HW-4

1) $p_{30} = \frac{\mu}{\lambda+\mu} = \frac{20}{20+5} = 0.8$, $q_{30} = 0.2$

$p_{20} = \frac{35}{15+35} = 0.7$, $q_{20} = 0.3$

$p_{50} = \frac{27}{3+27} = 0.9$, $q_{50} = 0.1$

Copatt table (when no units are added)

State	C_o	P_i	F_i
1	0	1	0
∞	∞	0	0

Add 20 MW

State	C_o	P_i	F_i
1	0	1	0
2	20	$1-p_{20} = 0.3$	$q_{20}\mu_{20} = 10.5$

Add 30 MW (2 Gen, 4 states)

$X = 20$, $C_i = 20$, $C_j = X - C_N = 20-30 = -10$

$P(20) = P(20)\,p_{30} + P(-10)\,q_{30}$

$= 0.3 \times 0.8 + 1 \times 0.2$

$= 0.44$

$F(20) = F(20)p_{30} + F(-10)q_{30} + [P(-10) - P(20)] q_{30} u_{30}$

$= 10.5 \times 0.8 + 0 + (1 - 0.3)\, 0.2 \times 20$

$= 11.2$

$\underline{X = 30}$, $C_i = 30$, $C_j' = 30 - 30 = 0$

$P(30) = P(30)p_{30} + P(0)q_{30}$

$= 0 + 1 \times 0.2$

$= 0.2$

$F(30) = F(30)p_{30} + F(0)q_{30} + [P(0) - P(30)] q_{30} u_{30}$

$= (1 - 0)\, 0.2 \times 20$

$= 4$

$\underline{X = 50}$, $C_i = 50$, $C_j = 50 - 30 = 20$

$P(50) = P(50)p_{30} + P(20)q_{30}$

$= 0.3 \times 0.2$

$= 0.06$

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$F(50) = F(50)p_{30} + F(20)q_{30} + [P(20) - P(50)]\,q_{30}\mu_{30}$

$= (10.5 \times 0.2) + (0.3 - 0)\,0.2 \times 20$

$= 3.3$

So, ~~then~~ COPAFT after adding 30 MW

State	C_0	P_i	F_i
1	0	1	0
2	20	0.44	11.2
3	30	0.2	4
4	50	0.06	3.3

Add 50 MW (3 Gen, 8 states)

$X = 20$, $C_i' = 20$, $C_j' = 20 - 50 = -30$

$P(20) = P(20)p_{50} + P(-30)q_{50}$

$= 0.44 \times 0.9 + 1 \times 0.1 = \underline{0.496}$

$F(20) = F(20)p_{50} + F(-30)q_{50} + [P(-30) - P(20)]\,q_{50}\mu_{50}$

$= 11.2 \times 0.9 + (1 - 0.44)\,0.1 \times 27$

$= \underline{11.592}$

$\underline{X=30}$, $C_i = 30$, $C_j = 30-50 = -20$

$P(30) = P(30)\,p_{50} + P(-20)\,q_{50}$

$= 0.2 \times 0.9 + (1 \times 0.1)$

$= 0.28$

$F(30) = F(30)\,p_{50} + F(-20)\,q_{50} + [P(-20) - P(30)]\,q_{50}\,\mu_{50}$

$= 4 \times 0.9 + [1 - 0.2]\,0.1 \times 27$

$= 5.76$

$\underline{X=50}$, $C_i = 50$, $C_j = 50-50 = 0$

$P(50) = P(50)\,p_{50} + P(0)\,q_{50}$

$= 0.06 \times 0.9 + 1 \times 0.1$

$= 0.154$

$F(50) = F(50)\,p_{50} + F(0)\,q_{50} + [P(0) - P(50)]\,q_{50}\,\mu_{50}$

$= 3.3 \times 0.9 + [1 - 0.06]\,0.1 \times 27$

$= 5.508$

$\underline{x = 70}$, $C_i = 70$, $C_j = 70 - 50 = 20$

$P(70) = P(70)p_{50} + P(20)q_{50}$

$= 0.44 \times 0.1$

$= 0.044$

$F(70) = F(70)p_{50} + F(20)q_{50} + [P(20) - P(70)]q_{50}\mu_{50}$

$= 11.2 \times 0.1 + [0.44 - 0]\, 0.1 \times 27$

$= 2.308$

$\underline{x = 80}$, $C_i = 80$, $C_j = 80 - 50 = 30$

$P(80) = P(80)p_{50} + P(30)q_{50}$

$= 0.2 \times 0.1 = 0.02$

$F(80) = F(80)p_{50} + F(30)q_{50} + [P(30) - P(80)]q_{50}\mu_{50}$

$= (4 \times 0.1) + (0.2 - 0)\, 0.1 \times 27$

$= 0.94$

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$\underline{X = 100}$, $C_i = 100$, $C_j = 100 - 50 = 50$

$P(100) = P(100) p_{50} + P(50) q_{50}$

$= 0.06 \times 0.1 = 0.006$

$F(100) = F(100) p_{50} + F(50) q_{50} + [P(50) - P(100)] q_{50} u_{50}$

$= (3.3 \times 0.1) + [0.06 - 0] 0.1 \times 27$

$= 0.492$

COPAFT table after adding 50 MW

State	C_0	P_i	F_i
1	0	~~0.496~~ 1	~~11.592~~ 0
2	20	0.496	11.592
3	30	0.28	5.76
4	50	0.154	5.508
5	70	0.044	2.308
6	80	0.02	0.94
7	100	0.006	0.492

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b) For load model, considering the 24 hrs of Monday of first week

$P_{annual} = 85$ MW

$P_{week1} = 86.2\%$ of $85 = 73.27$ MW

$P_{Monday} = 93\%$ of $73.27 = 68.141$ MW

24 hours of Monday

Hrs	Load	Hrs	Load
1	45.654	13	64.734
2	42.929	14	64.734
3	40.885	15	63.371
4	40.203	16	64.053
5	40.203	17	67.459
6	40.885	18	68.141
7	50.424	19	68.141
8	58.601	20	65.415
9	64.734	21	62.008
10	65.415	22	56.557
11	65.415	23	49.743
12	64.734	24	42.929

Load model considering only 10 MW intervals

State	$L = L_i$	$P_i = P(L > L_i)$	$f_i = f(L > L_i)/hr$
1	40	1	0
2	50	$\frac{16}{24} = 0.667$	$\frac{1}{24} = 0.0417$
3	60	$\frac{13}{24} = 0.542$	$\frac{1}{24} = 0.0417$
4	70	0	0

c) Generation Reserve Model

		Load			
Gen. cap	Outage cap	70	60	50	40
100	0	30	40	50	60
80	20	10	20	30	40
70	30	0	10	20	30
50	50	-20	-10	0	10
30	70	-40	-30	-20	-10
20	80	-50	-40	-30	-20
0	100	-70	-60	-50	-40

$M_{min} = -70$ & $C_c = 100$

$$P(-70) = \sum_{1}^{7} \left[P_G(C_j) - P_G(C_{j+1})\right] P_L\left(C_c - C_j - M_i\right)$$

$$= \left[P_G(C_1) - P_G(C_2)\right] P_L\left(100 - C_1 + 70\right)$$

$$+ \left[P_G(C_2) - P_G(C_3)\right] P_L\left(100 - (C_2 + 70)\right) + \cdots$$

$$\cdots + \left[P_G(C_7) - P_G(C_8)\right] P_L\left(100 - (C_7 + 70)\right)$$

$$P(-70) = 0$$

$$P(-60) = \left[P_G(C_1) - P_G(C_2)\right] P_L\left(100 - C_1 + 60\right)$$

$$+ \left[P_G(C_2) - P_G(C_3)\right] P_L\left(100 - (C_2 + 60)\right) + \cdots$$

$$\cdots + \left[P_G(C_7) - P_G(C_8)\right] P_L\left(100 - (C_7 + 60)\right)$$

$$= (0.006 - 0)\, 0.542.$$

$$P(-60) = 0.003252$$

$$P(-50) = P_G(100)\, P_L(50) = 0.006 \times 0.667$$

$$= 0.004$$

$$P(-40) = \left[P_G(C_5) - P_G(C_6)\right] P_L(70) + \left[P_G(C_6) - P_G(C_7)\right]$$

$$P_L(60) + \left[P_G(C_7) - P_G(C_8)\right] P_L(40)$$

$$= (0.044 - 0.02) \times 0 + (0.02 - 0.006) \times 0.542$$

$$+ (0.006 - 0) \times 1$$

$P(-40) = 0.013588$

$P(-30) = (0.044 - 0.02) \times 0.542 + (0.02 - 0.006) \times 0.667$

$+ 0.006 \times 1$

$P(-30) = 0.028346$

$P(-20) = (0.154 - 0.044) \times 0 + (0.044 - 0.02) \times 0.667$

$+ (0.02 - 0.006) \times 1 + (0.006 \times 1)$

$P(-20) = 0.036008$

$P(-10) = (0.154 - 0.044) \times 0.542 + (0.044 - 0.02) \times 1$

$+ (0.02 - 0.006) \times 1 + 0.006$

$P(-10) = 0.10362$

$P(0) = (0.154 - 0.044) \times 0.667 + (0.044 - 0.02) \times 1$

$+ (0.02 - 0.006) \times 1 + 0.006$

$P(0) = 0.11737$

Similarly, for $F_{\cdot}$

$$F(-70) = \sum_{j=1}^{7} \{ [F_G(C_j) - F_G(C_{j+1})] P_L(C_c - C_j - M_i)$$

$$+ [P_G(C_j) - P_G(C_{j+1})] F_L(C_c - C_j - M_i) \}$$

$$= 0$$

$$F(-60) = [F_G(100) - F_G 0] P_L(0) +$$

$$P_G(100) \times F_L(0)$$

$$= (0.492 \times 0.542) + (0.006 \times 0.0417)$$

$$= \cancel{6.672 \times 10^{-5}} \; 0.2669$$

$$F(-50) = (0.492 \times 0.669) + (0.006 \times 0.0417)$$

$$= \cancel{8.211 \times 10^{-5}} \; 0.3284$$

$$F(-40) = (0.94 - 0.492) \, 0.542 + (0.02 - 0.006)$$

$$\times 0.0417 + (0.492 \times 1) + (0.006 \times 0)$$

$$= 0.73539$$

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$$f(-30) = \left(\overset{2.308}{\cancel{0.044}} - \overset{0.94}{\cancel{0.02}}\right) \times 0.542 + (0.044 - 0.02) \times 0.0417$$

$$+ (0.94 - 0.492) \times 0.667 + (0.02 - 0.006) \times 0.94$$

$$+ \cancel{0.0}(0.492 \times 1)\cancel{0.0} + 0$$

$$f(-30) = \cancel{0.81798}\ 1.5464$$

$$f(-20) = \left(\overset{2.308}{\cancel{0.044}} - \overset{0.94}{\cancel{0.02}}\right) \times 0.667 + (0.044 - 0.02) \times 0.0417$$

$$+ (0.94 - 0.492) \times 1 + (0.02 - 0.006) \times 0$$

$$+ (0.492 \times 1) + 0$$

$$= \cancel{0.957}\ 1.8534$$

$$f(-10) = (5.508 - 2.308) \times 0.542 + (0.154 - 0.044) \times 0.0417$$

$$+ (2.308 - 0.94) \times 1 + 0 + (0.94 - 0.492) \times 1$$

$$+ (0.492 \times 1)$$

$$= 4.0469$$

d) $LOLP = P(-M_0) = P(-10) = 0.10362$

$LOLF = F(-M_0) = \frac{4.0469}{24} = 0.1686/day$

$$EPNS = \Delta M \left[\sum_{M=0}^{-60} P(M) - \frac{1}{2} \{ P(0) + P(-60) \} \right]$$

$$= 10 \left(0.306184 - \frac{1}{2} (0.11737 + 0.003252) \right)$$

$= 2.45873$ MW/day

$EUE = EPNS \times D = 2.45873 \times 24$

$= 59.0095$ MWh/d

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