Week 2 Homework - Summer 2020

This is a preview of the published version of the quiz

Started: May 9 at 12:02pm

Quiz Instructions

Please answer all the questions below.

Question 1
1 pts

(Lesson 2.1: Derivatives.) BONUS: If $f(x) = \ell n(2x-3)$, find the derivative $f'(x)$.

- a. $2x$
- b. $\frac{1}{2}\ell n(2x-3)$
- c. $2/(2x-3)$
- d. $x/2$

Question 2
1 pts

(Lesson 2.1: Derivatives.) BONUS: If $f(x) = \cos(1/x)$, find the derivative $f'(x)$.

- a. $\cos(1/x^2)$
- b. $\sin(1/x^2)$
- c. $-\frac{1}{x^2}\sin(1/x)$
- d. $\frac{1}{x^2}\sin(1/x)$

Question 3

1 pts

(Lesson 2.2: Finding Zeroes.) BONUS: Suppose that $f(x) = e^{4x} - 4e^{2x} + 4$. Use any method you want to find a zero of $f(x)$, i.e., x such that $f(x) = 0$.

- a. $x = 0$
- b. $x = 1$
- c. $x = \ell n(2) = 0.693$
- d. $x = \frac{1}{2}\ell n(2) = 0.347$

Question 4

1 pts

(Lesson 2.3: Integration.) BONUS: Find $\int_0^1 (2x+1)^2 dx$.

- a. 1/2
- b. 7/2
- c. 7/3
- d. 13/3

Question 5

1 pts

(Lesson 2.3: Integration.) BONUS: Find $\int_1^2 e^{2x} dx$.

- a. 1
- b. $e^2 - e$
- c. 23.6

d. 46.2

Question 6

1 pts

(Lesson 2.3: Integration.) BONUS: Find
$\lim_{x \to 0} \frac{\sin(x) - x}{x}$.

- a. 1
- b. 0
- c. ∞
- d. undetermined

Question 7

1 pts

(Lesson 2.4: Numerical Integration.) BONUS: Find the approximate value of the integral $\int_0^2 (x-1)^2 dx$ using the lesson's form of the Riemann sum with $f(x) = (x-1)^2, a = 0, b = 2$, and $n = 4$.

- a. -2
- b. 1/3
- c. 3/4
- d. 3

Question 8

1 pts

(Lesson 2.5: Probability Basics.) If $P(A) = P(B) = P(C) = 0.6$ and $A, B,$ and C are independent, find the probability that exactly one of $A, B,$ and C occurs.

- a. 0.144
- b. 0.288
- c. 0.576
- d. 0.6
- e. I'm from The University Of Georgia. Is the answer -3?

Question 9

1 pts

(Lesson 2.5: Probability Basics.) Toss 3 dice. What's the probability that a "4" will come up exactly twice?

- a. 5/72
- b. 1/2
- c. 13/16
- d. 1/8

Question 10

1 pts

(Lesson 2.6: Simulating Random Variables.) BONUS: Suppose U and V are independent Uniform(0,1) random variables. (You can simulate these using the RAND() function in Excel, for instance.) Consider the nasty-looking random variable

$$Z = \sqrt{-2\ell n(U)}\cos(2\pi V),$$

where the cosine calculation is carried out in radians (not degrees). Go ahead and calculate Z. . . don't be afraid. Now, repeat this task 1000 times (easy to do in Excel) and make a histogram of the 1000 Z's. What distribution does this look like?

- a. Normal
- b. Unif(0,1)
- c. Exponential
- d. Weibull

Question 11

1 pts

(Lesson 2.7: Great Expectations.) Suppose that X is a discrete random variable having $X = -1$ with probability 0.2, and $X = 3$ with probability 0.8. Find $\mathrm{E}[X]$.

- a. -1
- b. 3
- c. 1
- d. 2.2

Question 12

1 pts

(Lesson 2.7: Great Expectations.) Suppose that X is a discrete random variable having $X = -1$ with probability 0.2, and $X = 3$ with probability 0.8. Find $\mathrm{Var}[X]$.

- a. -1
- b. 1
- c. 2.56
- d. 5.12

Question 13

1 pts

(Lesson 2.7: Great Expectations.) Suppose that X is a discrete random variable having $X = -1$ with probability 0.2, and $X = 3$ with probability 0.8. Find $\mathrm{E}[3 - \frac{1}{X}]$.

- a. 3
- b. ∞
- c. -2
- d. 44/15

Question 14

1 pts

(Lesson 2.7: Great Expectations.) Suppose X is a continuous random variable with p.d.f. $f(x) = 4x^3$ for $0 \le x \le 1$. Find $\mathrm{E}[1/X^2]$.

- a. 2/3
- b. 1
- c. 3/2
- d. 2

Question 15

1 pts

(Lesson 2.8: Functions of a Random Variable.) Suppose X is the result of a 5-sided die toss having sides numbered $-2, -1, 0, 1, 2$. Find the probability mass function of $Y = X^2$.

- a. $\mathrm{P}(Y = 1) = \mathrm{P}(Y = 4) = 1/2$

b. $\mathrm{P}(Y=1)=\mathrm{P}(Y=2)=1/2$

c. $P\,(Y=0)=\frac{1}{5},\ and\ P\,(Y=1)=P\,(Y=4)\ =\frac{2}{5}$

d. $\mathrm{P}(Y=-2)=\mathrm{P}(Y=-1)=\mathrm{P}(Y=0)=\mathrm{P}(Y=1)=\mathrm{P}(Y=2)=1/5$

Question 16

1 pts

(Lesson 2.8: Functions of a Random Variable.) Suppose X is a continuous random variable with p.d.f. $f(x)=2x$ for $0<x<1$. Find the p.d.f. $g(y)$ of $Y=X^2$. (This may be easier than you think.)

a. $g(y)=1$, for $0<y<1$

b. $g(y)=y$, for $0<x<1$

c. $g(y)=y^2$, for $-1<y<1$

d. $g(y)=x^2$, for $0<y<1$

Question 17

1 pts

(Lesson 2.9: Jointly Distributed RVs.) Suppose that $f(x,y)=6x$ for $0\leq x\leq y\leq 1$. Find $\mathrm{P}(X<1/2 \text{ and } Y<1/2)$.

a. 1

b. 1/2

c. 1/4

d. 1/8

Question 18

1 pts

(Lesson 2.9: Jointly Distributed RVs.) Suppose that $f(x, y) = 6x$ for $0 \le x \le y \le 1$. Find the marginal p.d.f. $f_X(x)$ of X.

- a. $6x(1-x)$, for $0 \le x \le 1$
- b. $6x$, for $0 \le x \le 1$
- c. $6y$, for $0 \le x \le 1$
- d. $6x(1-y)$, for $0 \le x \le 1$

Question 19

1 pts

(Lesson 2.9: Jointly Distributed RVs.) YES or NO? Suppose X and Y have joint p.d.f. $f(x, y) = cxy/(1 + x^2 + y^2)$ for $0 < x < 1$, $0 < y < 1$, and whatever constant c makes the nasty mess integrate to 1. Are X and Y independent?

- a. Yes
- b. No

Question 20

1 pts

(Lesson 2.10: Conditional Expectation.) BONUS: Suppose that

$f(x, y) = 6x$ for $0 \le x \le y \le 1$. Hint (you may have seen this someplace): the marginal p.d.f. of X turns out to be

$f_X(x) = 6x(1-x)$ for $0 \le x \le 1$. Find the conditional p.d.f. of Y given that $X = x$.

- a. $f(y|x) = \frac{1}{1-x}, \quad 0 \le x \le y \le 1$

- b. $f(y|x) = \frac{1}{1-x}, \quad 0 \le x \le 1$
- c. $f(y|x) = \frac{1}{1-y}, \quad 0 \le y \le 1$
- d. $f(x|y) = \frac{1}{1-x}, \quad 0 \le x \le y \le 1$

Question 21

1 pts

(Lesson 2.10: Conditional Expectation.) BONUS: Again suppose that

$f(x, y) = 6x$ for $0 \le x \le y \le 1$. Hint (you may already have seen this someplace): the the marginal p.d.f. of X turns out to be $f_X(x) = 6x(1-x)$ for $0 \le x \le 1$. Find $\mathsf{E}[Y|X = x]$.

- a. $\mathsf{E}[Y|X = x] = 1/2, \quad 0 \le x \le 1$
- b. $\mathsf{E}[Y|X = x] = \frac{1+x}{2}, \quad 0 \le x \le 1$
- c. $\mathsf{E}[Y|X = x] = \frac{1+y}{2}, \quad 0 \le y \le 1$
- d. $\mathsf{E}[X|Y = y] = \frac{1+y}{2}, \quad 0 \le y \le 1$

Question 22

1 pts

(Lesson 2.10: Conditional Expectation.) BONUS: Yet again suppose that $f(x, y) = 6x$ for $0 \le x \le y \le 1$. Hint (you may already have seen this someplace): the the marginal p.d.f. of X turns out to be $f_X(x) = 6x(1-x)$ for $0 \le x \le 1$. Find $\mathsf{E}[\mathsf{E}[Y|X]]$.

- a. 1/2
- b. 2/3
- c. 3/4

d. 1

Question 23

1 pts

(Lesson 2.11: Covariance and Correlation.)
Suppose that the correlation between December snowfall and temperature in Siberacuse, NY is -0.5. Further suppose that $\text{Var}(S) =$ 100 in^2 and $\text{Var}(T) = 25$ (degrees F)^2. Find $\text{Cov}(S, T)$ (in units of degree inches, whatever those are).

a. -25

b. -5

c. 5

d. 25

Question 24

1 pts

(Lesson 2.11: Covariance and Correlation.) If X and Y both have mean -7 and variance 4, and $\text{Cov}(X, Y) = 1$, find $\text{Var}(3X - Y)$.

a. 34

b. 36

c. 40

d. 41

Question 25

1 pts

(Lesson 2.12: Probability Distributions.)

You may recall that the p.m.f. of the Geometric (p) distribution is
$f(x) = (1-p)^{x-1}p, x = 1, 2, \ldots$

If the number of orders at a production center this month is a Geom(0.7) random variable, find the probability that we'll have at most 3 orders.

- a. 0.027
- b. 0.14
- c. 0.86
- d. 0.973

Question 26

1 pts

(Lesson 2.12: Probability Distributions.) Suppose the SAT math score of a University of Georgia student can be approximated by a normal distribution with mean 400 and variance 225. Find the probability that the UGA Einstein will score at least a 415.

- a. 0.5
- b. 0.1587
- c. 0.975
- d. 0.8413

Question 27

1 pts

(Lesson 2.13: Limit Theorems.)
What is the most-important theorem in the universe?

- a. Eastern Limit Theorem
- b. Central Limit Theorem

c. Central Limit Serum

d. Central Simit Theorem (simit is a tasty Turkish bagel)

Question 28

1 pts

(Lesson 2.13: Limit Theorems.) If $X_1, \ldots, X_{400}$ are i.i.d. from some distribution with mean 1 and variance 400, find the approximate probability that the sample mean $\bar{X}$ is between 0 and 2.

a. 0.1587

b. 0.3174

c. 0.6826

d. 0.8413

Question 29

1 pts

(Lesson 2.14: Estimation.)
Suppose we collect the following observations: 7, -2, 1, 6. What is the sample variance?

a. 13

b. $\sqrt{13}$

c. 18

d. 28

Question 30

1 pts

(Lesson 2.14: Estimation.) BONUS: Consider two estimators, T_1 and T_2, for an unknown parameter θ. Suppose that the $\mathsf{Bias}(T_1) = 0$, $Bias\,(T_2) = \theta$, $\mathsf{Var}(T_1) = 4\theta^2$, and $\mathsf{Var}(T_2) = \theta^2$. Which estimator might you decide to use and why?

- a. T_1 - it has lower expected value
- b. T_1 - it has lower MSE
- c. T_2 - it has lower variance
- d. T_2 - it has lower MSE

Question 31

1 pts

(Lesson 2.15: Maximum Likelihood Estimation.) BONUS: Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. $\mathrm{Pois}(\lambda)$. Find $\hat{\lambda}$, the MLE of λ. (Don't panic --- it's not that difficult.)

- a. $\overline{X}$
- b. $1/\bar{X}$
- c. $n/\sum_{i=1}^{x} X_i$
- d. S^2

Question 32

1 pts

(Lesson 2.15: Maximum Likelihood Estimation.) BONUS: Suppose that we are looking at i.i.d. Exp(λ) customer service times. We observe times of 2, 4, and 9 minutes. What's the maximum likelihood estimator of λ^2?

- a. 5
- b. 1/5

c. 25

d. 1/25

Question 33

1 pts

(Lesson 2.16: Confidence Intervals.) BONUS: Suppose we collect the following observations: $7, -2, 1, 6$ (as in a previous question in this homework). Let's assume that these guys are i.i.d. from a normal distribution with $unknown$ variance σ^2. Give me a two-sided 95% confidence interval for the mean μ.

a. $[-2, 7]$

b. $[-3.75,\ 9.75]$

c. $[-6.75, 6.75]$

d. $[3.75, 9.75]$

Not saved Submit Quiz