```
> restart ;
  V := 10 ;
```

$$V := 10 \tag{1}$$

```
> with(VectorCalculus) :
  with(LinearAlgebra) :
> # choose Delimited, floating 32 bit when opening the file cert.csv
```

```
> coefficients := ImportData( )
```

$$coefficients := \begin{bmatrix} 362880 \text{ x } 19 \text{ Matrix} \\ \text{Data Type: } float_4 \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \tag{2}$$

```
> infeasibilityNOTproved := false;
  row := 0;
  r23 := 0;
  h1 := 1 + r21 + r31 + r41 + r51 + r61 + r71 + r81 + r91 + r101;
  h2 := r22 + r32 + r42 + r52 + r62 + r72 + r82 + r92 + r102;
  h3 := r33 + r43 + r53 + r63 + r73 + r83 + r93 + r103;
  f1 := r21^2 + r22^2 - r21;
  f2[1] := r31^2 + r32^2 + r33^2 - r31;
  f2[2] := r31^2 + r32^2 + r33^2 - r21*r31 - r22*r32 - r23*r33;
  f3[1] := r41^2 + r42^2 + r43^2 - r41;
  f3[2] := r41^2 + r42^2 + r43^2 - r21*r41 - r22*r42 - r23*r43;
  f3[3] := r41^2 + r42^2 + r43^2 - r31*r41 - r32*r42 - r33*r43;
  f4[1] := r51^2 + r52^2 + r53^2 - r51;
  f4[2] := r51^2 + r52^2 + r53^2 - r21*r51 - r22*r52 - r23*r53;
  f4[3] := r51^2 + r52^2 + r53^2 - r31*r51 - r32*r52 - r33*r53;
  f4[4] := r51^2 + r52^2 + r53^2 - r41*r51 - r42*r52 - r43*r53;
  f5[1] := r61^2 + r62^2 + r63^2 - r61;
  f5[2] := r61^2 + r62^2 + r63^2 - r21*r61 - r22*r62 - r23*r63;
  f5[3] := r61^2 + r62^2 + r63^2 - r31*r61 - r32*r62 - r33*r63;
  f5[4] := r61^2 + r62^2 + r63^2 - r41*r61 - r42*r62 - r43*r63;
  f5[5] := r61^2 + r62^2 + r63^2 - r51*r61 - r52*r62 - r53*r63;
  f6[1] := r71^2 + r72^2 + r73^2 - r71;
  f6[2] := r71^2 + r72^2 + r73^2 - r21*r71 - r22*r72 - r23*r73;
```

```
f6[3] := r71^2 + r72^2 + r73^2 - r31 * r71 - r32 * r72 - r33 * r73;
f6[4] := r71^2 + r72^2 + r73^2 - r41 * r71 - r42 * r72 - r43 * r73;
f6[5] := r71^2 + r72^2 + r73^2 - r51 * r71 - r52 * r72 - r53 * r73;
f6[6] := r71^2 + r72^2 + r73^2 - r61 * r71 - r62 * r72 - r63 * r73;
f7[1] := r81^2 + r82^2 + r83^2 - r81;
f7[2] := r81^2 + r82^2 + r83^2 - r21 * r81 - r22 * r82 - r23 * r83;
f7[3] := r81^2 + r82^2 + r83^2 - r31 * r81 - r32 * r82 - r33 * r83;
f7[4] := r81^2 + r82^2 + r83^2 - r41 * r81 - r42 * r82 - r43 * r83;
f7[5] := r81^2 + r82^2 + r83^2 - r51 * r81 - r52 * r82 - r53 * r83;
f7[6] := r81^2 + r82^2 + r83^2 - r61 * r81 - r62 * r82 - r63 * r83;
f7[7] := r81^2 + r82^2 + r83^2 - r71 * r81 - r72 * r82 - r73 * r83;
f8[1] := r91^2 + r92^2 + r93^2 - r91;
f8[2] := r91^2 + r92^2 + r93^2 - r21 * r91 - r22 * r92 - r23 * r93;
f8[3] := r91^2 + r92^2 + r93^2 - r31 * r91 - r32 * r92 - r33 * r93;
f8[4] := r91^2 + r92^2 + r93^2 - r41 * r91 - r42 * r92 - r43 * r93;
f8[5] := r91^2 + r92^2 + r93^2 - r51 * r91 - r52 * r92 - r53 * r93;
f8[6] := r91^2 + r92^2 + r93^2 - r61 * r91 - r62 * r92 - r63 * r93;
f8[7] := r91^2 + r92^2 + r93^2 - r71 * r91 - r72 * r92 - r73 * r93;
f8[8] := r91^2 + r92^2 + r93^2 - r81 * r91 - r82 * r92 - r83 * r93;
f9[1] := r101^2 + r102^2 + r103^2 - r101;
f9[2] := r101^2 + r102^2 + r103^2 - r21 * r101 - r22 * r102 - r23 * r103;
f9[3] := r101^2 + r102^2 + r103^2 - r31 * r101 - r32 * r102 - r33 * r103;
f9[4] := r101^2 + r102^2 + r103^2 - r41 * r101 - r42 * r102 - r43 * r103;
f9[5] := r101^2 + r102^2 + r103^2 - r51 * r101 - r52 * r102 - r53 * r103;
f9[6] := r101^2 + r102^2 + r103^2 - r61 * r101 - r62 * r102 - r63 * r103;
f9[7] := r101^2 + r102^2 + r103^2 - r71 * r101 - r72 * r102 - r73 * r103;
f9[8] := r101^2 + r102^2 + r103^2 - r81 * r101 - r82 * r102 - r83 * r103;
f9[9] := r101^2 + r102^2 + r103^2 - r91 * r101 - r92 * r102 - r93 * r103;

for f2index to 2 do
for f3index to 3 do
for f4index to 4 do
for f5index to 5 do
for f6index to 6 do
for f7index to 7 do
for f8index to 8 do
for f9index to 9 do
row := row + 1;
f := expand(coefficients[row, 11] * f1 + coefficients[row, 12] * f2[f2index] + coefficients[row, 13] * f3[f3index] + coefficients[row, 14] * f4[f4index]
    + coefficients[row, 15] * f5[f5index] + coefficients[row, 16] * f6[f6index] + coefficients[row, 17] * f7[f7index] + coefficients[row, 18] * f8[f8index]
```

```
    + coefficients[row, 19] * f9[f9index] + mu1 * h1 + mu2 * h2 + mu3 * h3);
H := Hessian(f, [r21, r31, r41, r51, r61, r71, r81, r91, r101, r22, r32, r42, r52, r62, r72, r82, r92, r102, r33, r43, r53, r63, r73, r83, r93, r103]);
 HPositiveDefinite := false;
if min(Vector([Determinant(H), Determinant(SubMatrix(H, 1..25, 1..25)), Determinant(SubMatrix(H, 1..24, 1..24)), Determinant(SubMatrix(H, 1..23, 1
    ..23)), Determinant(SubMatrix(H, 1..22, 1..22)), Determinant(SubMatrix(H, 1..21, 1..21)), Determinant(SubMatrix(H, 1..20, 1..20)),
    Determinant(SubMatrix(H, 1..19, 1..19)), Determinant(SubMatrix(H, 1..18, 1..18)), Determinant(SubMatrix(H, 1..17, 1..17)),
    Determinant(SubMatrix(H, 1..16, 1..16)), Determinant(SubMatrix(H, 1..15, 1..15)), Determinant(SubMatrix(H, 1..14, 1..14)),
    Determinant(SubMatrix(H, 1..13, 1..13)), Determinant(SubMatrix(H, 1..12, 1..12)), Determinant(SubMatrix(H, 1..11, 1..11)),
    Determinant(SubMatrix(H, 1..10, 1..10)), Determinant(SubMatrix(H, 1..9, 1..9)), Determinant(SubMatrix(H, 1..8, 1..8)),
    Determinant(SubMatrix(H, 1..7, 1..7)), Determinant(SubMatrix(H, 1..6, 1..6)), Determinant(SubMatrix(H, 1..5, 1..5)), Determinant(SubMatrix(H, 1
    ..4, 1..4)), Determinant(SubMatrix(H, 1..3, 1..3)), Determinant(SubMatrix(H, 1..2, 1..2)), Determinant(SubMatrix(H, 1..1, 1..1)) ])) > 0
   then HPozitivDefinit := true;
end if;
 if HPositiveDefinite = true then
dfdr21 := diff(f, r21);
dfdr31 := diff(f, r31);
dfdr41 := diff(f, r41);
dfdr51 := diff(f, r51);
dfdr61 := diff(f, r61);
dfdr71 := diff(f, r71);
dfdr81 := diff(f, r81);
dfdr91 := diff(f, r91);
dfdr101 := diff(f, r101);
dfdr22 := diff(f, r22);
dfdr32 := diff(f, r32);
dfdr42 := diff(f, r42);
dfdr52 := diff(f, r52);
dfdr62 := diff(f, r62);
dfdr72 := diff(f, r72);
dfdr82 := diff(f, r82);
dfdr92 := diff(f, r92);
dfdr102 := diff(f, r102);
dfdr33 := diff(f, r33);
dfdr43 := diff(f, r43);
dfdr53 := diff(f, r53);
dfdr63 := diff(f, r63);
dfdr73 := diff(f, r73);
dfdr83 := diff(f, r83);
dfdr93 := diff(f, r93);
dfdr103 := diff(f, r103);
S := solve({dfdr101 = 0, dfdr21 = 0, dfdr31 = 0, dfdr41 = 0, dfdr51 = 0, dfdr61 = 0, dfdr71 = 0, dfdr81 = 0, dfdr91 = 0, dfdr102 = 0, dfdr22 = 0, dfdr32 = 0,
```

```
    dfdr42 = 0, dfdr52 = 0, dfdr62 = 0, dfdr72 = 0, dfdr82 = 0, dfdr92 = 0, dfdr103 = 0, dfdr33 = 0, dfdr43 = 0, dfdr53 = 0, dfdr63 = 0, dfdr73 = 0, dfdr83 = 0,
    dfdr93 = 0, h1 = 0, h2 = 0, h3 = 0}, {mu1, mu2, mu3, r101, r21, r31, r41, r51, r61, r71, r81, r91, r102, r22, r32, r42, r52, r62, r72, r82, r92, r103, r33, r43,
    r53, r63, r73, r83, r93});
r21star := eval(r21, S);
r31star := eval(r31, S);
r41star := eval(r41, S);
r51star := eval(r51, S);
r61star := eval(r61, S);
r71star := eval(r71, S);
r81star := eval(r81, S);
r91star := eval(r91, S);
r101star := eval(r101, S);
r22star := eval(r22, S);
r32star := eval(r32, S);
r42star := eval(r42, S);
r52star := eval(r52, S);
r62star := eval(r62, S);
r72star := eval(r72, S);
r82star := eval(r82, S);
r92star := eval(r92, S);
r102star := eval(r102, S);
r33star := eval(r33, S);
r43star := eval(r43, S);
r53star := eval(r53, S);
r63star := eval(r63, S);
r73star := eval(r73, S);
r83star := eval(r83, S);
r93star := eval(r93, S);
r103star := eval(r103, S);
fmin := eval(subs({r101 = r101star, r21 = r21star, r31 = r31star, r41 = r41star, r51 = r51star, r61 = r61star, r71 = r71star, r81 = r81star, r91 = r91star, r102
    = r102star, r22 = r22star, r32 = r32star, r42 = r42star, r52 = r52star, r62 = r62star, r72 = r72star, r82 = r82star, r92 = r92star, r103 = r103star, r33
    = r33star, r43 = r43star, r53 = r53star, r63 = r63star, r73 = r73star, r83 = r83star, r93 = r93star}, expand(coefficients[row, 11] * f1 + coefficients[row, 12]
    * f2[f2index] + coefficients[row, 13] * f3[f3index] + coefficients[row, 14] * f4[f4index] + coefficients[row, 15] * f5[f5index] + coefficients[row, 16]
    * f6[f6index] + coefficients[row, 17] * f7[f7index] + coefficients[row, 18] * f8[f8index] + coefficients[row, 19] * f9[f9index])));
 if fmin > 0 then
   else print([f2index, f3index, f4index, f5index, f6index, f7index, f8index, f9index, "non-positive lower bound!"]);
      infeasibilityNOTproved := true;
 end if;
 end if;
 end do;
 end do;
```

```
 end do;
 end do;
 end do;
 end do;
 end do;
end do;
print("Is there a system with nonpositive lower bound?");
print(infeasibilityNOTproved);
```

$$infeasibilityNOTproved := false$$

$$row := 0$$

$$r23 := 0$$

$$h1 := 1 + r21 + r31 + r41 + r51 + r61 + r71 + r81 + r91 + r101$$

$$h2 := r22 + r32 + r42 + r52 + r62 + r72 + r82 + r92 + r102$$

$$h3 := r33 + r43 + r53 + r63 + r73 + r83 + r93 + r103$$

$$f1 := r21^2 + r22^2 - r21$$

$$f2_1 := r31^2 + r32^2 + r33^2 - r31$$

$$f2_2 := r31^2 + r32^2 + r33^2 - r21\,r31 - r22\,r32$$

$$f3_1 := r41^2 + r42^2 + r43^2 - r41$$

$$f3_2 := r41^2 + r42^2 + r43^2 - r21\,r41 - r22\,r42$$

$$f3_3 := r41^2 + r42^2 + r43^2 - r31\,r41 - r32\,r42 - r33\,r43$$

$$f4_1 := r51^2 + r52^2 + r53^2 - r51$$

$$f4_2 := r51^2 + r52^2 + r53^2 - r21\,r51 - r22\,r52$$

$$f4_3 := r51^2 + r52^2 + r53^2 - r31\,r51 - r32\,r52 - r33\,r53$$

$$f4_4 := r51^2 + r52^2 + r53^2 - r41\,r51 - r42\,r52 - r43\,r53$$

$$f5_1 := r61^2 + r62^2 + r63^2 - r61$$

$$f5_2 := r61^2 + r62^2 + r63^2 - r21\,r61 - r22\,r62$$

$$f5_3 := r61^2 + r62^2 + r63^2 - r31\,r61 - r32\,r62 - r33\,r63$$

$$f5_4 := r61^2 + r62^2 + r63^2 - r41\,r61 - r42\,r62 - r43\,r63$$

$$f5_5 := r61^2 + r62^2 + r63^2 - r51\,r61 - r52\,r62 - r53\,r63$$

$$f6_1 := r71^2 + r72^2 + r73^2 - r71$$

$$f6_2 := r71^2 + r72^2 + r73^2 - r21\,r71 - r22\,r72$$

$$f6_3 := r71^2 + r72^2 + r73^2 - r31\,r71 - r32\,r72 - r33\,r73$$

$$f6_4 := r71^2 + r72^2 + r73^2 - r41\,r71 - r42\,r72 - r43\,r73$$

$$f6_5 := r71^2 + r72^2 + r73^2 - r51\,r71 - r52\,r72 - r53\,r73$$

$$f6_6 := r71^2 + r72^2 + r73^2 - r61\,r71 - r62\,r72 - r63\,r73$$

$$f7_1 := r81^2 + r82^2 + r83^2 - r81$$

$$f7_2 := r81^2 + r82^2 + r83^2 - r21\,r81 - r22\,r82$$

$$f7_3 := r81^2 + r82^2 + r83^2 - r31\,r81 - r32\,r82 - r33\,r83$$

$$f7_4 := r81^2 + r82^2 + r83^2 - r41\,r81 - r42\,r82 - r43\,r83$$

$$f7_5 := r81^2 + r82^2 + r83^2 - r51\,r81 - r52\,r82 - r53\,r83$$

$$f7_6 := r81^2 + r82^2 + r83^2 - r61\,r81 - r62\,r82 - r63\,r83$$

$$f7_7 := r81^2 + r82^2 + r83^2 - r71\,r81 - r72\,r82 - r73\,r83$$

$$f8_1 := r91^2 + r92^2 + r93^2 - r91$$

$$f8_2 := r91^2 + r92^2 + r93^2 - r21\,r91 - r22\,r92$$

$$f8_3 := r91^2 + r92^2 + r93^2 - r31\,r91 - r32\,r92 - r33\,r93$$

$$f8_4 := r91^2 + r92^2 + r93^2 - r41\,r91 - r42\,r92 - r43\,r93$$

$$f8_5 := r91^2 + r92^2 + r93^2 - r51\,r91 - r52\,r92 - r53\,r93$$

$$f8_6 := r91^2 + r92^2 + r93^2 - r61\,r91 - r62\,r92 - r63\,r93$$

$$f8_7 := r91^2 + r92^2 + r93^2 - r71\,r91 - r72\,r92 - r73\,r93$$

$$f8_8 := r91^2 + r92^2 + r93^2 - r81\,r91 - r82\,r92 - r83\,r93$$

$$f9_1 := r101^2 + r102^2 + r103^2 - r101$$

$$f9_2 := r101^2 + r102^2 + r103^2 - r21\,r101 - r22\,r102$$

$$f9_3 := r101^2 + r102^2 + r103^2 - r31\,r101 - r32\,r102 - r33\,r103$$

$$f9_4 := r101^2 + r102^2 + r103^2 - r41\,r101 - r42\,r102 - r43\,r103$$

$$f9_5 := r101^2 + r102^2 + r103^2 - r51\,r101 - r52\,r102 - r53\,r103$$

$$f9_6 := r101^2 + r102^2 + r103^2 - r61\,r101 - r62\,r102 - r63\,r103$$

$$f9_7 := r101^2 + r102^2 + r103^2 - r71\,r101 - r72\,r102 - r73\,r103$$

$$f9_8 := r101^2 + r102^2 + r103^2 - r81\,r101 - r82\,r102 - r83\,r103$$

$$f9_9 := r101^2 + r102^2 + r103^2 - r91\,r101 - r92\,r102 - r93\,r103$$

"Is there a system with nonpositive lower bound?"

false

(3

>