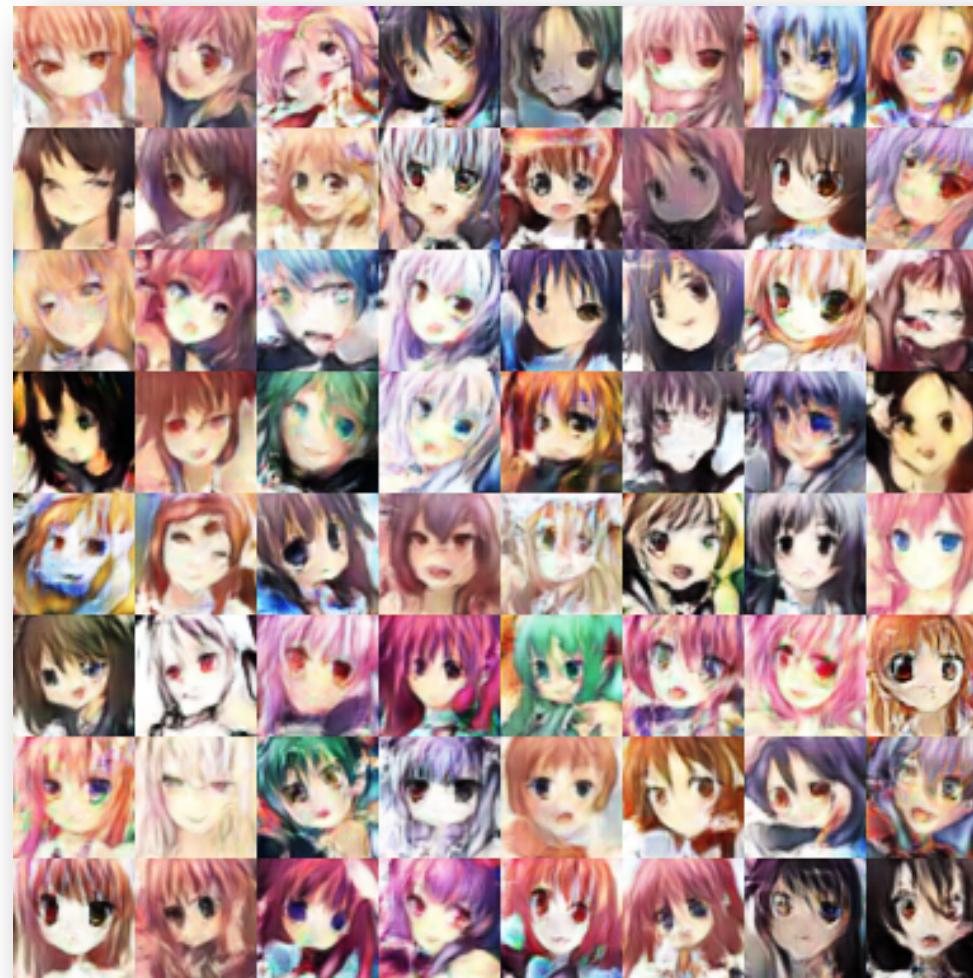
Training GANs: Iterations

10000 rounds



Training GANs: Iterations

20000 rounds



Advantages with GANs

- Generation and sampling is straightforward.
- Robust to Overfitting since Generator never sees the training data.
- Empirically, GANs are good at capturing the modes of the distribution.

Challenges with GANs

- Training is hard:
 - **Non-convergence:** the model parameters oscillate and never converge.
 - **Mode-collapse:** the generator (G) produces limited varieties of samples.
 - **Diminished-gradients:** the discriminator (D) gets too successful that the generator's gradients vanish.

Training GANs: Non-convergence

- GANs involve two (or more) players:
 - Discriminator (D) tries to maximize its reward.
 - Generator (G) tries to minimize the Discriminator's reward.

$$\min_G \max_D V(D, G) = \mathbb{E}_{x \sim p(x)} [\log D(x)] + \mathbb{E}_{z \sim q(z)} [\log(1 - D(G(z)))]$$

- We might not converge to the Nash equilibrium:

$$P_{data}(x) = P_{gen}(x) \quad \forall x$$

Training GANs: Non-convergence

$$\min_x \max_y V(x, y)$$

Let $V(x, y) = xy$

- State 1:

$x > 0$	$y > 0$	$V > 0$
---------	---------	---------

Increase y	Decrease x
------------	------------

- State 2:

$x < 0$	$y > 0$	$V < 0$
---------	---------	---------

Decrease y	Decrease x
------------	------------

- State 3:

$x < 0$	$y < 0$	$V > 0$
---------	---------	---------

Decrease y	Increase x
------------	------------

- State 4 :

$x > 0$	$y < 0$	$V < 0$
---------	---------	---------

Increase y	Increase x
------------	------------

- State 5:

$x > 0$	$y > 0$	$V > 0$
---------	---------	---------

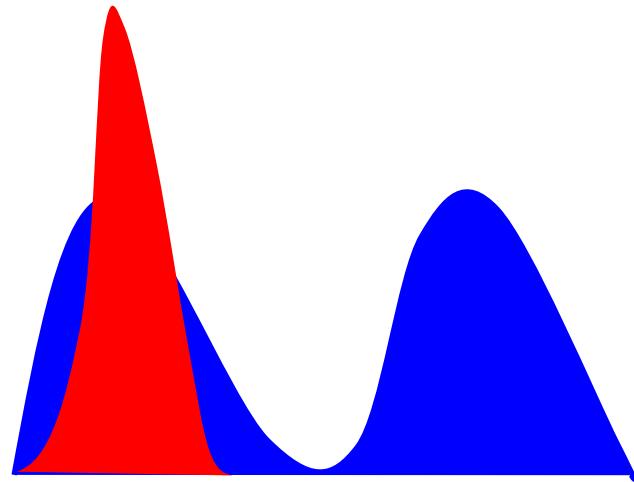
Increase y	Decrease x
------------	------------

== State 1

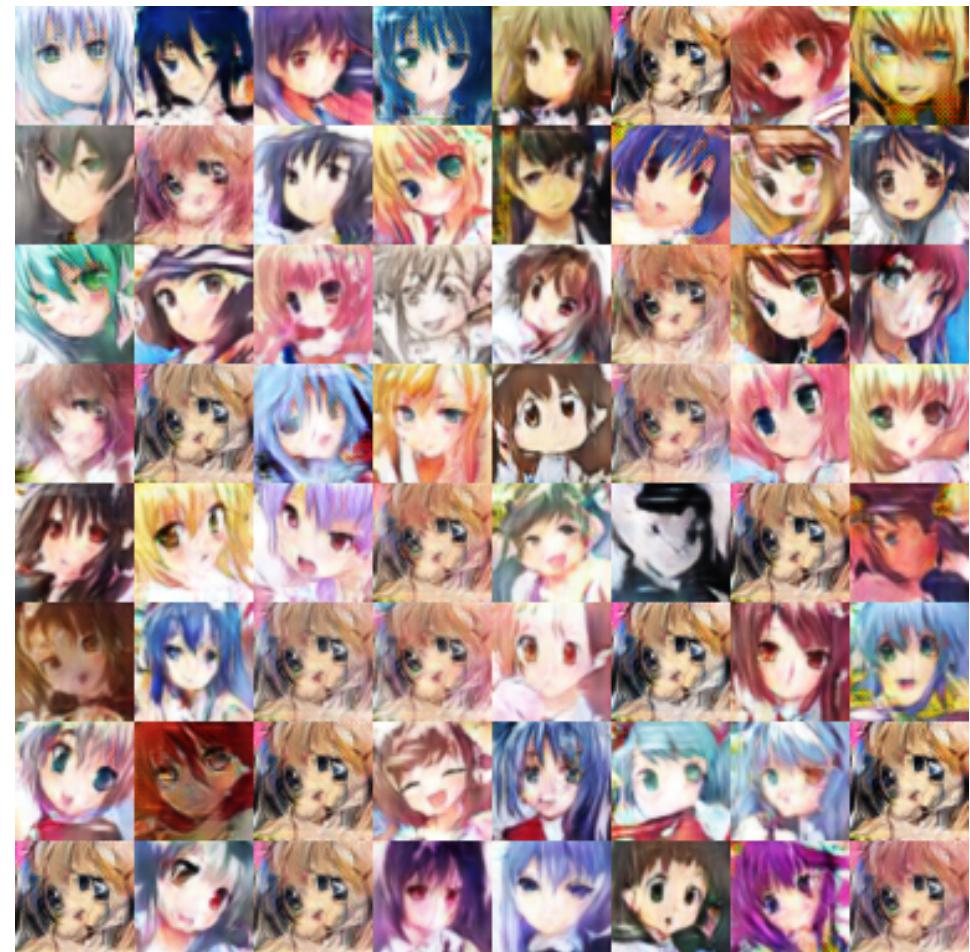
Training GANs: Mode-collapse

Converge to same faces

Generated
Distribution



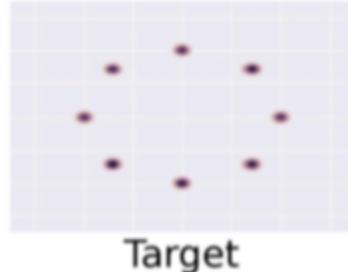
Data
Distribution



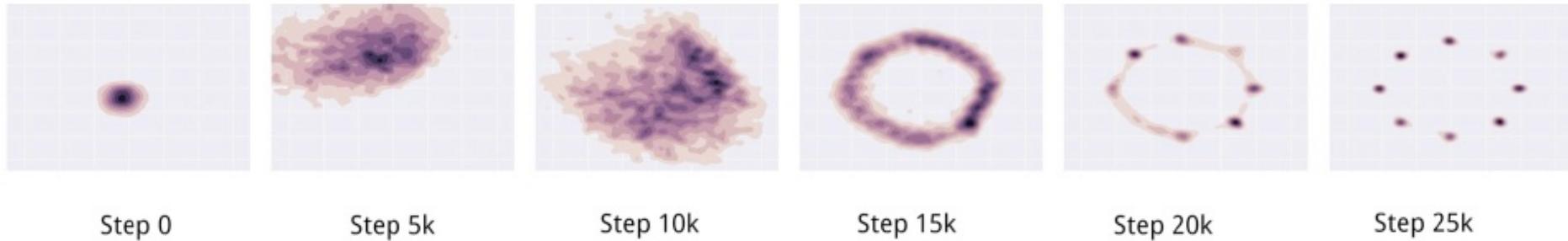
Sometimes, this is hard to tell since
Generative Adversarial Networks
one sees only what's generated, but not what's missed.

Training GANs: Mode-collapse

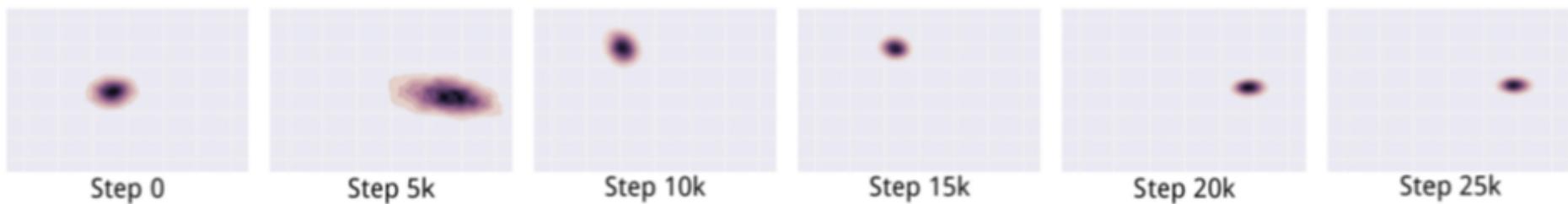
8 Gaussian distributions:



What we want ...



In reality ...

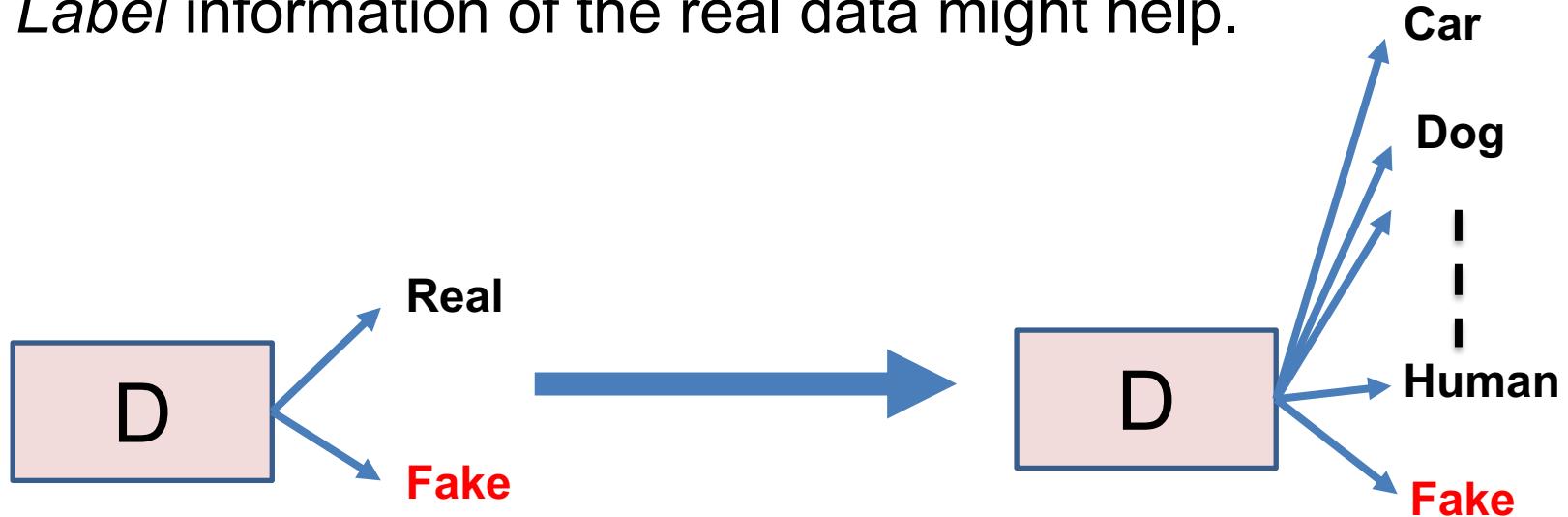


Training GANs: Tipps (Mini-batches)

- At mode collapse:
 - Generator (G) produces good samples, but a very few of them.
 - Thus, Discriminator (D) can't tag them as fake.
- To address this problem:
 - Let the Discriminator know about this edge-case.
- More formally:
 - Let the Discriminator look at the entire batch instead of single examples
 - If there is lack of diversity, it will mark the examples as fake

Training GANs: Tipps (Multi-class)

- *Label* information of the real data might help.



- Empirically generates much better samples

Training GANs: Tipps (Loss)

$$\min_G \max_D V(D, G)$$

$$V(D, G) = \mathbb{E}_{x \sim p(x)}[\log D(x)] + \mathbb{E}_{z \sim q(z)}[\log(1 - D(G(z)))]$$

$$D^* = \operatorname{argmax}_D V(D, G)$$

$$G^* = \operatorname{argmin}_G V(D, G)$$

- In this formulation, Discriminator's strategy was $D(x) \rightarrow 1$, $D(G(z)) \rightarrow 0$
- Alternatively, we can flip the binary classification labels i.e. **Fake = 1, Real = 0**

$$V(D, G) = \mathbb{E}_{x \sim p(x)}[\log(1 - D(x))] + \mathbb{E}_{z \sim q(z)}[\log(D(G(z)))]$$
- In this new formulation, Discriminator's strategy will be $D(x) \rightarrow 0$, $D(G(z)) \rightarrow 1$

Training GANs: Tipps (Loss)

- If all we want to encode is $D(x) \rightarrow 0, D(G(z)) \rightarrow 1$

$$D^* = \operatorname{argmax}_D \mathbb{E}_{x \sim p(x)} [\log(1 - D(x))] + \mathbb{E}_{z \sim q(z)} [\log(D(G(z)))]$$

We can use this

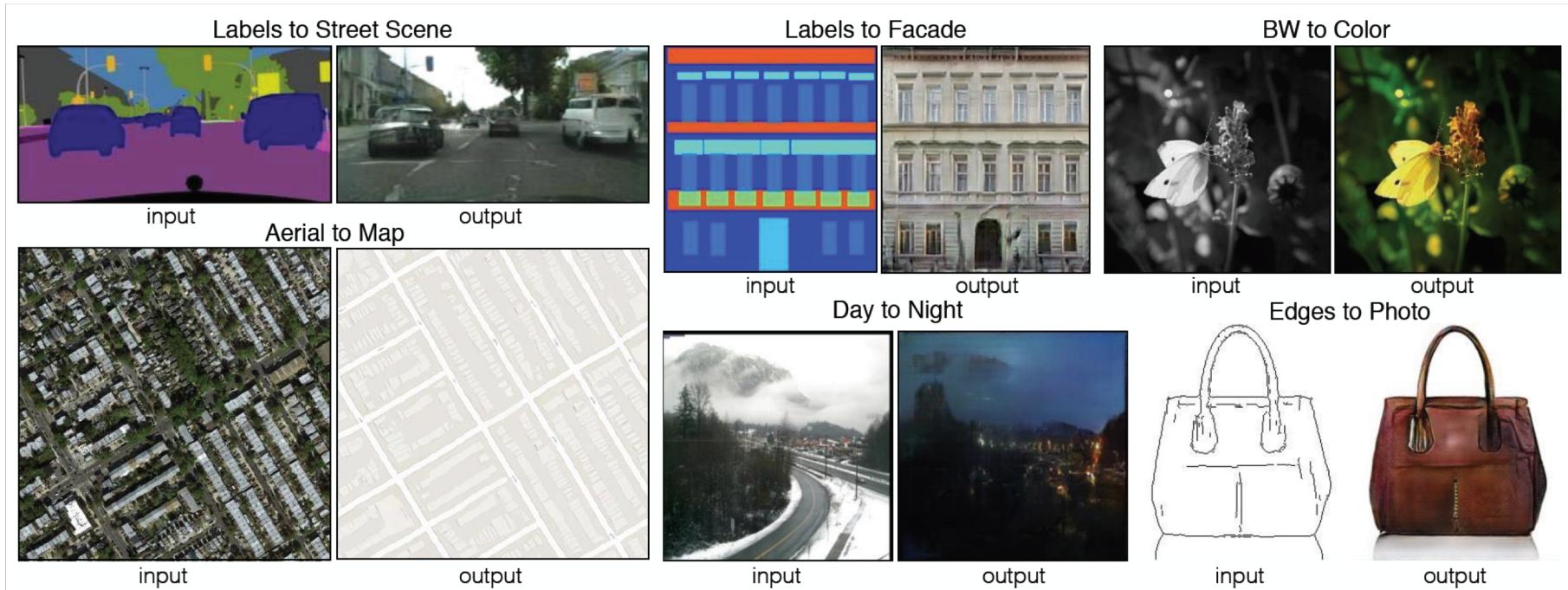
$$D^* = \operatorname{argmin}_D \mathbb{E}_{x \sim p(x)} \log(D(x)) + \mathbb{E}_{z \sim q(z)} [\log(1 - D(G(z)))]$$

- Now, we can replace cross-entropy with any loss function (**Hinge Loss**)

$$D^* = \operatorname{argmin}_D \mathbb{E}_{x \sim p(x)} D(x) + \mathbb{E}_{z \sim q(z)} \max(0, m - D(G(z)))$$

- And thus, instead of outputting probabilities, Discriminator just has to output:
 - High values for fake samples
 - Low values for real samples

Image-To-Image Translation



[Link to an interactive demo of this paper](#)

Isola, P., Zhu, J. Y., Zhou, T., & Efros, A. A. “**Image-to-image translation with conditional adversarial networks**”. arXiv preprint arXiv:1611.07004. (2016).

Image-To-Image Translation

■ Architecture

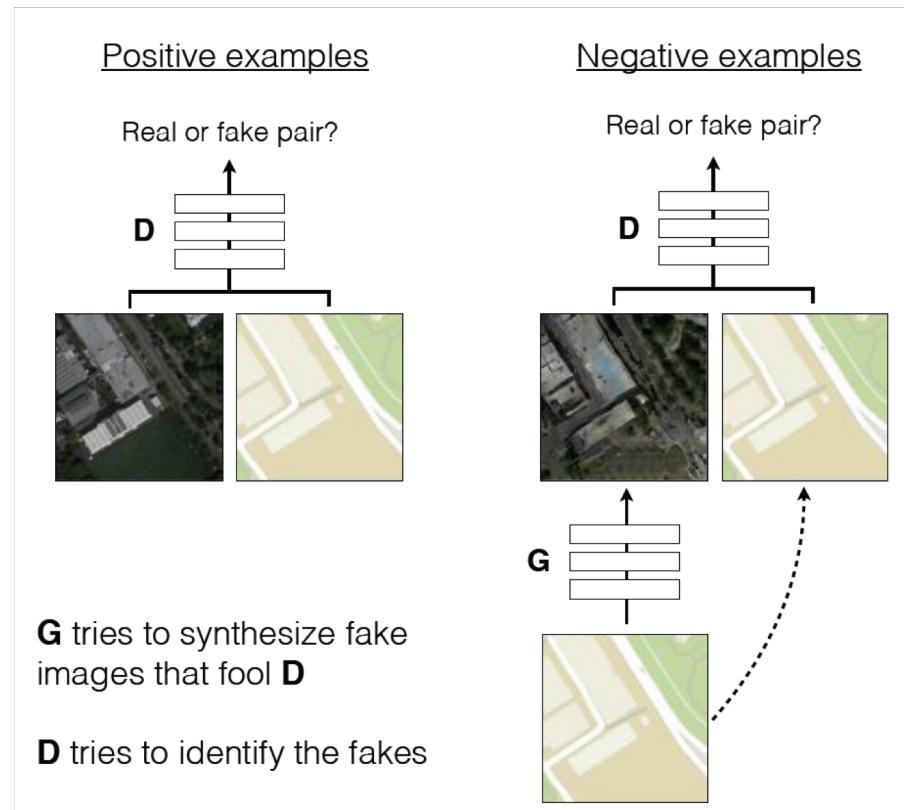
- DCGAN-based.

■ Training:

- Conditioned on images from the source domain.

■ Benefits:

- Effective way of handling different domains.
- No need of **structural loss**.



Text-To-Image Synthesis

■ Motivation

- Given a text description, generate closely related images.

■ Training:

- Use CGAN with **D** and **G** conditioned on dense “text” embedding.

this small bird has a pink breast and crown, and black primaries and secondaries.



this magnificent fellow is almost all black with a red crest, and white cheek patch.



the flower has petals that are bright pinkish purple with white stigma



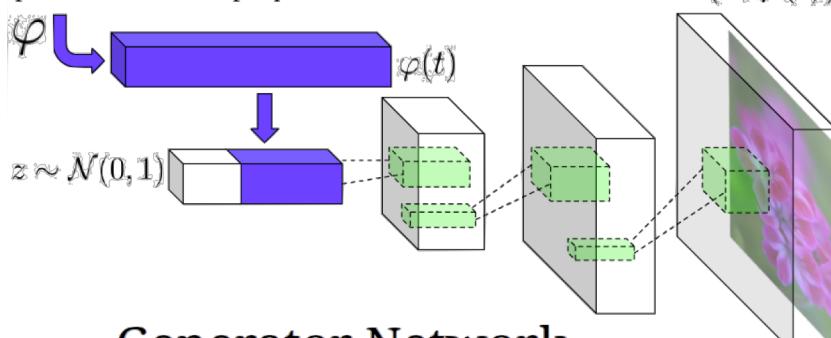
this white and yellow flower have thin white petals and a round yellow stamen



Reed, S., Akata, Z., Yan, X., Logeswaran, L., Schiele, B., & Lee, H. “**Generative adversarial text to image synthesis**”. ICML (2016).

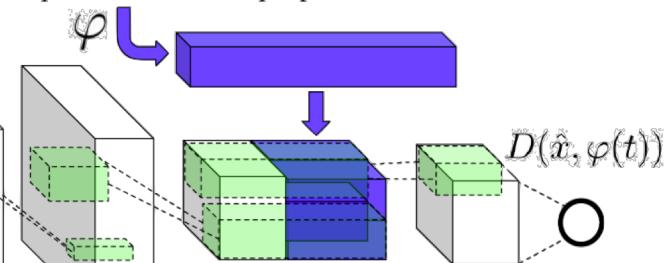
Text-To-Image Synthesis

This flower has small, round violet petals with a dark purple center



Generator Network

This flower has small, round violet petals with a dark purple center



Discriminator Network

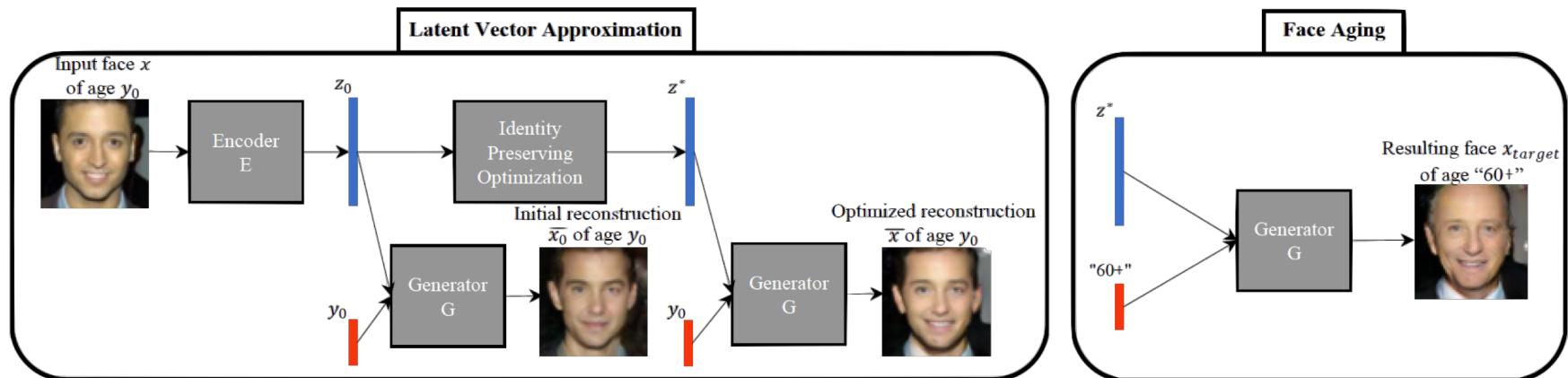
Positive Samples:

{Real Image, Right Text}

Negative Samples:

{Real Image, Wrong Text}
 {Fake Image, Right Text}

Facial Aging

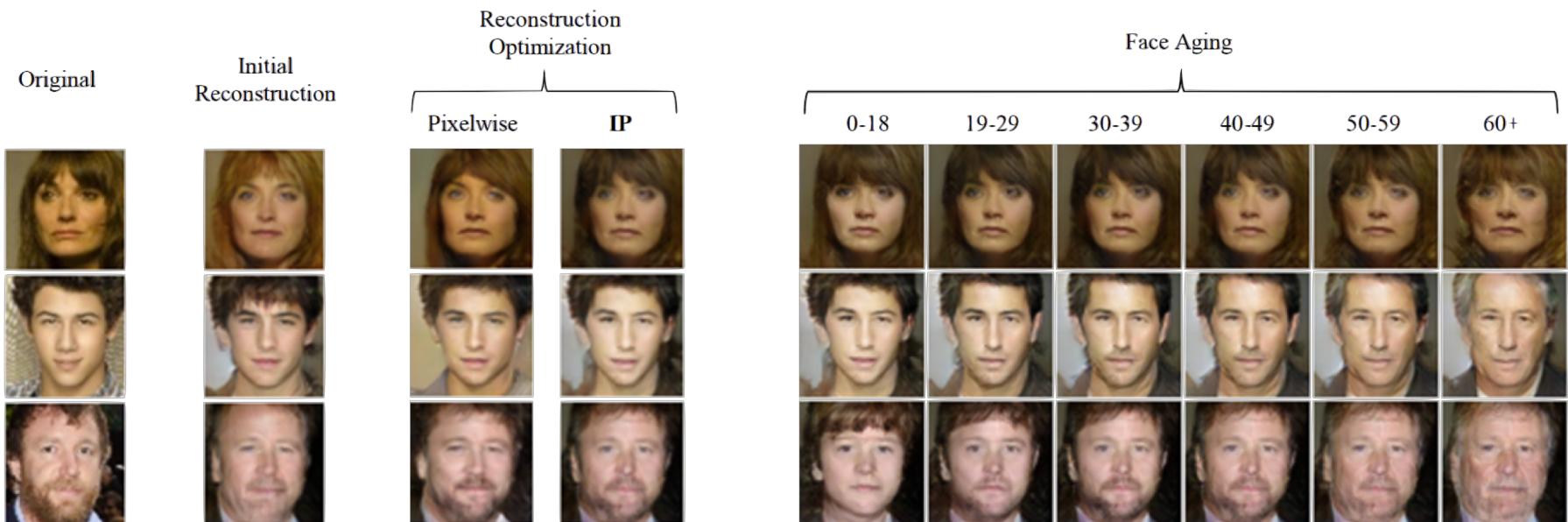


- **Architecture:**
 - Given a text description, generate closely related images.

- **Training:**
 - Latent code conditioned on a discrete embedding of age categories.

Antipov, G., Baccouche, M., & Dugelay, J. L. (2017). **“Face Aging With Conditional Generative Adversarial Networks”**. arXiv preprint arXiv:1702.01983.

Facial Aging





Generative Adversarial Networks (GANs)

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Arequipa, 9th January 2019